OUTDEGREE PRIME PAIR LABELING OF DIRECTED GRAPHS

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ABSTRACT:

Let D(p,q) be a digraph. A function f : V→{1,2,…,p+q} is said to be an outdegree prime pair labeling of D, if at each $u \in V(D)$, $\gcd[\,f(v), f(w)] = 1$, $\forall\, v, w \in N^{+}(u)$, where $N^{+}(u) = \{w \in V(D) | uw \in A(D)\}$. In this paper, we tried for outdegree prime pair labeling of some digraphs obtained by orienting the graphs such as Wheel W_n, Book With Triangular Page B_3^n, Double Triangular Snake D(T_n) and Directed Triangular snake TS_n.

Keywords: Outdegree prime pair labeling

AMS Subject Classification: 05C78

1. INTRODUCTION

A directed graph or digraph D consists of a finite set V of vertices (points) and a collection of ordered pairs of distinct vertices. Any such pair (u, v) is called an arc or directed line and will usually be denoted by $\overrightarrow{uv}$. The arc $\overrightarrow{uv}$ goes from u to v and incident with u and v, we also say u is adjacent to v and v is adjacent from u. A digraph D with p vertices and q arcs is denoted by D (p, q).A wheel graph $W_n$is a graph formed by connecting a single vertex to all vertices of a cycle. A directed triangular snake TS_n is a connected digraph, all of whose blocks are directed triangles and whose block-cut points form a directed path.One edge union of cycles of same length is called a Book. The common edge is called as the Base of the Book. If we consider n copies of cycles of length 3, then the book is called Book with Triangular page and is denoted by B_3^n [4]. A Double Triangular Snake D(Tn) consists of two triangular snakes that have a common path [6]. The indegree $d^-(v)$ of a vertex v in a digraph D is the number of arcs having v as its terminal vertex. The outdegree $d^+(v)$ of v is the number of arcs having v as its initial vertex [1].A labeling of a graph G is an assignment of integers to either the vertices or the edges or both subject to certain conditions. The notion of prime labeling for graphs originated with Roger Entringer and was introduced in the paper by Tout et al [7] in the early 1980's and since then it is an active field of research for many scholars. Motivated by the definition of prime labeling, indegree prime labeling of digraphs [2] and the indegree prime pair labeling [5] we define a new concept called an outdegree prime pair labeling and we tried for some digraphs obtained by orienting the graphs such as Wheel W_n, Book with triangular page B_3^n,Double Triangular Snake D(T_n) and Directed triangular snake TS_n. For more reference on labeling, we refer [3].

2. MAIN RESULTS

Definition 2.1: Let D(p,q) be a digraph. A function f : V→{1,2,…,p+q} is said to be an outdegree prime pair labeling of D, if at each $u \in V(D)$, $\gcd[f(v), f(w)] = 1, \forall\, v, w \in N^+(u)$, where $N^+(u) = \{w \in V(D) | uw \in A(D)\}$.

THEOREM 2.2:

Let D_1 be the digraph obtained from the wheel graph by orienting the arcs of C_n in the clockwise direction and the arcs from the central vertex in the inward direction as shown in the figure 2.1. Then, D_1 admits outdegree prime pair labeling.

PROOF:

Let $V(D_1) = \{u, u_i \,|1 \le i \le n\}$ be the vertex set and

$A(D_1) = \{\overrightarrow{u_i u} \,|1 \le i \le n\} \cup \{\overrightarrow{u_i u_{i+1}} \,|1 \le i \le n-1\} \cup \{\overrightarrow{u_n u_1}\}$ be the arc set of D_1 as in figure 2.1.

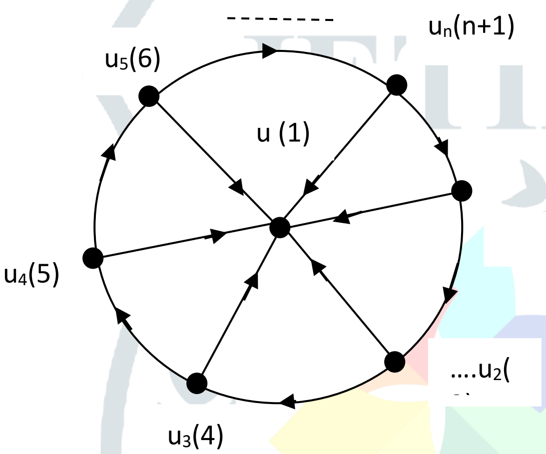

Figure 2.1, Wheel W_n.

The digraph D_1 has n+1 vertices and 2n arcs.

Therefore, p+q =3n+1

Define a function $f: V \to \{1,2, \dots, 3n+1\}$ by $f(u) = 1, f(u_i) = i+1, 1 \le i \le n$

Now, $N^+(u) = \emptyset$ ⟶ (1)

$N^+(u_i) = \{u, u_{i+1}\}, \forall\, 1 \le i \le n-1$

$N^+(u_n) = \{u, u_1\}$

Also, $\gcd[f(v), f(w)] = \gcd[1, i+1] = 1, \forall\, v, w \in N^+(u_i),\ i = 1 \; to \; n-1$ ⟶ (2)

and $\gcd[f(v), f(w)] = \gcd[1,2] = 1, \forall\, v, w \in N^+(u_n)$ ⟶ (3)

From (1), (2) & (3), f admits an outdegree prime pair labeling of D_1.

REMARK 2.3:

(1) If in D_1 the direction of arcs of C_n are reversed, then also D_1 admits outdegree prime pair labeling.

(2) Let D_2 be the digraph obtained from D_1 by reversing the direction of the arcs at the central vertex as in figure 2.2.

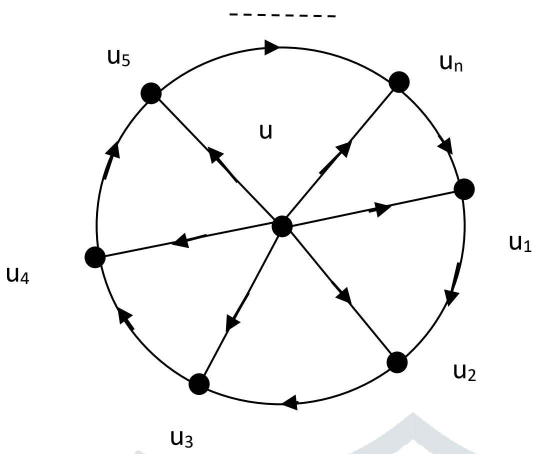

Figure 2.2, Wheel W_n

Then the above labeling of D_1 is not an outdegree prime pair labeling of D_2. The following theorem states a necessary and sufficient for the existence of an outdegree prime pair labeling of D_2.

THEOREM 2.4:

Let D_2 be the digraph obtained from D_1 by reversing the direction of the arcs at the central vertex as in figure 2.2. Then, D_2 admits outdegree prime pair labeling iff n ≤ 12.

PROOF:

Let n ≤ 12. Then there are atleast n prime numbers ≤ 3n+1.

Therefore, assigning $f(u) = 1$ & $f(u_i)$ = a prime number ≤ 3n+1, we get an outdegree prime pair labeling of D_2.

Now to prove the converse,

Clearly in D_2, every vertex in the cycle gives outdegree to the central vertex u. Therefore, if D_2 admits an outdegree prime labeling, then gcd$[f(v_i), f(v_j)] = 1, \forall\, v_i, v_j \in C$, where C is the outer cycle of the wheel.

This is possible only when $f(v_i)$ is prime $\forall\, v_i \in C$

Also p+q = 3n+1.

Therefore, D_2 admits an outdegree prime pair labeling only when there are atleast n primes ≤ 3n+1

Again, this is possible only when n ≤ 12 (manually checked).

Therefore, D_2 admits an outdegree prime pair labeling only if n ≤ 12.

REMARK 2.5:

A graph may exhibit both indegree and outdegree prime pair labeling. But they need not be the same. For example, the outdegree and indegree prime pair labeling of D_1 and D_2 $(n \leq 12)$ are completely different. Further, D_1 exhibits outdegree prime pair labeling for all n, and indegree prime pair labeling only for $n \leq 12$. Reverse is the case with D_2.

THEOREM 2.6:

The directed triangular snake TS_n admits outdegree prime pair labeling.

PROOF:

Let V(TS$_n$) =$\{v_1,v_2,\ldots,v_{2n-2},v_{2n-1}\}$ be the vertex set

and A(TS$_n$)=$\{\overrightarrow{v_iv_{i+1}}$ | 1 ≤ i ≤ 2n-2} ∪ $\{\overrightarrow{v_iv_{i+2}}$ | i = 1,3,5,…,2n-3} be the arc set, where the vertices with odd suffixes lie along the path and the vertices with even suffixes are on the top of the triangles. Then the directed triangular snake is as in the figure 2.3.

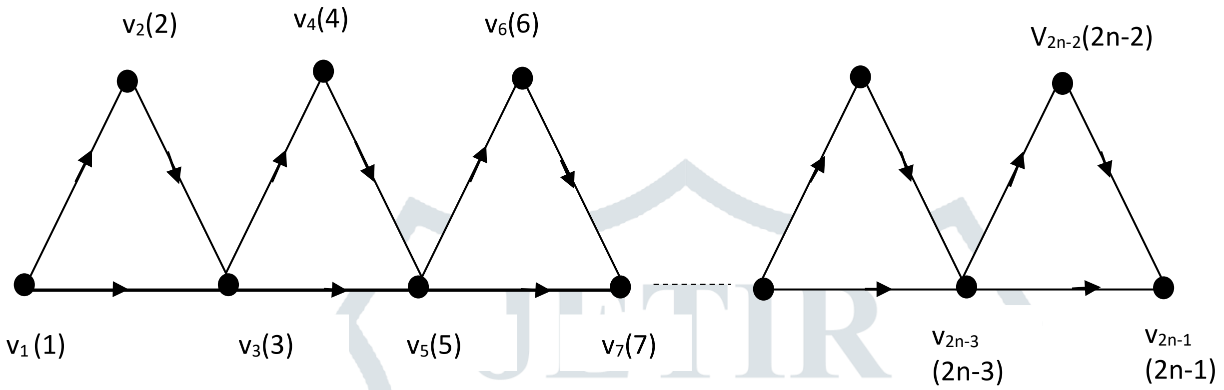

Figure 2.3, Directed Triangular Snake TS_n

Hence, the directed triangular snake has 2n-1 vertices and 3n-3 arcs.

Define a function f : V→{1,2,3,…,5n-4} by $f(v_i) = i$, ∀ $1 \le i \le 2n-1$.

i.e., The vertices on the horizontal path get the consecutive odd numbers and the upper vertices in the triangles get the consecutive even numbers.

Now,$N^+(v_{2i}) = \{v_{2i+1}\}$, 1 ≤ i ≤ n-1 ⟶ (1)

$N^+(v_{2i-1}) = \{v_{2i}, v_{2i-1}\}$, 1 ≤ i ≤ n-1

$N^+(v_{2n-1}) = \emptyset$ ⟶ (2)

Therefore, gcd$[f(u), f(v)]$ = gcd$(i, i+1) = 1$, ∀ $u, v \in N^+(v_{2i-1})$, 1 ≤ i ≤ n-1 ⟶ (3)

From (1),(2) & (3), f admits an outdegree prime pair labeling.

REMARK 2.7:

Suppose D_3 is the digraph obtained by reorienting the arcs of TS_n in theorem 2.6, as in figure 2.4,

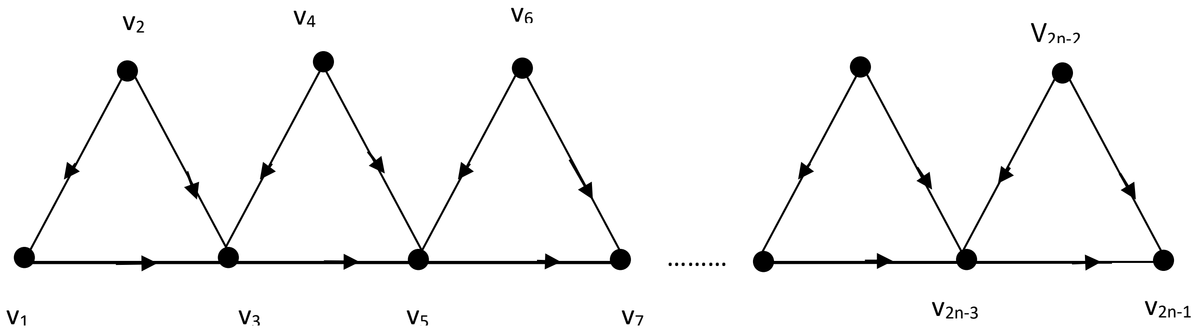

Figure 2.4, Directed Triangular Snake TS_n

then also D_3 admits an outdegree prime pair labeling with labeling as in theorem 2.6. But if the direction of the arcs $v_i v_{i-1}$ & $v_i v_{i+1}$ are reversed the resulting graph is not an outdegree prime pair labeling with labeling as in theorem 2.6.

THEOREM 2.8:

Suppose D_4 is the digraph obtained by orienting the arcs of the book with triangular page B_3^n as in figure 2.5. Then D_4 admits outdegree prime pair labeling when n ≤ 10.

PROOF:

Let $V(D_4) = \{u, v, u_i \mid 1 \le i \le n\}$ be the vertex set and

$A(D_4) = \{\overrightarrow{uu_i}, \overrightarrow{u_i v}, \overrightarrow{uv} \mid 1 \le i \le n\}$ be the arc set of D_4 as in figure 2.5.

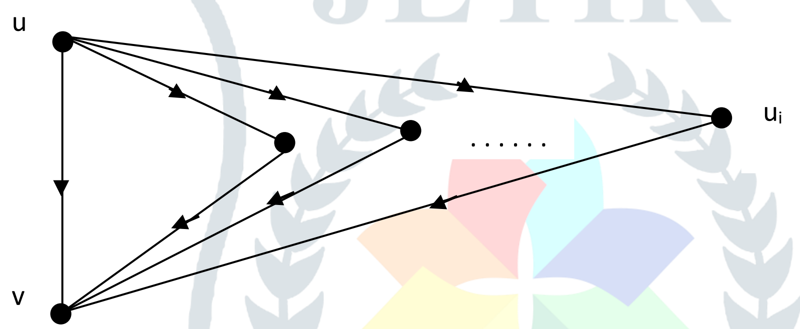

Figure 2.5, book with triangular page B_3^n

The digraph D_4 has n+2 vertices and 2n+1 arcs.

Therefore, p+q =3n+3.

List out the prime numbers ≤ 3n+3.

Define $f(u_i) = i^{th}$ prime number ≤ 3n+3 for all u_i.

$f(u) = i+1^{th}$ prime number

f(v) = 1.

Now, $N^+(v) = \emptyset$ ⟶ (1)

$N^+(u_i) = \{v\}, \forall\ 1 \le i \le n$ ⟶ (2)

$N^+(u) = \{v, u_i\}, \forall\ 1 \le i \le n$

Since $f(v_i)$& $f(w_i)$ are prime $\forall i$,

$\gcd[f(v_i), f(w_i)] = 1, \forall\ v_i, w_i \in N^+(u),\ i = 1\ to\ n$ ⟶ (3)

From (1), (2) & (3), f admits an outdegree prime pair labeling of D_4.

REMARK 2.9:

The digraphs in figures 2.3 & 2.5 exhibit the same in and out degree prime pair labeling.

THEOREM 2.10:

Suppose D_5 be the digraph obtained by orienting the arcs of the Double Triangular Snake $D(T_n)$ as in figure 2.6. Then D_5 admits outdegree prime pair labeling.

PROOF:

Let V(D₅) $=\{u_i, w_i \mid 1 \le i \le n-1\} \cup \{v_i \mid 1 \le i \le n\}$ be the vertex set

and A(D₅)=$\{\overrightarrow{v_i v_{i+1}} \mid 1 \le i \le n-1\} \cup \{\overrightarrow{u_i v_i}, \overrightarrow{w_i v_i} \mid 1 \le i \le n-1\} \cup \{\overrightarrow{u_i v_{i+1}}, \overrightarrow{w_i v_{i+1}} \mid 1 \le i \le n-1\}$ be the arc set of D₅.

Then the double triangular snake is as in the figure 2.6.

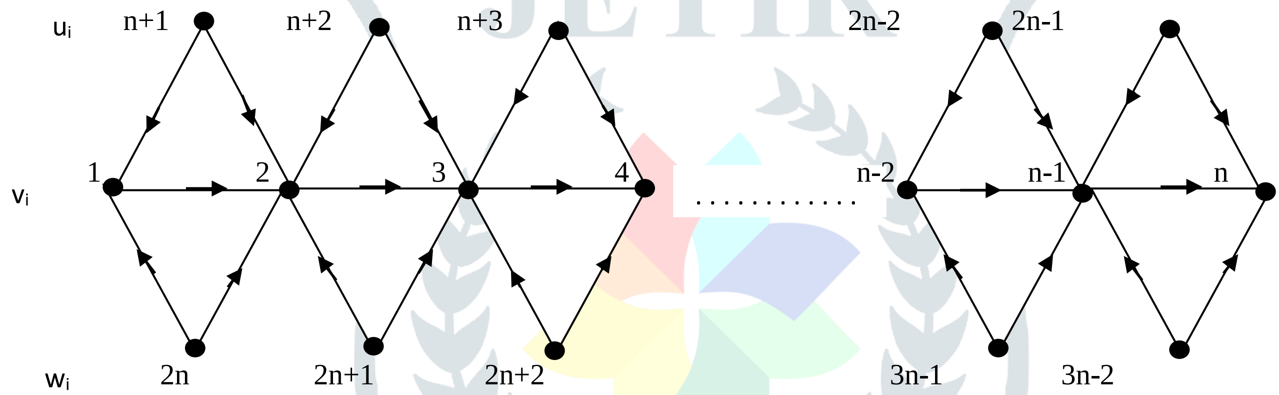

Figure 2.6, Double Triangular Snake D(T_n)

Hence, the double triangular snake has 3n-2 vertices and 5n-5 arcs.

Define a function f : V→{1,2,3,…,8n-2} by

$f(v_i) = i, \forall\, 1 \le i \le n.$

$f(u_i) = i + 1, \forall\, 1 \le i \le n-1.$

$f(w_i) = 2n - 1 + i, \forall\, 1 \le i \le n-1.$

Now, $N^+(u_i) = N^+(w_i) = \{v_i, v_{i+1}\}, 1 \le i \le n-1$ ⟶ (1)

$N^+(v_i) = \{v_{i+1}\}$, $1 \le i \le n-1$.

Therefore, $\gcd[f(u), f(v)] = \gcd(i, i+1) = 1, \forall\, u, v \in N^+(u_i) \& N^+(w_i)$, $1 \le i \le n-1$ ⟶ (2)

From (1) & (2), f admits an outdegree prime pair labeling.

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