BI-IDEALS IN C_1 AND C_2 NEAR SUBTRACTION SEMIGROUPS

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Abstract:

In this paper we introduce the notation of bi-ideals in C_1 and C_2 near-subtraction semigroup and study some of their properties.

Key words:

Near subtraction semigroups, IFP, strong IFP, GNF, bi-ideals in C_1 and C_2 near-subtraction semigroup.

1. Introduction

B.M.Schein [7] considered systems of the form (X; o;/), where X is a set of functions closed under the composition “o” of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction “/” (and hence (X;/) is a subtraction algebra in the sense of [1]). Y.B.Jun et al[4] introduced the notation of ideals in subtraction algebras and discussed the characterization of ideals. For basic definition one may refer to Pilz[6]. Mahalakshmi et al. [5] studied the notation of bi-ideals in near subtraction semigroups. The purpose of this paper is to introduce the notation of bi-ideals in C_1 and C_2 near-subtraction semigroups. We investigate some basic results, examples and properties.

2.Preliminaries

Definition:2.1. A nonempty set X together with binary operations **‘‘−’’** is said to be **subtraction algebra** if it satisfies the following conditions

(i) $x-(y-x) = x$.

(ii) $x-(x-y) = y-(y-x)$.

(iii) $(x-y)-z = (x-z)-y$, for every $x, y, z \in X$.

Definition:2.2. A nonempty set X together with two binary operations **‘‘−’’** and **‘‘•’’** is said to be a **subtraction semigroup** if it satisfies the following conditions

(i) (X,−) is a subtraction algebra.

(ii) (X,•) is a semigroup.

(iii) $x(y-z) = xy-xz$ and $(x-y)z = xz-yz$, for every $x, y, z \in X$.

Definition:2.3. A non empty set X together with two binary operations "−" and "•" is said to be a **right near subtraction semigroup** if it satisfies the following conditions

(i) (X, −) is a subtraction algebra.
(ii) (X, •) is a semigroup.
(iii) (x − y) z = xz – yz, for every x, y, z∈X.

It is clear that 0x = 0, for all x∈X. Similarly we can define a left near- subtraction semigroup. Here after a near–subtraction semigroup means only a right near-subtraction semigroup.

Definition:2.4. A nonempty subset S of a subtraction semigroup X is said to be a s**ubalgebra** of X, if x–y ∈S, for allx, y∈S.

Definition:2.5. Let (X, - , .) be a near – subtraction semigroup. A nonempty subset I of X is called

(i) A **left ideal** if I is a subalgebra of (X , –) and xi – x (y – i) ∈ I for all x , y ∈ X and i ∈ I .

(ii) A **right ideal** I is a subalgebra of (X , –) and IX ⊆ I.

(iii) If I is both a left and right ideal then, it is called a two-sided ideal (simply, ideal) of X.

Definition:2.6. A near subtraction semigroup X is said to be **Zero – symmetric** if x0 = 0 for every x ∈ X.

Definition:2.7. An element e ∈ X is said to be **idempoten**t if for each e ∈ X, e^2 = e.

Definition:2.8. A subalgebra Q of (X , −) is said to be a **quasi-ideal** of zero-symmetric near subtraction semigroup of X if QX∩XQ ⊆ Q.

Definition: 2.9. A subalgebra B of (X , −) is said to be a **bi-ideal** of zero-symmetric near subtraction semigroup of X if BXB ⊆ B.

Definition:2.10. We say that X is an **s (s') near subtraction semigroup** if a∈Xa(aX), for all a∈X.

Definition:2.11. A s-near subtraction semigroup X is said to be a **$\bar{s}$-near subtraction semigroup** if x ∈ xX, for all a∈X.

Definition:2.12. A near subtraction semigroup X is said to be **sub commutative** if aX = Xa, for every a∈X.

Definition:2.13. A near subtraction semigroup X is said to be **left-bipotent** if Xa = X a^2, for every a∈X.

Definition:2.14. An element a∈X is said to be **regular** if for each a∈X, a = aba, for some b∈X.

Definition:2.15. An element a∈X is said to be **strongly regular** if for each a∈X, a = b a^2, for some b ∈X.

Definition:2.16. X is said to fulfill the **insertion-of factors property** (**IFP**) provided for all a, b, n ∈X, ab = 0 ⇒anb = 0.

Definition:2.17. X has **strong IFP** , if for all ideals I of X and for all a, b ∈X, ab ∈I ⇒ anb ∈I.

3. Bi-ideals in C_1 and C_2 near-subtraction semigroup

In this section we define bi-ideals in C_1 and C_2 near-subtraction semigroups and give some examples of these new concepts.

Definition:3.1. Let X be a right near subtraction semigroup. If for all x∈ X, xX = xXx then we say X is a C_1 near subtraction semigroup.

Definition:3.2. Let X be a right near subtraction semigroup. If for all x∈ X, Xx = xXx then we say X is a C_2 near subtraction semigroup.

Example 3.3. Let X = {0,a,b,c}be the Klein's four group.Define subtraction and multiplication in X as follows:

–	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	0
c	c	0	c	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	c	b
c	0	a	b	c

Here (X, –, .) is a near subtraction semigroup (see [[6], pg.407] scheme 4 (0, 14, 2, 1)).

Then X is a C_1 near subtraction semigroup. But X is not a C_2 near subtraction semigroup. Since Xa ≠ aXa.

Example 3.4. Let X = {0,a,b,c} be the Klein's four group. Define subtraction and multiplication in X as follows:

-	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	0
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	c
b	0	0	0	0
c	0	a	0	c

Here (X, –, .) is a near subtraction semigroup (see [[6], pg.407] scheme 14 (0, 7, 0, 9)).

Then X is a C_2 near subtraction semigroup. But X is not a C_1 near subtraction semigroup. Since cX ≠ cXc.

Remark:3.5. Neither $C_1 \Longrightarrow C_2$ near subtraction semigroup.

Lemma:3.6. Let X is a C_1 *and* C_2 near subtraction semigroup ⟺ X is C_1 near subtraction semigroup and every idempotent is central.

Proof: To prove that X is C_1 near subtraction semigroup and every idempotent is central.

Since e ∈ E and X is C_1 near subtraction semigroup. ∴ eX = e X e for some x ∈ X.

⟹ exe = eu and ex = eve for some u, v in X.

Now exe = (ex) e = (eve) e = eve = ex ………..(1)

Since X is a C_2 near subtraction semigroup. ∴ Xe = eXe for some x ∈ X.

⟹ exe = ue and xe = eve for some u, v in X.

Now exe = e (xe) = e (evbe) = eve = xe ……….(2)

From (1) and (2) we get ex = exe = xe . Hence E ⊆ C (X).

Conversely, assume that X is C_1 near subtraction semigroup and every idempotent is central. (ie) ex = xe .

Let u ∈ eXe ⟹ u = e x' e = x' e e = x' e ∈ Xe , for all x' ∈ X. ∴ eXe ⊆ Xe ……….(3)

Let u ∈ Xe ⟹ u = xe = ex ∈ eX = eXe, for all x ∈ X. ∴ Xe ⊆ e Xe ……….(4)

From (1) and (2) we get Xe = eXe . Hence X is a C_2 near subtraction semigroup.

Theorem:3.7. Let X be a C_1 near subtraction semigroup then X has strong IFP.

Proof: Let I be an ideal of C_1 near subtraction semigroup of X.

Assume that ab ∈ I, for all a, b ∈ X.

For n ∈ X, an ∈ aX = aXa ⟹ an = a n' a , for all n' ∈ X.

anb = a n'ab ∈ XI ⊆ I ⟹ anb ∈ I. Hence X has strong IFP.

Theorem:3.8. If X is a zero symmetric C_1 near subtraction semigroup then X has an IFP.

Proof: Let X is a zero symmetric C_1 near subtraction semigroup.

If ab = 0, for some a, b ∈ X and an ∈ aX = aXa, for some n ∈ X.

⟹ an = a n' a , for some n' ∈ X.

⟹ anb = a n' a b = a n'. 0 = 0 ⟹ anb = 0. Hence X has an IFP.

Theorem:3.9. If X is a C_2 near subtraction semigroup then X has an IFP.

Proof: Let X is a C_2 near subtraction semigroup.

If ab = 0, for some a, b ∈ X and nb ∈ Xb = bXb, for some n ∈ X.

⟹ nb= b n' b , for some n' ∈ X

⟹ anb = ab n' b = 0. n' b = 0 ⟹ anb = 0. Hence X has an IFP.

Theorem:3.10. Let X be a C_1 be a near subtraction semigroup , if xX = yX then Xx = Xy, for all a, b ∈ X.

Proof: Let X be a C_1 be a near subtraction semigroup.

If xX = yX, for some x, y ∈ X.

Let x ∈ xX = Yx = yXy ⟹ x = y n' y, for some n' ∈ X [∴ C_1 is near subtraction semigroup]

⟹ nx = n y n' y ⟹ Xx ⊆ Xy ………..(1)

Similarly, let y ∈ yX = xX = xXx ⟹ y = x n' x, for some n' ∈ X

[∴ C_1 is near subtraction semigroup]

⟹ ny = n x n' x ⟹ Xy ⊆ Xx ………..(2)

From (1) and (2), hence Xx = Xy.

Theorem:3.11. Let X be a C_2 be a near subtraction semigroup , if Xx = Xy then xX = yX, for all a, b ∈ X.

Proof: Let X be a C_2 be a near subtraction semigroup.

Assume Xx = Xy, for some x, y ∈ X.

Let x ∈ xX = xY = yXy ⟹ x = y n' y, for some n' ∈ X [∴ C_2 is near subtraction semigroup]

⟹ xn = y n' yn ⟹ xX ⊆ yX ………..(1)

Similarly, let y ∈ Xy = Xx = xXx ⟹ y = x n' x, for some n' ∈ X

[∴ C_2 is near subtraction semigroup]

⟹ yn = x n' xn ⟹ yX ⊆ xX ………..(2)

From (1) and (2), hence xX =y X.

Corollary:3.11.1 If X is a C_1 *and* C_2 near subtraction semigroup then Xx = Xy ⟺ Xx = Yx, for all x, y ∈ X.

Theorem:3.12. Let X is a $\bar{s}$, C_1 *and* C_2 near subtraction semigroup then $M_1 \cap M_2$ = M_1M_2 for any two left N-subgroup M_1 and M_2 of X.

Proof: Let X is a $\bar{s}$, C_1 *and* C_2 near subtraction semigroup and let x ∈ $M_1 \cap M_2$, then

x ∈ M_1 and x ∈ M_2.

Let x ∈ Xx = xXx [since C_2 is near subtraction semigroup]

= xxXx ∈ XM_1XM_2 ⊆ M_1M_2

Therefore $M_1 \cap M_2 \subseteq M_1M_2$ ………..(1)

If x ∈ M_1M_2 then x = yz, for some y∈ M_1 and z ∈ M_2.

Now x = yz ∈ yX = yXy ∈ X $M_1 \subseteq M_1$ [since C_1 is near subtraction semigroup]

Also x = yz ∈ Xz = zXz ∈ X $M_2 \subseteq M_2$ [since C_2 is near subtraction semigroup]

Therefore $M_1M_2 \subseteq M_1 \cap M_2$ ……….(2)

From (1) and (2), hence $M_1 \cap M_2$ = M_1M_2.

Remark: 3.13. Let X be a $\bar{S}$ either C_1 *or* C_2 near subtraction semigroup then X is regular.

Proof: For every x ∈ xX = x X x [∴ C_1 *is* near subtraction semigroup]

For every x ∈ Xx = x X x [∴ C_2 is near subtraction semigroup]

Hence X is regular.

Remark: 3.14. Let s is a C_1 near subtraction semigroup then X is strongly regular.

Proof: For all x ∈ X, let x ∈ xX = x X x = xXxx ∈ Xx^2 ⟹ x ∈ Xx^2

Hence X is strongly regular.

Theorem:3.15. Let X be a s, C_1 near subtraction semigroup. Then X is strongly regular iff

B = BXB, for every bi-ideal B of X.

Proof: Let X is a s, C_1 - near subtraction semigroup and X is strongly regular. Then X is regular. Let B be bi-ideal of X. Now for b ∈ B, b = bab ∈ BXB, for some a ∈ X.

Thus B ⊆ BXB, for every bi –ideal B of X.

Conversely, let a ∈ X, Xa is a bi-ideal of X. Since X is a s-near subtraction semigroup.

Let a ∈ Xa = XaXXa ⊆ XaXa = XaXaa ∈ Xa^2.Therefore a ∈ Xa^2.

Hence X is strongly regular.

Theorem:3.16. Let X be a s, C_1 near subtraction semigroup. Then B = BXB, for every bi-ideal B of X iff X is a left bi-potent near subtraction semigroup.

Proof: Let X is a left bi-potent near subtraction semigroup. Let B be a bi-ideal of X. Since X is a s-near subtraction semigroup. let b∈ bX = bXb = bXbb ∈ X b^2 . Thus X is styrongly regular. By theorem 3.15, B = BXB for every bi-ideal B of X.

Conversely let a ∈ X. Then Xa is a bi-ideal of X.

Let y ∈ Xa = XaXXa ⊆ XaXa = XaXaa ∈ Xa^2. Therefore Xa ⊆ Xa^2.

But Xa^2 ⊆ Xa, for all a ∈ X. Therefore Xa^2 = Xa .

Hence X is a left bi-potent near subtraction semigroup.

Lemma: 3.17. [Refer V.Mahalakshmi [5]]

Let X be a zero – symmetric near subtraction semigroup. If L = { 0 }, then

en = ene, for 0 ≠ e ∈ E and n ∈ X.

Lemma: 3.18. [Refer V.Mahalakshmi [5]]

If X has the condition, eX = eXe = Xe, for all e ∈ E, then E ⊆ C(X).

Remark: 3.19. [Refer V.Mahalakshmi [5]]. The following are equivalent

- **(i)** X is a GNF.
- **(ii)** X is regular and each idempotent is central.
- **(iii)** X is regular and subcommutative.

Theorem:3.20. Let X be a left permutable s- near subtraction semigroup then the following are equivalent.

- **(i)** X is C_1 - near subtraction semigroup.
- **(ii)** B = BXB for every bi-ideal B of X.
- **(iii)** Q = QXQ for every quasi-ideal Q of X.
- **(iv)** X is left bi-potent near subtraction semigroup.

(v) X is regular.

(vi) aXa = Xa = Xa^2 for all a ∈ X.

(vii) X is a C_2 near subtraction semigroup and X is strongly regular.

(viii) X is GNF.

Proof: (i) ⇒ (ii) Assume that X is C_1 - near subtraction semigroup.

By remark:3.14, X is strongly regular.

By theorem:3.16, B=BXB for every bi-ideal B of X.

(ii) ⇒ (iii) Since every quasi-ideal is also a bi-ideal. We have Q = QXQ for every quasi-ideal Q of X.

(iii) ⇒ (iv) Let a ∈ X then Xa is a quasi-ideal of X.

If x ∈ Xa = XaXXa ⊆ XaXa then x = x_1 a x_2 a, for some x_1, x_2 ∈ X. Since X be a left permutable s- near subtraction semigroup. Therefore x = x_1 a x_2 a = x_1 x_2 a^2 ∈ X a^2.

That is Xa ⊆ X a^2. But X a^2 ⊆ Xa, for all a ∈ X. Therfore Xa^2 = Xa .

Hence X is left bi-potent near subtraction semigroup.

(iv) ⇒ (v) Let a ∈ Xa = X a^2.This implies that X is strongly regular. Then X is regular.

(v) ⇒ (vi) Let x ∈ aXa. Since X is left permutable s- near subtraction semigroup.

Now x = a x_1 a = x_1 a^2 ∈ Xa^2 ⇒ aXa ⊆ X a^2 ………(1)

Simillarly x ∈ X a^2 = Xaa then x = x_1 aa = ax_1a ∈ aXa ⇒ X a^2 ⊆ aXa ………..(2)

From (1) and (2), we get aXa = X a^2. Since X is regular for every a ∈ X, there exists

x ∈ X such that a = axa. Thus Xa = Xaxa ∈ XaXa . That is for every y ∈ Xa,

y = x_1 a x_2 a = x_1 x_2 a^2 ∈ X a^2. Therefore Xa ⊆ X a^2. But X a^2 ⊆ Xa, for all a ∈ X.

Hence aXa = X a^2 = Xa, for every a ∈ X.

(vi) ⇒ (vii) Obviously true. Therefore X is a C_2 near subtraction semigroup and

X is strongly regular. Since x is strongly regular. Let a ∈ X a^2 then L = { 0}.

Then by Lemma:3.17, eXe = eX, for every e ∈ E. Since X is C_2 near subtraction semigroup. Therefore Xe = eXe and so eX = eXe = Xe for every e ∈ E.

Again by the Lemma: 3.18, E ⊆ C(X). Hence X is GNF.

(viii) ⇒ (i) Let X is a GNF, by Remark:3.19, X is regular and sub commutative for a ∈ X.

Now aX = axaX = axXa ∈ aXXa ⊆ aXa. This implies aX ⊆ aXa ………..(1)

Obviously, aXa ⊆ aX ………(2).

From (1) and (2), we get aX = aXa. Hence X is C_1 near subtraction semigroup.

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