Interval-Valued Neutrosophic Bipolar Vague Sets

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Abstract: In this work, the new generalized concept of interval valued neutrosophic bipolar vague sets are introduced which is the combination of bipolar sets, vague sets and interval neutrosophic sets. Moreover, some remarkable properties like complement, union and intersection of interval valued neutrosophic bipolar vague sets are investigated and the proposed concepts are illustrated with suitable examples.

Keywords: Interval Valued Neutrosophic Bipolar Vague Set, Interval Neutrosophic Sets, Operations.

1. Introduction

In order to handle uncertainty many fields are successfully modeled by the classical mathematics, since concept of uncertainty is too complicate and not clearly defined object. But they can be modeled a number of different approaches including the probability theory, fuzzy set theory, intuitionistic fuzzy set, bipolar set theory, neutrosophic set theory and some other mathematical tools. Bipolar fuzzy sets are extension of fuzzy sets whose membership degree ranges from $[-1, 1]$. The membership degree $(0, 1]$ represents that an object satisfies a certain property whereas the membership degree $[-1, 0)$ represents that the element satisfies the implicit counter-property. The positive information indicates that the consideration to be possible and negative information indicates that the consideration is granted to be impossible. Nowadays, a variety of decision making problems are based on two-sided bipolar judgements on a positive side and a negative side. Bipolar fuzzy sets are playing a substantial role in chemistry, economics, computer science, engineering, medicine and decision making problems (for more details see [4–6, 10, 11, 20] and references therein). Akram [2] introduced bipolar fuzzy graphs and discuss its various properties and several new concepts on bipolar neutrosophic graphs. The generalization fuzzy set theory is the vague set theory and Vague sets are regarded as a special case of context-dependent fuzzy sets. In order to handle the indeterminate and inconsistent information, the neutrosophic set is introduced by the author Smarandache and studied extensively about neutrosophic set [3, 5, 6, 14–19]. Neutrosophic sets are the more generalized sets, one can deal with uncertain informations in a more successful way when contrasted with fuzzy sets and all other versions of fuzzy sets and it have the greater adaptability, accuracy and similarity to the framework when contrasted with past existing fuzzy models. The neutrosophic set has three completely independent parts, which are truth-membership degree, indeterminacy-membership degree and falsity-membership degree with the sum of these values lies between 0 and 3; therefore, it is applied to many different areas, such as algebra [8, 9] and decision-making problems (see [20] and references therein). From academic point of view, the neutrosophic set and operators need to be specified because is hard to be applied to the real applications. So the concept of interval valued neutrosophic bipolar sets are investigated which can represent uncertain, imprecise, incomplete and inconsistent information. In the application point of view, recent works are studied extensively about the development of interval valued neutrosophic graph structures. In [16], he established the certain notions including strong neutrosophic soft graphs and complete neutrosophic soft graphs. The concept of neutrosophic vague soft expert set which is a combination of neutrosophic vague set and soft expert set to improve the reasonability of decision making in reality. Motivation of the mentioned works as earlier [7], weintroduce the concept of interval valued neutrosophic bipolar vague sets. The major contribution in this work as follows:

- Newly introduced interval valued neutrosophic bipolar vague set with the operations like union and intersection of interval neutrosophic bipolar vague sets are provided.
- Further we presented the concept with suitable examples and a short conclusion is given with future directions. The obtained results will generalise the existing result [7].

2. Preliminaries

Definition 2.1. *[17] A vague set A on a non empty set X is a pair (T_A, F_A), where $T_A : X \to [0,1]$ and $F_A : X \to [0,1]$ are true membership and false membership functions, respectively, such that*

$$0 \leq T_A(x) + F_A(y) \leq 1, \text{ for any } x \in X.$$

Let X and Y be two non-empty sets. A vague relation R of X to Y is a vague set R on $X \times Y$, that is, $R = (T_R, F_R)$, where $T_R : X \times Y \to [0,1], F_R : X \times Y \to [0,1]$ which satisfies the condition:

$$0 \leq T_R(x,y) + F_R(x,y) \leq 1, \text{ for any } x \in X.$$

Definition 2.2. *[16] A Neutrosophic set A is contained in another neutrosophic set B, (i.e) $A \subseteq B$, if $\forall x \in X, T_A(x) \leq T_B(x), I_A(x) \geq I_B(x)$and $F_A(x) \geq F_B(x)$.*

Definition 2.3. *[4] Let X be a space of points (objects), with a generic elements in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership-function $F_A(x)$.*

For each point x in X, $T_A(x), F_A(x), I_A(x) \in [0,1]$,

$$A = \{\langle x, T_A(x), F_A(x), I_A(x)\rangle\} \text{ and } 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

Definition 2.4. *[20] A bipolar neutrosophic set A in X is defined as an object of the form $A = \{< x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x) >: x \in X\}$, where $T^P, I^P, F^P : X \to [0,1]$ and $T^N, I^N, F^N : X \to [-1,0]$. The Positive membership degree $T^P(x), I^P(x), F^P(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^N(x), I^N(x), F^N(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.*

Definition 2.5. *[2] Let X be a non-empty set. Then we call*

$$A = \{\langle x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x)\rangle, x \in X\}$$

as a bipolar single valued neutrosophic relation on X such that $T_A^P(x,y) \in [0,1], I_A^P(x,y) \in [0,1]$, $F_A^P(x,y) \in [0,1]$ and $T_A^N(x,y) \in [-1,0], I_A^N(x,y) \in [-1,0], F_A^N(x,y) \in [-1,0]$.

Definition 2.6. *[2] Let $A = (T_P^A, I_P^A, F_P^A, T_N^A, I_N^A, F_N^A)$ and $B = (T_P^B, I_P^B, F_P^B, T_N^B, I_N^B, F_N^B)$ be bipolar single valued neutrosophic set on X. If $B = (T_P^B, I_P^B, F_P^B, T_N^B, I_N^B, F_N^B)$ is a bipolar single valued neutrosophic relation on $A = (T_P^A, I_P^A, F_P^A, T_N^A, I_N^A, F_N^A)$ then*

$$\begin{aligned} T_B^P(xy) &\leq \min(T_A^P(x), T_A^P(y)), & T_B^N(xy) &\geq \max(T_A^N(x), T_A^N(y)) \\ I_B^P(xy) &\geq \max(I_A^P(x), I_A^P(y)), & I_B^N(xy) &\leq \min(I_A^N(x), I_A^N(y)) \\ F_B^P(xy) &\geq \max(F_A^P(x), F_A^P(y)), & F_B^N(xy) &\leq \min(F_A^N(x), F_A^N(y)) \end{aligned}$$

A bipolar single valued neutrosophic relation B on X is called symmetric, if $T_B^P(xy) = T_B^P(yx), I_B^P(xy) = I_B^P(yx), F_B^P(xy) = F_B^P(yx)$ and $T_B^N(xy) = T_B^N(yx), I_B^N(xy) = I_B^N(yx), F_B^N(xy) = F_B^N(yx)$ for all $xy \in X$.

Definition 2.7. *[1] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV} = \{\langle x, \hat{T}_{A_{NV}}(x), \hat{I}_{A_{NV}}(x), \hat{F}_{A_{NV}}(x)\rangle, x \in X\}$ whose truth-membership, indeterminacy membership and falsity-membership function is defined as*

$$\hat{T}_{A_{NV}}(x) = [\hat{T}^{-}(x), \hat{T}^{+}(x)], [\hat{I}^{-}(x), \hat{I}^{+}(x)], [\hat{F}^{-}(x), \hat{F}^{+}(x)],$$

where $T^{+}(x) = 1 - F^{-}(x), F^{+}(x) = 1 - T^{-}(x)$, and $0 \leq T^{-}(x) + I^{-}(x) + F^{-}(x) \leq 2$.

Definition 2.8. *[7] A Interval neutrosophic set A_{INS} (INS in short) on the universe of discourse X defines as follows:*

$$A_{IN} = \{\langle x, ({}^{L}T_{A_{IN}}(x), {}^{U}T_{A_{IN}}(x)), ({}^{L}I_{A_{IN}}(x), {}^{U}IA_{IN}(x)), ({}^{L}F_{A_{IN}}(x), {}^{U}F_{A_{IN}}(x)), \rangle, x \in X\}$$

Where for each point $x \in X$. we have $T_A(x) \in [0,1], I_A(x) \in [0,1], F_A(x) \in [0,1]$ and $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

Definition 2.9. *[7] A Interval neutrosophic vague set A_{INV} (INVS in short) on the universe of discourse X written as*

$$A_{INV} = \{\langle x, ({}^{L}\tilde{T}_{A_{INBV}}(x), {}^{U}\tilde{T}_{A_{INBV}}(x)), ({}^{L}\tilde{I}_{A_{INBV}}(x), {}^{U}\tilde{I}_{A_{INBV}}(x)), ({}^{L}\tilde{F}_{A_{INBV}}(x), {}^{U}\tilde{F}_{A_{INBV}}(x)), \rangle, x \in X\}$$

whose truth-membership, indeterminacy membership and falsity-membership function is defined as

$$\{{}^{L}\tilde{T}_{A_{INV}}(x)\} = [{}^{L}(T_A^{-})(x), {}^{L}(T_A^{+})(x)], \{{}^{U}\tilde{T}_{A_{INV}}(x)\} = [{}^{U}(T_A^{-})(x), {}^{U}(T_A^{+})(x)]$$
$$\{{}^{L}\tilde{I}_{A_{INV}}(x)\} = [{}^{L}(I_A^{-})(x), {}^{L}(I_A^{+})(x)], \{{}^{U}\tilde{I}_{A_{INV}}(x)\} = [{}^{U}(I_A^{-})(x), {}^{U}(I_A^{+})(x)]$$
$$\{{}^{L}\tilde{F}_{A_{INV}}(x)\} = [{}^{L}(F_A^{-})(x), {}^{L}(F_A^{+})(x)], \{{}^{U}\tilde{F}_{A_{INV}}(x)\} = [{}^{U}(F_A^{-})(x), {}^{U}(F_A^{+})(x)],$$

where $({}^{L}T_A^{+})(x) = 1 - ({}^{L}F_A^{-})(x), ({}^{U}T_A^{+})(x) = 1 - ({}^{U}F_A^{-})(x)$,
$({}^{L}F_A^{+})(x) = 1 - ({}^{L}T_A^{-})(x), ({}^{U}F_A^{+})(x) = 1 - ({}^{U}T_A^{-})(x)$, and

$$0 \leq ({}^{L}T_A^{-})(x) + ({}^{U}T_A^{-})(x) + ({}^{L}I_A^{-})(x) + ({}^{U}I_A^{-})(x) + ({}^{L}F_A^{-})(x) + ({}^{U}F_A^{-})(x) \leq 4.$$
$$0 \leq ({}^{L}T_A^{+})(x) + ({}^{U}T_A^{+})(x) + ({}^{L}I_A^{+})(x) + ({}^{U}I_A^{+})(x) + ({}^{L}F_A^{+})(x) + ({}^{U}F_A^{+})(x) \leq 4.$$

An INVS A_{INV}, when X is continuous is presented as follows:

$$A_{INV} = \int_X \Big\{ \frac{({}^{L}\tilde{T}_{A_{INV}}(x), {}^{U}\tilde{T}_{A_{INV}}(x)), ({}^{L}\tilde{I}_{A_{INBV}}(x), {}^{U}\tilde{I}_{A_{INBV}}(x)), ({}^{L}\tilde{F}_{A_{INV}}(x), {}^{U}\tilde{F}_{A_{INV}}(x))}{x} : x \in X \Big\}$$

and when X is discrete function an INVS A_{INV} can be presented as follows:

$$A_{INV} = \sum_{i=n}^{n} \Big\{ \frac{({}^{L}\tilde{T}_{A_{INV}}(x), {}^{U}\tilde{T}_{A_{INV}}(x)), ({}^{L}\tilde{I}_{A_{INBV}}(x), {}^{U}\tilde{I}_{A_{INBV}}(x)), ({}^{L}\tilde{F}_{A_{INV}}(x), {}^{U}\tilde{F}_{A_{INV}}(x))}{x_i} : x \in X \Big\}$$

$0 \leq (\sup \tilde{T}_A(x) + \sup \tilde{I}_A(x) + \sup \tilde{F}_A(x)) \leq 2.$

3. INTERVAL NEUTROSOPHIC BIPOLAR VAGUE SET

Definition 3.1. *A Interval neutrosophic bipolar vague set A_{INBV} (INBVS in short) on the universe of discourse X written as*

$$A_{INBV} = \{\langle x, ({}^{L}\tilde{T}^{P}_{A_{INBV}}(x), {}^{U}\tilde{T}^{P}_{A_{INBV}}(x)), ({}^{L}\tilde{I}^{P}_{A_{INBV}}(x), {}^{U}\tilde{I}^{P}_{A_{INBV}}(x)), ({}^{L}\tilde{F}^{P}_{A_{INBV}}(x), {}^{U}\tilde{F}^{P}_{A_{INBV}}(x)),$$
$$({}^{L}\tilde{T}^{N}_{A_{INBV}}(x), {}^{U}\tilde{T}^{N}_{A_{INBV}}(x)), ({}^{L}\tilde{I}^{N}_{A_{INBV}}(x), {}^{U}\tilde{I}^{N}_{A_{INBV}}(x)), ({}^{L}\tilde{F}^{N}_{A_{INBV}}(x), {}^{U}\tilde{F}^{N}_{A_{INBV}}(x))\rangle, x \in X\}$$

whose truth-membership, indeterminacy membership and falsity-membership function is defined as

$$\{{}^{L}\tilde{T}^{P}_{A_{INBV}}(x)\} = [{}^{L}(T_A^{-})^{P}(x), {}^{L}(T_A^{+})^{P}(x)], \{{}^{U}\tilde{T}^{P}_{A_{INBV}}(x)\} = [{}^{U}(T_A^{-})^{P}(x), {}^{U}(T_A^{+})^{P}(x)]$$
$$\{{}^{L}\tilde{I}^{P}_{A_{INBV}}(x)\} = [{}^{L}(I_A^{-})^{P}(x), {}^{L}(I_A^{+})^{P}(x)], \{{}^{U}\tilde{I}^{P}_{A_{INBV}}(x)\} = [{}^{U}(I_A^{-})^{P}(x), {}^{U}(I_A^{+})^{P}(x)]$$
$$\{{}^{L}\tilde{F}^{P}_{A_{INBV}}(x)\} = [{}^{L}(F_A^{-})^{P}(x), {}^{L}(F_A^{+})^{P}(x)], \{{}^{U}\tilde{F}^{P}_{A_{INBV}}(x)\} = [{}^{U}(F_A^{-})^{P}(x), {}^{U}(F_A^{+})^{P}(x)],$$

where $({}^{L}T_A^{+})^P(x) = 1 - ({}^{L}F_A^{-})^P(x), ({}^{U}T_A^{+})^P(x) = 1 - ({}^{U}F_A^{-})^P(x)$,
$({}^{L}F_A^{+})^P(x) = 1 - ({}^{L}T_A^{-})^P(x), ({}^{U}F_A^{+})^P(x) = 1 - ({}^{U}T_A^{-})^P(x)$.
Also we have,

$$\{{}^{L}\tilde{T}^N_{A_{INBV}}(x)\} = [{}^{L}(T_A^{-})^N(x),\ {}^{L}(T_A^{+})^N(x)], \{{}^{U}\tilde{T}^N_{A_{INBV}}(x)\} = [{}^{U}(T_A^{-})^N(x),\ {}^{U}(T_A^{+})^N(x)]$$
$$\{{}^{L}\tilde{I}^N_{A_{INBV}}(x)\} = [{}^{L}(I_A^{-})^N(x),\ {}^{L}(I_A^{+})^N(x)], \{{}^{U}\tilde{I}^N_{A_{INBV}}(x)\} = [{}^{U}(I_A^{-})^N(x),{}^{U}(I_A^{+})^N(x)]$$
$$\{{}^{L}\tilde{F}^N_{A_{INBV}}(x)\} = [{}^{L}(F_A^{-})^N(x),\ {}^{L}(F_A^{+})^N(x)], \{{}^{U}\tilde{F}^N_{A_{INBV}}(x)\} = [{}^{U}(F_A^{-})^N(x),{}^{U}(F_A^{+})^N(x)],$$

where $({}^{L}T_A^{+})^N(x) = -1 - ({}^{L}F_A^{-})^N(x), ({}^{U}T_A^{+})^N(x) = -1 - ({}^{U}F_A^{-})^N(x)$,
$({}^{L}F_A^{+})^N(x) = -1 - ({}^{L}T_A^{-})^N(x), ({}^{U}F_A^{+})^N(x) = -1 - ({}^{U}T_A^{-})^N(x)$, *and with the condition,*

$$0 \geq ({}^{L}T_A^{-})^N(x) + ({}^{U}T_A^{-})^N(x) + ({}^{L}I_A^{-})^N(x) + ({}^{U}I_A^{-})^N(x) + ({}^{L}F_A^{-})^N(x) + ({}^{U}F_A^{-})^N(x) \geq -4.$$
$$0 \geq ({}^{L}T_A^{+})^N(x) + ({}^{U}T_A^{+})^N(x) + ({}^{L}I_A^{+})^N(x) + ({}^{U}I_A^{+})^N(x) + ({}^{L}F_A^{+})^N(x) + ({}^{U}F_A^{+})^N(x) \geq -4.$$
$$0 \leq ({}^{L}T_A^{-})^P(x) + ({}^{U}T_A^{-})^P(x) + ({}^{L}I_A^{-})^P(x) + ({}^{U}I_A^{-})^P(x) + ({}^{L}F_A^{-})^P(x) + ({}^{U}F_A^{-})^P(x) \leq 4.$$
$$0 \leq ({}^{L}T_A^{+})^P(x) + ({}^{U}T_A^{+})^P(x) + ({}^{L}I_A^{+})^P(x) + ({}^{U}I_A^{+})^P(x) + ({}^{L}F_A^{+})^P(x) + ({}^{U}F_A^{+})^P(x) \leq 4.$$

An (INBVS) A_{INBV} *when* E *is continuous is presented as follows.*

$$A_{INBV} = \int_x \Big\{({}^{L}\tilde{T}^P_{A_{INBV}}(x),\ {}^{U}\tilde{T}^P_{A_{INBV}}(x)), ({}^{L}\tilde{I}^P_{A_{INBV}}(x),\ {}^{U}\tilde{I}^P_{A_{INBV}}(x)), ({}^{L}\tilde{F}^P_{A_{INBV}}(x),{}^{U}\tilde{F}^P_{A_{INBV}}(x)),$$
$$({}^{L}\tilde{T}^N_{A_{INBV}}(x),\ {}^{U}\tilde{T}^N_{A_{INBV}}(x)), ({}^{L}\tilde{I}^N_{A_{INBV}}(x),\ {}^{U}\tilde{I}^N_{A_{INBV}}(x)), ({}^{L}\tilde{F}^N_{A_{INBV}}(x),\ {}^{U}\tilde{F}^N_{A_{INBV}}(x))/x : x \in X.\Big\}$$

and when X *is discrete an INBVS* A_{INBV} *can be presented as follows:*

$$A_{INBV} = \sum_{i=1}^{n} \Big\{({}^{L}\tilde{T}^P_{A_{INBV}}(x),\ {}^{U}\tilde{T}^P_{A_{INBV}}(x)), ({}^{L}\tilde{I}^P_{A_{INBV}}(x),\ {}^{U}\tilde{I}^P_{A_{INBV}}(x)), ({}^{L}\tilde{F}^P_{A_{INBV}}(x),\ {}^{U}\tilde{F}^P_{A_{INBV}}(x)),$$
$$({}^{L}\tilde{T}^N_{A_{INBV}}(x),\ {}^{U}\tilde{T}^N_{A_{INBV}}(x)), ({}^{L}\tilde{I}^N_{A_{INBV}}(x),\ {}^{U}\tilde{I}^N_{A_{INBV}}(x)), ({}^{L}\tilde{F}^N_{A_{INBV}}(x),\ {}^{U}\tilde{F}^N_{A_{INBV}}(x))/x_i : x_i \in X\Big\}$$

$$0 \leq (\sup \tilde{T}^P_A(x) + \sup \tilde{I}^P_A(x) + \sup \tilde{F}^P_A(x)) \leq 2.$$
$$-2 \leq (\inf \tilde{T}^N_A(x) + \inf \tilde{I}^N_A(x) + \inf \tilde{F}^N_A(x)) \leq -2.$$

Example 3.1. *Let* $X = \{x_1, x_2, x_3\}$ *be a set of universe we define the INBV set* A_{INBV} *as follows*

$$A_{INBV} = \Big\{x_1/T\{[0.2, 0.5], [0.2, 0.3]\}, I\{[0.1, 0.6], [0.3, 0.6]\}, F\{[0.5, 0.8], [0.7, 0.8]\},$$
$$T\{[-0.3, -0.5], [-0.3, -0.3]\}, I\{[-0.1, -0.6], [-0.3, -0.6]\}, F\{[-0.5, -0.7], [-0.7, -0.7]\},$$
$$\{x_2/T\{[0.4, 0.5], [0.1, 0.7]\}, I\{[0.1, 0.7], [0.5, 0.5]\}, F\{[0.5, 0.6], [0.3, 0.9]\},$$
$$T\{[-0.4, -0.5], [-0.1, -0.7]\}, I\{[-0.1, -0.7], [-0.5, -0.5]\}, F\{[-0.5, -0.6], [-0.3, -0.9]\},$$
$$\{x_3/T\{[0.6, 0.9], [0.2, 0.5]\}, I\{[0.3, 0.7], [0.4, 0.6]\}, F\{[0.1, 0.5], [0.5, 0.8]\},$$
$$T\{[-0.5, -0.5], [-0.6, -0.3]\}, I\{[-0.4, -0.3], [-0.6, -0.3]\}, F\{[-0.5, -0.5], [-0.7, -0.4]\}\Big\}$$

Definition 3.2. *The complement of INBVS* A_{INBV} *is denoted by* A^c_{INBV} *and it is defined by*

$$({}^{L}\tilde{T}^c_{A_{INBV}}(x))^P = \{(1 - {}^{L}T_A^{+}(x))^P, (1 - {}^{L}T_A^{-}(x))^P\},$$
$$({}^{U}\tilde{T}^c_{A_{INBV}}(x))^P = \{(1 - {}^{U}T_A^{+}(x))^P, (1 - {}^{U}T_A^{-}(x))^P\}$$
$$({}^{L}\tilde{I}^c_{A_{INBV}}(x))^P = \{(1 - {}^{L}I_A^{+}(x))^P, (1 - {}^{L}I_A^{-}(x))^P\},$$
$$({}^{U}\tilde{I}^c_{A_{INBV}}(x))^P = \{(1 - {}^{U}I_A^{+}(x))^P, (1 - {}^{U}I_A^{-}(x))^P\}$$

$$({}^L\tilde{F}^c_{A_{INBV}}(x))^P = \{(1-\ {}^LF^+_A(x))^P, (1-\ {}^LF^-_A(x))^P\},$$
$$({}^U\tilde{F}^c_{A_{INBV}}(x))^P = \{(1-\ {}^UF^+_A(x))^P, (1-\ {}^UF^-_A(x))^P\}$$
$$({}^L\tilde{T}^c_{A_{INBV}}(x))^N = \{(-1-\ {}^LT^+_A(x))^N, (-1-\ {}^LT^-_A(x))^N\},$$
$$({}^U\tilde{T}^c_{A_{INBV}}(x))^N = \{(-1-\ {}^UT^+_A(x))^N, (-1-\ {}^UT^-_A(x))^N\}$$
$$({}^L\tilde{I}^c_{A_{INBV}}(x))^N = \{(-1-\ {}^LI^+_A(x))^N, (-1-\ {}^LI^-_A(x))^N\},$$
$$({}^U\tilde{I}^c_{A_{INBV}}(x))^N = \{(-1-\ {}^UI^+_A(x))^N, (-1-\ {}^UI^-_A(x))^N\}$$
$$({}^L\tilde{F}^c_{A_{INBV}}(x))^N = \{(-1-\ {}^LF^+_A(x))^N, (-1-\ {}^LF^-_A(x))^N\},$$
$$({}^U\tilde{F}^c_{A_{INBV}}(x))^N = \{(-1-\ {}^UF^+_A(x))^N, (-1-\ {}^UF^-_A(x))^N\}.$$

Example 3.2. *Considering above example we have*

$$A^c_{INBV} = \Big\{x_1/T\{[0.8,0.5],[0.8,0.7]\}, I\{[0.9,0.4],[0.7,0.4]\}, F\{[0.5,0.2],[0.3,0.2]\},$$
$$T\{[-0.7,-0.5],[-0.7,-0.7]\}, I\{[-0.9,-0.4],[-0.7,-0.4]\}, F\{[-0.5,-0.3],[-0.3,-0.3]\},$$
$$\{x_2/T\{[0.6,0.5],[0.9,0.3]\}, I\{[0.9,0.3],[0.5,0.5]\}, F\{[0.5,0.4],[0.7,0.1]\},$$
$$T\{[-0.6,-0.5],[-0.9,-0.3]\}, I\{[-0.9,-0.3],[-0.5,-0.5]\}, F\{[-0.5,-0.4],[-0.7,-0.1]\},$$
$$\{x_3/T\{[0.4,0.1],[0.8,0.5]\}, I\{[0.7,0.3],[0.6,0.4]\}, F\{[0.9,0.5],[0.5,0.2]\},$$
$$T\{[-0.5,-0.5],[-0.4,-0.7]\}, I\{[-0.6,-0.7],[-0.4,-0.7]\}, F\{[-0.5,-0.5],[-0.3,-0.6]\}\Big\}$$

Definition 3.3. *Two INBVSs A_{INBV} and B_{INBV} of the universe X are said to be equal, if for all $x_i \in X$,*

$$({}^L\tilde{T}_{A_{INBV}})^P(x_i) = ({}^L\tilde{T}_{B_{INBV}})^P(x_i), ({}^U\tilde{T}_{A_{INBV}})^P(x_i) = ({}^U\tilde{T}_{B_{INBV}})^P(x_i),$$
$$({}^L\tilde{I}_{A_{INBV}})^P(x_i) = ({}^L\tilde{I}_{B_{INBV}})^P(x_i), ({}^U\tilde{I}_{A_{INBV}})^P(x_i) = ({}^U\tilde{I}_{B_{INBV}})^P(x_i),$$
$$({}^L\tilde{F}_{A_{INBV}})^P(x_i) = ({}^L\tilde{F}_{B_{INBV}})^P(x_i), ({}^U\tilde{F}_{A_{INBV}})^P(x_i) = ({}^U\tilde{F}_{B_{INBV}})^P(x_i),$$

and

$$({}^L\tilde{T}_{A_{INBV}})^N(x_i) = ({}^L\tilde{T}_{B_{INBV}})^N(x_i), ({}^U\tilde{T}_{A_{INBV}})^N(x_i) = ({}^U\tilde{T}_{B_{INBV}})^N(x_i),$$
$$({}^L\tilde{I}_{A_{INBV}})^N(x_i) = ({}^L\tilde{I}_{B_{INBV}})^N(x_i), ({}^U\tilde{I}_{A_{INBV}})^N(x_i) = ({}^U\tilde{I}_{B_{INBV}})^N(x_i),$$
$$({}^L\tilde{F}_{A_{INBV}})^N(x_i) = ({}^L\tilde{F}_{B_{INBV}})^N(x_i), ({}^U\tilde{F}_{A_{INBV}})^N(x_i) = ({}^U\tilde{F}_{B_{INBV}})^N(x_i),$$

Definition 3.4. *Let A_{INBV} and B_{INBV} be two INBVSs of the universe X. If for all $x_i \in X$,*

$$({}^L\tilde{T}_{A_{INBV}})^P(x_i) \le ({}^L\tilde{T}_{B_{INBV}})^P(x_i), ({}^U\tilde{T}_{A_{INBV}})^P(x_i) \le ({}^U\tilde{T}_{B_{INBV}})^P(x_i),$$
$$({}^L\tilde{I}_{A_{INBV}})^P(x_i) \ge ({}^L\tilde{I}_{B_{INBV}})^P(x_i), ({}^U\tilde{I}_{A_{INBV}})^P(x_i) \ge ({}^U\tilde{I}_{B_{INBV}})^P(x_i),$$
$$({}^L\tilde{F}_{A_{INBV}})^P(x_i) \ge ({}^L\tilde{F}_{B_{INBV}})^P(x_i), ({}^U\tilde{F}_{A_{INBV}})^P(x_i) \ge ({}^U\tilde{F}_{B_{INBV}})^P(x_i),$$

and

$$({}^L\tilde{T}_{A_{INBV}})^N(x_i) \ge ({}^L\tilde{T}_{B_{INBV}})^N(x_i), ({}^U\tilde{T}_{A_{INBV}})^N(x_i) \ge ({}^U\tilde{T}_{B_{INBV}})^N(x_i),$$
$$({}^L\tilde{I}_{A_{INBV}})^N(x_i) \le ({}^L\tilde{I}_{B_{INBV}})^N(x_i), ({}^U\tilde{I}_{A_{INBV}})^N(x_i) \le ({}^U\tilde{I}_{B_{INBV}})^N(x_i),$$
$$({}^L\tilde{F}_{A_{INBV}})^N(x_i) \le ({}^L\tilde{F}_{B_{INBV}})^N(x_i), ({}^U\tilde{F}_{A_{INBV}})^N(x_i) \le ({}^U\tilde{F}_{B_{INBV}})^N(x_i),$$

then the NBVS $(A_{INBV})^P$ are included by $(B_{INBV})^P$, denoted by $(A_{INBV})^P \subseteq (B_{INBV})^P$ where $1 \le i \le n$ and $(A_{INBV})^N$ are included by $(B_{INBV})^N$, denoted by $(A_{INBV})^N \subseteq (B_{INBV})^N$ where $1 \le i \le n$.

Definition 3.5. *The union of two INVSs A_{INBV} and B_{INBV} is a NBVSs, C_{INBV}, written as $C_{INBV} = A_{INBV} \cup B_{INBV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of A_{INBV} and B_{INBV} by*

$$({}^L\tilde{T}_{C_{INBV}})^P(x) = [\max(({}^LT^-_{A_{INBV}})^P(x), ({}^LT^-_{B_{INBV}})^P(x)), \max(({}^LT^+_{A_{INBV}})^P(x), ({}^LT^+_{B_{INBV}})^P(x))]$$

$$({}^U\tilde{T}_{C_{INBV}})^P(x) = [\max(({}^UT^-_{A_{INBV}})^P(x), ({}^UT^-_{B_{INBV}})^P(x)), \max(({}^UT^+_{A_{INBV}})^P(x), ({}^UT^+_{B_{INBV}})^P(x))]$$
$$({}^L\tilde{I}_{C_{INBV}})^P(x) = [\min(({}^LI^-_{A_{INBV}})^P(x), ({}^LI^-_{B_{INBV}})^P(x)), \min(({}^LI^+_{A_{INBV}})^P(x), ({}^LI^+_{B_{INBV}})^P(x))]$$
$$({}^U\tilde{I}_{C_{INBV}})^P(x) = [\min(({}^UI^-_{A_{INBV}})^P(x), ({}^UI^-_{B_{INBV}})^P(x)), \min(({}^UI^+_{A_{INBV}})^P(x), ({}^UI^+_{B_{INBV}})^P(x))]$$
$$({}^L\tilde{F}_{C_{INBV}})^P(x) = [\min(({}^LF^-_{A_{INBV}})^P(x), ({}^LF^-_{B_{INBV}})^P(x)), \min(({}^LF^+_{A_{INBV}})^P(x), ({}^LF^+_{B_{INBV}})^P(x))]$$
$$({}^U\tilde{F}_{C_{INBV}})^P(x) = [\min(({}^UF^-_{A_{INBV}})^P(x), ({}^UF^-_{B_{INBV}})^P(x)), \min(({}^UF^+_{A_{INBV}})^P(x), ({}^UF^+_{B_{INBV}})^P(x))], \text{ and}$$
$$({}^L\tilde{T}_{C_{INBV}})^N(x) = [\min(({}^LT^-_{A_{INBV}})^N(x), ({}^LT^-_{B_{INBV}})^N(x)), \min(({}^LT^+_{A_{INBV}})^N(x), ({}^LT^+_{B_{INBV}})^N(x))]$$
$$({}^U\tilde{T}_{C_{INBV}})^N(x) = [\min(({}^UT^-_{A_{INBV}})^N(x), ({}^UT^-_{B_{INBV}})^N(x)), \min(({}^UT^+_{A_{INBV}})^N(x), ({}^UT^+_{B_{INBV}})^N(x))]$$
$$({}^L\tilde{I}_{C_{iNBV}})^N(x) = [\max(({}^LI^-_{A_{INBV}})^N(x), ({}^LI^-_{B_{INBV}})^N(x)), \max(({}^LI^+_{A_{INBV}})^N(x), ({}^LI^+_{B_{INBV}})^N(x))]$$
$$({}^U\tilde{I}_{C_{INBV}})^N(x) = [\max(({}^UI^-_{A_{INBV}})^N(x), ({}^UI^-_{B_{INBV}})^N(x)), \max(({}^UI^+_{A_{INBV}})^N(x), ({}^UI^+_{B_{INBV}})^N(x))]$$
$$({}^L\tilde{F}_{C_{INBV}})^N(x) = [\max(({}^LF^-_{A_{INBV}})^N(x), ({}^LF^-_{B_{INBV}})^N(x)), \max(({}^LF^+_{A_{INBV}})^N(x), ({}^LF^+_{B_{INBV}})^N(x))]$$
$$({}^U\tilde{F}_{C_{INBV}})^N(x) = [\max(({}^UF^-_{A_{INBV}})^N(x), ({}^UF^-_{B_{INBV}})^N(x)), \max(({}^UF^+_{A_{INBV}})^N(x), ({}^UF^+_{B_{INBV}})^N(x))]$$

Definition 3.6. *The intersection of two INVSs A_{INBV} and B_{INBV} is a INBVSs C_{INBV}, written as $C_{INBV} = A_{INBV} \cap B_{INBV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of A_{INBV} and B_{INBV} by*

$$({}^L\tilde{T}_{C_{INBV}})^P(x) = [\min(({}^LT^-_{A_{INBV}})^P(x), ({}^LT^-_{B_{INBV}})^P(x)), \min(({}^LT^+_{A_{INBV}})^P(x), ({}^LT^+_{B_{INBV}})^P(x))]$$
$$({}^U\tilde{T}_{C_{INBV}})^P(x) = [\min(({}^UT^-_{A_{INBV}})^P(x), ({}^UT^-_{B_{INBV}})^P(x)), \min(({}^UT^+_{A_{INBV}})^P(x), ({}^UT^+_{B_{INBV}})^P(x))]$$
$$({}^L\tilde{I}_{C_{INBV}})^P(x) = [\max(({}^LI^-_{A_{INBV}})^P(x), ({}^LI^-_{B_{INBV}})^P(x)), \max(({}^LI^+_{A_{INBV}})^P(x), ({}^LI^+_{B_{INBV}})^P(x))]$$
$$({}^U\tilde{I}_{C_{INBV}})^P(x) = [\max(({}^UI^-_{A_{INBV}})^P(x), ({}^UI^-_{B_{INBV}})^P(x)), \max(({}^UI^+_{A_{INBV}})^P(x), ({}^UI^+_{B_{INBV}})^P(x))]$$
$$({}^L\tilde{F}_{C_{INBV}})^P(x) = [\max(({}^LF^-_{A_{INBV}})^P(x), ({}^LF^-_{B_{INBV}})^P(x)), \max(({}^LF^+_{A_{INBV}})^P(x), ({}^LF^+_{B_{INBV}})^P(x))]$$
$$({}^U\tilde{F}_{C_{INBV}})^P(x) = [\max(({}^UF^-_{A_{INBV}})^P(x), ({}^UF^-_{B_{INBV}})^P(x)), \max(({}^UF^+_{A_{INBV}})^P(x), ({}^UF^+_{B_{INBV}})^P(x))], \text{ and}$$
$$({}^L\tilde{T}_{C_{INBV}})^N(x) = [\max(({}^LT^-_{A_{INBV}})^N(x), ({}^LT^-_{B_{INBV}})^N(x)), \max(({}^LT^+_{A_{INBV}})^N(x), ({}^LT^+_{B_{INBV}})^N(x))]$$
$$({}^U\tilde{T}_{C_{INBV}})^N(x) = [\max(({}^UT^-_{A_{INBV}})^N(x), ({}^UT^-_{B_{INBV}})^N(x)), \max(({}^UT^+_{A_{INBV}})^N(x), ({}^UT^+_{B_{INBV}})^N(x))]$$
$$({}^L\tilde{I}_{C_{iNBV}})^N(x) = [\min(({}^LI^-_{A_{INBV}})^N(x), ({}^LI^-_{B_{INBV}})^N(x)), \min(({}^LI^+_{A_{INBV}})^N(x), ({}^LI^+_{B_{INBV}})^N(x))]$$
$$({}^U\tilde{I}_{C_{INBV}})^N(x) = [\min(({}^UI^-_{A_{INBV}})^N(x), ({}^UI^-_{B_{INBV}})^N(x)), \min(({}^UI^+_{A_{INBV}})^N(x), ({}^UI^+_{B_{INBV}})^N(x))]$$
$$({}^L\tilde{F}_{C_{INBV}})^N(x) = [\min(({}^LF^-_{A_{INBV}})^N(x), ({}^LF^-_{B_{INBV}})^N(x)), \min(({}^LF^+_{A_{INBV}})^N(x), ({}^LF^+_{B_{INBV}})^N(x))]$$
$$({}^U\tilde{F}_{C_{INBV}})^N(x) = [\min(({}^UF^-_{A_{INBV}})^N(x), ({}^UF^-_{B_{INBV}})^N(x)), \min(({}^UF^+_{A_{INBV}})^N(x), ({}^UF^+_{B_{INBV}})^N(x))]$$

Example 3.3. *Let U be a set of universe and let A_{INBV} and B_{INBV} be INBVSs, then the union $A_{INBV} \cup B_{INBV}$ is defined as follows:*

$$\begin{aligned} A_{INBV} = \Big\{ & x_1/T\{[0.2,0.5],[0.2,0.3]\}, I\{[0.1,0.6],[0.3,0.6]\}, F\{[0.5,0.8],[0.7,0.8]\}, \\ & T\{[-0.3,-0.5],[-0.3,-0.3]\}, I\{[-0.1,-0.6],[-0.3,-0.6]\}, F\{[-0.5,-0.7],[-0.7,-0.]\}, \\ & \{x_2/T\{[0.4,0.5],[0.1,0.7]\}, I\{[0.1,0.7],[0.5,0.5]\}, F\{[0.5,0.6],[0.3,0.9]\}, \\ & T\{[-0.4,-0.5],[-0.1,-0.7]\}, I\{[-0.1,-0.7],[-0.5,-0.5]\}, F\{[-0.5,-0.6],[-0.3,-0.9]\}, \\ & \{x_3/T\{[0.6,0.9],[0.2,0.5]\}, I\{[0.3,0.7],[0.4,0.6]\}, F\{[0.1,0.5],[0.5,0.8]\}, \\ & T\{[-0.5,-0.5],[-0.6,-0.3]\}, I\{[-0.4,-0.3],[-0.6,-0.3]\}, F\{[-0.5,-0.5],[-0.7,-0.4]\}\Big\} \end{aligned}$$

$$B_{INBV} = \Big\{x_1/T\{[0.1, 0.4], [0.3, 0.2]\}, I\{[0.2, 0.7], [0.4, 0.6]\}, F\{[0.6, 0.9], [0.8, 0.7]\},$$
$$T\{[-0.2, -0.6], [-0.4, -0.9]\}, I\{[-0.5, -0.5], [-0.3, -0.6]\}, F\{[-0.4, -0.8], [-0.1, -0.6]\},$$
$$\{x_2/T\{[0.4, 0.5], [0.1, 0.7]\}, I\{[0.2, 0.7], [0.4, 0.4]\}, F\{[0.5, 0.6], [0.3, 0.9]\},$$
$$T\{[-0.2, -0.6], [-0.2, -0.3]\}, I\{[-0.2, -0.6], [-0.5, -0.5]\}, F\{[-0.4, -0.8], [-0.7, -0.8]\},$$
$$\{x_3/T\{[0.4, 0.9], [0.2, 0.5]\}, I\{[0.1, 0.8], [0.2, 0.4]\}, F\{[0.1, 0.6], [0.5, 0.8]\},$$
$$T\{[-0.6, -0.9], [-0.2, -0.5]\}, I\{[-0.3, -0.7], [-0.4, -0.6]\}, F\{[-0.1, -0.4], [-0.5, -0.8]\}\Big\}$$

by using Definition 3.7 we obtain INBV union $H_{INBV} = A_{INBV} \cup B_{INBV}$ presented as follows:

$$A_{INBV} \cup B_{INBV} = H_{INBV}$$
$$= \Big\{x_1/T\{[0.2, 0.5], [0.3, 0.3]\}, I\{[0.1, 0.6], [0.3, 0.6]\}, F\{[0.5, 0.8], [0.7, 0.7]\},$$
$$T\{[-0.3, -0.6], [-0.4, -0.9]\}, I\{[-0.1, -0.6], [-0.3, -0.6]\}, F\{[-0.4, -0.7], [-0.1, -0.6]\},$$
$$\{x_2/T\{[0.4, 0.5], [0.1, 0.7]\}, I\{[0.1, 0.7], [0.4, 0.5]\}, F\{[0.1, 0.4], [0.3, 0.8]\},$$
$$T\{[-0.4, -0.6], [-0.2, -0.7]\}, I\{[-0.1, -0.6], [-0.5, -0.5]\}, F\{[-0.4, -0.6], [-0.3, -0.8]\},$$
$$\{x_3/T\{[0.6, 0.9], [0.2, 0.5]\}, I\{[0.1, 0.7], [0.2, 0.6]\}, F\{[0.1, 0.4], [0.5, 0.8]\},$$
$$T\{[-0.6, -0.9], [-0.6, -0.5]\}, I\{[-0.3, -0.3], [-0.4, -0.3]\}, F\{[-0.1, -0.4], [-0.5, -0.4]\}\Big\}$$

moreover, by using Definition 3.8 we obtained INBV intersection $D_{INBV} = A_{INBV} \cap B_{INBV}$ presented as follows:

$$A_{INBV} \cap B_{INBV} = D_{INBV}$$
$$= \Big\{x_1/T\{[0.1, 0.4], [0.2, 0.2]\}, I\{[0.2, 0.7], [0.5, 0.6]\}, F\{[0.6, 0.9], [0.8, 0.8]\},$$
$$T\{[-0.2, -0.5], [-0.3, -0.3]\}, I\{[-0.5, -0.6], [-0.3, -0.6]\}, F\{[-0.5, -0.8], [-0.7, -0.7]\},$$
$$\{x_2/T\{[0.4, 0.5], [0.1, 0.7]\}, I\{[0.2, 0.7], [0.5, 0.5]\}, F\{[0.5, 0.6], [0.3, 0.9]\},$$
$$T\{[-0.2, -0.5], [-0.1, -0.3]\}, I\{[-0.2, -0.7], [-0.5, -0.5]\}, F\{[-0.5, -0.8], [-0.7, -0.9]\},$$
$$\{x_3/T\{[0.3, 0.9], [0.2, 0.5]\}, I\{[0.3, 0.8], [0.4, 0.6]\}, F\{[0.1, 0.6], [0.5, 0.8]\},$$
$$T\{[-0.5, -0.5], [-0.2, -0.3]\}, I\{[-0.4, -0.7], [-0.6, -0.6]\}, F\{[-0.5, -0.5], [-0.7, -0.8]\}\Big\}$$

4. Conclusion

The new generalized concept of interval valued neutrosophic bipolar vague sets have been introduced and some remarkable properties like complement, union and intersection of interval valued neutrosophic bipolar vague sets have been investigated. The proposed concepts were illustrated with suitable examples. Further we can extend to investigate this concept into interval valued neutrosophic bipolar vague graphs with regular and isomorphic properties.

References

[1] Abdel-Basset M.,Saleh., Gamal A and Smarandache F., An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number, Applied Soft Computing, 77 (2019): 438-452.
[2] Akram M., Bipolar fuzzy graphs, Information Sciences, 181(24) (2011): 5548-5564.

[3] Broumi S., Deli I and Smarandache F., Neutrosophic refined relations and their properties, Neutrosophic refined relations and their properties Neutrosophic Theory and Its Applications,(2014): 228-248.
[4] Deli I and Broumi S., Neutrosophic soft relations and some properties, Annals of Fuzzy Mathematics and Informatics (AFMI), (2014): 1-14.
[5] Deli I., Ali M and Smarandache F., Bipolar neutrosophic sets and their application based on multi-criteria decision making problems, 2015 International Conference on Advanced Mechatronic Systems (ICAMechS), (2015): 249-254 DOI:10.1109/icamechs.2015.7287068.
[6] Hashim R., Gulistan M and Smarandache F., Applications of neutrosophic bipolar fuzzy sets in HOPE foundation for planning to build a children hospital with different types of similarity measures. Symmetry, 10(8) (2018): 331.
[7] Hashim H., Abdullah L and Al-Quran A., Interval Neutrosophic Vague Sets. Neutrosophic Sets and Systems, (25): 2019.
[8] Jun Y.B., Smarandache F., Song S.Z and Khan M., Neutrosophic positive implicative N-ideals in BCK-algebras, Axioms, 7(1) (2018): 1-3.
[9] Jun Y.B., Kim S and Smarandache F., Interval neutrosophic sets with applications in BCK/BCI-algebra. Axioms, 7(2) (2018): 1-23.
[10] Jun Y. B and Hur K., Bipolar-valued fuzzy subalgebras based on bipolar-valued fuzzy points. Annals of Fuzzy Mathematics and Informatics, 12(6) (2016), 893-902.
[11] Rashmanlou H., Jun Y. B and Borzooei R. A., More results on highly irregular bipolar fuzzy graphs. Annals of Fuzzy Mathematics and Informatics, 8(1) (2014): 149-168.
[12] Satham Hussain S., Jahir Hussain R and Smarandache F., On Neutrosophic Vague Graphs, Neutrosophic Sets and Systems, 2019 (Submitted).
[13] Satham Hussain S., Jahir Hussain R., Jun B Y and Smarandache F., Neutrosophic Bipolar Vague Set and its Application to Neutrosophic Bipolar Vague Graphs, Neutrosophic Sets and Systems, 2019 (Submitted).
[14] Smarandache F., A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press, 1999.
[15] Smarandache F., Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics, 24 (2005): 287-297.
[16] Ali M and Smarandache F., Complex Neutrosophic Set, Neural Computing and Applications, 28(7) 2017: 1817-1834.
[17] Ali M., Deli I and Smarandache F., The Theory of Neutrosophic Cubic Sets and Their Applications in Pattern Recognition, Journal of Intelligent and Fuzzy Systems, (In press).
[18] Broumi S and Smarandache F., Intuitionistic neutrosophic soft set, Journal of Information and Computer Science, 8(2) (2013): 130-140.
[19] Broumi S., Smarandache F., Talea M and Bakali A., Single Valued Neutrosophic Graphs: Degree,Order and Size, 2016 IEEE International Conference on Fuzzy Systems, 2016.
[20] Ulucay V., Deli I and Sahin M., Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making, Neural Computing and Applications, 29(3)(2018): 739-748.