EFFECT OF INTERPHASE MASS TRANSFER ON UNSTEADY CONVECTIVE DIFFUSION OF BLOOD FLOW THROUGH A POROUS MEDIUM WITH MAGNETIC FIELD.

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Abstract : The aim of the present study is to dispersion of solute undergoing an irreversible fist-order chemical reaction at the bounding walls of a parallel channel. Convection coefficient and dispersion coefficient are influenced by the couple stress parameter arising due to suspension in the fluid. The exchange coefficient arises mainly due to the interphase mass transfer and it is independent of the solvent fluid velocity. The wall catalyzed reaction also influence the convection and dispersion coefficients. The study of solute dispersion may be applied to understand the transport of drug(nutrients) in plasma in blood flow through porous medium. The boundary absorption plays an important role in cardiovascular flow. Results reveal that transport coefficients are enormously affected by wall absorption.

I Introduction

Biomagnetic Fluid Dynamics (BFD) is a relatively new area in fluid dynamics and during the last decades an extensive research work has been done on the fluid dynamics of biological fluids in the presence of magnetic field. In biological science, the investigation for diffusivity of nutrients, metabolic products, drugs and other solutes are of most important. In many situation material mixed in the blood reach to different parts of the body by the process of diffusion. There are a lot of applications of this research field in bioengineering and medicine and the research work in this subject is rapidly growing(Carlton et al.,(2001), Voltairas et al.,(2002) and Ganguly et al.,(2005)). Particularly, interphase mass transfer plays an important role in physiological situations. It is necessary to develop a technique for handling such problems, which involve interphase mass transport. Investigations on the blood flow characteristics have been done by Bali and Awasthi(2007). Several authors focused on dispersion to understand the transport of nutrients in blood and different artificial devices (Middleman(1972), Lightfoot(1974), Cooney(1976), Jayaraman et al.,(1981)). Taylor(1953,1954) studied the dispersion process in Newtonian flow and discussed the effective dispersion coefficient with respect to the average speed of the flow, the radius of the tube and molecular diffusion coefficient.

Dispersion of a non-uniform initial distribution in time-variable isothermal laminar flow in a tube with a first-order rate process at the tube wall is analyzed by Sankarasubramanian and Gill (1973). They studied miscible dispersion in laminar flow in a tube in the presence of interfacial transport due to an irreversible first-order reaction at the tube wall by an exact procedure. The new concept, namely the exchange coefficients and a general expression are derived showing the time-dependent nature of these coefficients. The exchange coefficient reflects the interphase process and it enables to determine the average concentration distribution in terms of tabulated functions. The analysis conducted was confined to the case of dispersion in a fully developed steady flow. Interphase mass transfer can be applied to physiological problems, where a first-order chemical reaction occurs at the tube wall. One such situation is transport of oxygen and nutrients to tissue cells and removal of metabolic waste products from tissue cells. It also takes place in pulmonary capillaries, where the carbon dioxide is removed from the blood and oxygen is taken up by the blood.

During the last decade broad research work has been done on the fluid dynamics of biological fluids under the influence of magnetic field. The artificial organs implanted or extracorporeal, designed utilizing metals cause various forms of blood damage due to lack of biocompatibility of smooth(rough) surfaces. It is hazardous because they produce stress leading to the force. This force eventually damages the erythrocytes (red blood cell) and leads to the loss of hemoglobin which is known as haemolysis. One of the most important reason for the blood damages may also be due to physiological and chemical reasons. Korchevskii et al.,(1965) studied the influence of magnetic field in human system with a motivation to regulate the movement of blood. Rudraiah et al.,(1988) described that self generated electric field reduces the concentration of erythrocytes and hence increase dispersion. Higashi et al.,(1993), Haik et al.,(2001) and Tzirtzilakis (2005) investigated the flow of biomagnetic fluid under the action of an applied magnetic field.

The objective of this paper is to consider the effects of couple stress and magnetic field on the unsteady convective diffusion with interphase mass transfer using the generalized dispersion model of Sankarasubramanian and Gill (1973). Convection coefficient K_1 and dispersion coefficient K_2 are influenced by the couple stress parameter arising due to suspension in the fluid, magnetic field and porous parameter. The exchange coefficient K_0 arises mainly due to the interphase mass transfer and it is independent of the solvent fluid velocity. The wall catalyzed reaction also influence the convection and dispersion coefficients. The study of solute dispersion may be applied to understand the transport of drug(nutrients) in blood flow through the porous medium. The boundary absorption plays an important role in cardiovascular flow. Interphase mass transfer occurs at the wall of permeable blood vessels.

II Mathematical Formulation

The blood flow is considered to be a steady, fully developed (unidirectional), incompressible and homogeneous fluid. The fluid is bounded by porous layers separated by a distance 2*h*. Flow region may be classified into two sub-regions: fluid film and porous tissue (Figure 1).

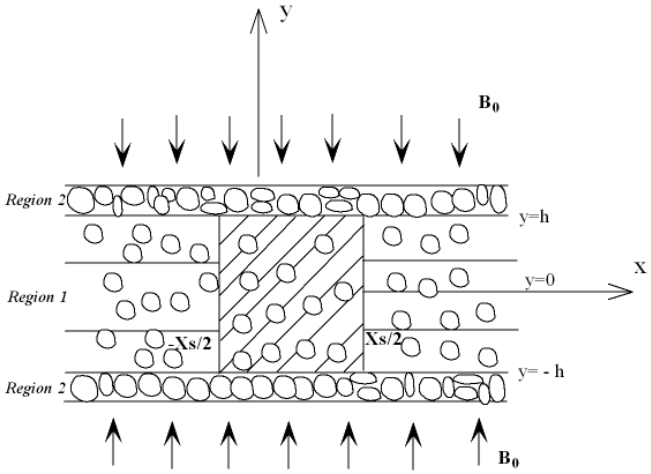

Figure 1: Physical configuration

In deriving the governing equations and the corresponding boundary conditions, the following assumptions are made:
(i) The induced magnetic field and the electric field produced by the motion of blood are negligible (since blood has low magnetic Reynolds number)
(ii) A uniform magnetic field B0 is applied in the y−direction to the flow of blood.
(iii) The solute diffuses in a fully developed flow through the porous medium in channel bounded by porous beds.
(iv) A slug is introduced for concentration C which is a function of time (t) and coordinates x and y .
Using above assumption the governing equations for incompressible flow of non-Newtonian fluid in cartesian coordinates are:

Region 1: Fluid Film Region

$$\frac{\partial u}{\partial x}=0$$

$$-\frac{\partial p^*}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}-\lambda\frac{\partial^4 u}{\partial y^4}-B_0^2\sigma_0 u-\frac{\mu}{k}u=0 \quad (1)$$

$$\frac{\partial p^*}{\partial y}=0 \quad (2)$$

The concentration C satisfying the convective diffusion equation gives

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \quad (3)$$

Region 2:Porous Tissue Region

$$\frac{\partial u_p}{\partial x}=0$$

$$-\frac{\partial p^*}{\partial x}-\frac{\mu}{k}(1+\beta_1)u_p=0 \quad (4)$$

$$\frac{\partial p^*}{\partial y} = 0 \tag{5}$$

The boundary conditions on the velocity are,

$$\frac{\partial u}{\partial y} = -\frac{\alpha}{\sqrt{k}}(u - u_p) \qquad at \qquad y = h \tag{6}$$

$$\frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{k}}(u - u_p) \qquad at \qquad y = -h \tag{7}$$

The couple stress conditions,

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad at \qquad y = \pm h \tag{8}$$

The initial and boundary conditions on concentration

With interphase mass transfer

The initial distribution assumed to be in a variable separable form given by

$$C = C_0 \psi_1(x) Y_1(y) \qquad at \qquad t = 0 \tag{9}$$

The heterogeneous reaction conditions are:

$$\left.\begin{aligned} -D\frac{\partial C}{\partial y}(t, x, h) &= K_s C \\ D\frac{\partial C}{\partial y}(t, x, -h) &= K_s C \end{aligned}\right\} \tag{10}$$

As the amount of solute in the system is finite,

$$C(t, \infty, y) = \frac{\partial C}{\partial x}(t, \infty, y) = 0 \tag{11}$$

where u is the x component of velocity, p^* is the pressure, μ is the viscosity of the fluid, λ is the couple stress parameter, B0 is the applied magnetic field, _0 is the electrical conductivity, t is the time, D is the molecular diffusivity. Equation (4) is the modified Darcy equation, modified in the sense of incompressible couple stress parameter _1 in to the Darcy equation, k is the permeability of the porous medium and up is the Darcy velocity, _ is the slip parameter, C0 is the reference concentration and Ks is the reaction rate constant catalyzed by the walls. Equations (6) and (7) are Beavers and Joseph(BJ)(1967) slip condition at the lower and upper permeable surfaces. Equation (8) specifies the vanishing of the couple stress.

Introducing the non-dimensional quantities

$$u = \frac{u^*}{\bar{u}},\ u_p = \frac{u_p^*}{\bar{u}},\ \eta = \frac{y}{h},\ \xi' = \frac{x}{h\,Pe},\ \xi_s = \frac{x_s}{h\,Pe},\ Pe = \frac{\bar{u}\,h}{D},\ p^* = \frac{p}{\rho\,\bar{u}^2},\ \tau = \frac{Dt}{h^2},\ \theta = \frac{C}{C_0},\ \beta = \frac{k_s h}{D}$$

Equations (1) to (5) in non-dimensional form are

Region 1:**Fluid Film Region**

$$\frac{\partial^4 U}{\partial \eta^4} - a^2\frac{\partial^2 U}{\partial \eta^2} + a^2(M^2 + a^2)U = a^2 P \tag{12}$$

and

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial \xi'} = \frac{1}{Pe^2}\left(\frac{\partial^2 \theta}{\partial \xi'^2} + \frac{\partial^2 \theta}{\partial \eta^2}\right) \tag{13}$$

we define the axial coordinate moving with the average velocity of flow as $x_1 = x - \tau \bar{u}$ which is in dimensionless form

$\xi = \xi' - \tau$, where $\xi' = \frac{x_1}{hPe}$. .

Then equation (13) becomes

$$\frac{\partial \theta}{\partial \tau} + U' \frac{\partial \theta}{\partial \xi'} = \frac{1}{Pe^2}\left(\frac{\partial^2 \theta}{\partial \xi'^2} + \frac{\partial^2 \theta}{\partial \eta^2}\right) \tag{14}$$

where, $U' = \frac{U - \bar{U}}{\bar{U}}$ (non-dimensional velocity in a moving coordinate system)

Region 2: Porous Tissue Region

$$U_p = \frac{P}{\sigma^2(1+\beta_1)} \tag{15}$$

The initial and boundary conditions (6) to (11) in dimensionless form

$$\frac{\partial U}{\partial \eta} = -\alpha\sigma(U - U_p) \qquad at \qquad \eta = 1 \tag{16}$$

$$\frac{\partial U}{\partial \eta} = \alpha\sigma(U - U_p) \qquad at \qquad \eta = -1 \tag{17}$$

The couple stress conditions,

$$\frac{\partial^2 U}{\partial \eta^2} = 0 \qquad at \qquad \eta = \pm 1 \tag{18}$$

With interphase mass transfer

$$\theta = \psi(\xi) Y(\eta) \qquad at \qquad \tau = 0 \tag{19}$$

The heterogeneous reaction conditions are:

$$\left.\begin{aligned} \frac{\partial \theta}{\partial \eta}(\tau, \xi, 1) &= -\beta\theta \\ \frac{\partial \theta}{\partial \eta}(\tau, \xi, -1) &= \beta\theta \end{aligned}\right\} \tag{20}$$

As the amount of solute in the system is finite,

$$\theta(\tau, \infty, \eta) = \frac{\partial \theta}{\partial \xi}(\tau, \infty, \eta) = 0 \tag{21}$$

where $a = \frac{h}{l_1}$ is the couple stress parameter, $l = \sqrt{\frac{\lambda}{\mu}}$ is the material constant characterizing the couple stress property of the fluid, $M^2 = \frac{B_0^2 \sigma_0 h^2}{\mu}$ is the square of the Hartmann number, $P = -\frac{\mathrm{Re}}{Pe}\frac{\partial p}{\partial \xi'}$, $\mathrm{Re} = \frac{\rho U h}{\mu}$ is the Reynolds number,

$Pe = \frac{\bar{u}\, h}{D}$ is the Peclet number, $\sigma = \frac{h}{\sqrt{k}}$ is the porous parameter, K_s is the reaction rate constant catalyzed by the walls.

III Method of solution

3.1 Velocity distribution

The solution to equation (12), satisfying the boundary conditions (16) to (18),
we obtain the velocity of blood as

$$u = 2C_1\, Coshm_1\eta + 2C_3\, Coshm_3\eta + \frac{P}{M^2 + \sigma^2} \tag{22}$$

where C1 ,C2 ,C3 and C4 are constants given in Appendix 1.
The normalized axial components of velocity is

$$U' = \frac{U - \bar{U}}{\bar{U}} = \frac{2}{A_1}\left[C_1 Coshm_1\eta + C_3 Coshm_3\eta - \left(\frac{C_1 Sinhm_1}{m_1} + \frac{C_3 Sinhm_3}{m_3} \right) \right] \tag{23}$$

where

$$\bar{U} = \frac{1}{2}\int_{-1}^{1} U(\eta) d\eta = \frac{2C_1 Sinhm_1}{m_1} + \frac{2C_3 Sinhm_3}{m_3} + \frac{P}{M^2 + \sigma^2} \tag{24}$$

Generalized Dispersion model

The solution of (14) is obtained using the generalized dispersion model of Gill and Sankarasubramanian(1970) formulated as a series expansion in the form

$$\theta(\tau, \xi, \eta) = \theta_m(\tau, \xi) + f_1(\tau, \eta)\frac{\partial \theta_m}{\partial \xi} + f_2(\tau, \eta)\frac{\partial^2 \theta_m}{\partial \xi^2} + \ldots \tag{25}$$

where, θ_m is the dimensionless cross sectional average concentration, given by

$$\theta(\tau, \xi) = \frac{1}{2}\int_{-1}^{1} \theta(\tau, \xi, \eta) d\eta \tag{26}$$

Integrating equation (14) with respect to _ in [−1, 1] and using the equation (55), we get

$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2}\frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{2}\int_{-1}^{1} \frac{\partial^2 \theta}{\partial \eta^2} d\eta - \frac{1}{2}\frac{\partial}{\partial \xi}\int_{-1}^{1} U' \theta \, d\eta \tag{27}$$

Substituting equation (25) in (27), we obtain

$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2}\frac{\partial^2 \theta}{\partial \xi^2} - \frac{1}{2}\frac{\partial}{\partial \xi}\int_{-1}^{1} U' \left(\theta_m(\tau, \xi) + f_1(\tau, \eta)\frac{\partial \theta_m}{\partial \xi} + f_2(\tau, \eta)\frac{\partial^2 \theta_m}{\partial \xi^2} + \ldots \right) d\eta \tag{28}$$

In this model we write

$$\frac{\partial \theta_m}{\partial \tau} = \sum_{k=0}^{\infty} K_k(\tau)\frac{\partial^k \theta}{\partial \xi^k} \tag{29}$$

where the dispersion coefficient, $K_k(\tau)$ Substituting the Equation (29) in (28) we obtain

$$K_0\theta_m + K_1(\tau)\frac{\partial \theta}{\partial \xi} + K_2(\tau)\frac{\partial^2 \theta}{\partial \xi^2} + K_3(\tau)\frac{\partial^3 \theta}{\partial \xi^3} + \ldots\ldots = \frac{1}{Pe^2}\frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{2}\left[\frac{\partial}{\partial \eta}\left(f_0\theta_m + f_1\frac{\partial \theta_m}{\partial \xi} \right) \right]_{-1}^{1}$$
$$- \frac{1}{2}\frac{\partial}{\partial \xi}\int_{-1}^{1} U' \left(f_0(\tau, \eta)\theta_m(\tau, \xi) + f_1(\tau, \eta)\frac{\partial \theta_m}{\partial \xi} + f_2(\tau, \eta)\frac{\partial^2 \theta_m}{\partial \xi^2} + \ldots \right) d\eta$$

Equating the coefficient $\frac{\partial \theta_m}{\partial \xi}, \frac{\partial^2 \theta_m}{\partial \xi^2}, \ldots,$. we get,

$$K_i(\tau) = \frac{\delta_{ij}}{Pe^2} + \frac{1}{2}\frac{\partial f_i}{\partial \xi}(\tau, 1) - \frac{1}{2}\int_{-1}^{1} U \, f_{i-1}(\tau, \eta) \, d\eta \qquad (i = 1,2,3,\ldots.) \tag{30}$$

where $f_{-1} = 0$

The exchange coefficient $K_0(\tau)$ accounts for the non-zero solute flux at the channel wall, and negative sign indicates the depletion of solute in the system with time caused by the irreversible reaction, which occurs at the channel wall. The presence of non-zero solute flux at the walls of the channel, also affects the higher order Ki due to the explicit appearance of $\frac{\partial f_1}{\partial \eta}(\tau, 1)$ in equation (30). Equation (30) can be truncated after the term involving K_2 without causing serious error, because $K_3, K_4,$, etc. become negligibly small compared to K_2.
The resulting model for the mean concentration is

$$\frac{\partial \theta_m}{\partial \tau} = K_0 \theta_m + K_1(\tau)\frac{\partial \theta}{\partial \xi} + K_2(\tau)\frac{\partial^2 \theta}{\partial \xi^2} \tag{31}$$

Substituting (25) in (14) and using the generalized dispersion model of Gill and Sankarasubramanian in the resulting equation, we get the equation for f_k from the differential equations of the form

$$\frac{\partial f_k}{\partial \tau} = \frac{\partial^2 f_k}{\partial \eta^2} - U' f_{k-1} + \frac{1}{Pe^2} f_{k-2} = \sum_{i=0}^{k} K_i f_{k-i} \qquad (k = 0,1,2,\ldots) \tag{32}$$

where, $f_{-1} = f_{-2} = 0$

Since θ_m is chosen to satisfy the initial and boundary conditions on θ from equations (19) to (20) conditions on the f_k function becomes

$$f_k = finite \quad at \quad \tau = 0 \tag{33}$$

$$\frac{\partial}{\partial \eta} f_k(\tau,1) = -\beta f_k \tag{34}$$

$$\frac{\partial}{\partial \eta} f_k(\tau,-1) = \beta f_k \tag{35}$$

Also, from equation (26) we have

$$\frac{1}{2}\int_{-1}^{1} f_k(\tau,\eta)d\eta = \delta_{k0}, (k = 0,1,2) \tag{36}$$

To evaluate $K_i's$, we need to know the $f_k's$'s which are obtained by solving (32) for $f_k's$ subject to the boundary conditions,

The function f_0 and the exchange coefficient K_0 are independent of the velocity and can be solved easily. Substituting k = 0 in equation (32) we get the differential equation for f_0 as

$$\frac{\partial f_0}{\partial \tau} = \frac{\partial^2 f_0}{\partial \eta^2} - f_0 K_0 \tag{37}$$

For i = 0 in (30) we have

$$K_0(\tau) = \frac{1}{2}\left[\frac{\partial f_0}{\partial \eta}\right]_{-1}^{1} - f_0 K_0 \tag{38}$$

A simultaneous solution has to be obtained from these two equations (37) and (38) with an initial condition for f_0 using (26) by taking $\tau = 0$ in that equation to get

$$\theta_m(0,\xi) = \frac{1}{2}\int_{-1}^{1} \theta_m(0,\xi,\eta)d\eta \tag{39}$$

Substituting $\tau = 0$ in (25) and setting $f_k(\eta) = 0\,(k = 1,2,3)$ gives the initial condition for f_0 as

$$f_0(0,\eta) = \frac{\theta(0,\xi,\eta)}{\theta_m(0,\xi)} \tag{40}$$

Left hand side of (40) is a function of η only and the right hand side is a function of both ξ and η . Thus the initial concentration distribution is a separable function of ξ and η . Substituting equation (34) and (35) into equation (40), we get

$$f_0(0,\eta) = \frac{\psi(\eta)}{\frac{1}{2}\int_{-1}^{1} \psi(\eta)d\eta} \tag{41}$$

The solution of the reaction diffusion equation (37) with these conditions are formulated as

$$f_0(\tau,\eta) = g_0(\tau,\eta)\exp\left[-\int_{-1}^{1} K_{0I}(\eta)d\eta\right] \tag{42}$$

from which it follows that $g_0(\tau,\eta)$ has to satisfy

$$\frac{\partial g_0}{\partial \tau} = \frac{\partial^2 g_0}{\partial \eta^2} \tag{43}$$

with conditions

$$f_0 = g_0 = \frac{\psi(\eta)}{\frac{1}{2}\int_{-1}^{1}\psi(\eta)d\eta} \quad at \quad \tau = 0 \tag{44}$$

$$g_0 = finite \quad at \quad \eta = 0 \tag{45}$$

$$\frac{\partial g_0}{\partial \eta} = -\beta g_0 \quad at \quad \eta = 1 \tag{46}$$

The solution of (43) subject to conditions (44) to (46) is

$$g_0(\tau,\eta) = \sum_{n=0}^{\infty} A_n e^{-\mu_n^2 \tau} Cos(\mu_n \eta) \tag{47}$$

where μ_n's are the roots of

$$\mu_n \tan \mu_n = \beta, \quad n = 0,1,2,\ldots \tag{48}$$

and An 's are given by

$$A_n = \frac{2\int_{-1}^{1}\psi(\eta)\cos\mu_n\eta\, d\eta}{\left(1+\frac{\cos 2\mu_n}{2\mu_n}\right)\int_{-1}^{1}\psi(\eta)d\eta} \tag{49}$$

From(42), it follows that

$$f_0(\tau,\eta) = \frac{2g_0(\tau,\eta)}{\int_{-1}^{1} g_0(\tau,\eta)d\eta} = \frac{\sum_{n=0}^{\infty} A_n e^{-\mu_n^2\tau} Cos(\mu_n\eta)}{\sum_{n=0}^{\infty}\frac{A_n}{\mu_n} e^{-\mu_n^2\tau} Sin\,\mu_n} \tag{50}$$

The first ten roots of the transcendental equation (48) are obtained using MATHEMATICA 8.0 and are given in Table 1. These ten roots ensure the convergence of the series in the expansions of f_0 and K_0. Having obtained f_0, we get K_0 from (43) in the form

$$K_0(\infty) = \frac{\sum_{n=0}^{\infty} A_n \mu_n e^{-\mu_n^2\tau} Sin(\mu_n\eta)}{\sum_{n=0}^{\infty}\frac{A_n}{\mu_n} e^{-\mu_n^2\tau} Sin\,\mu_n} \tag{51}$$

Here $K_0(\tau)$ is independent of velocity distribution.

As $\tau \to \infty$, we get the asymptotic solution for K_0 from (51) as

$$K_0(\infty) = -\mu_0^2 \tag{52}$$

where μ_0 is the first root of the equation (48). Physically, this represents first order chemical reaction coefficient to obtain $K_0(\infty)$. We get $K_1(\infty)$, from (30) (with i = 1) knowing $f_0(\infty,\eta)$ and $f_1(\infty,\eta)$. Likewise, $K_2(\infty), K_3(\infty),\ldots,$

require the knowledge of $K_0, K_1, f_0, f_1,$ and f_2 . Equation (50) in the limit $\tau \to \infty,$ reduces to

$$f_0(\infty,\eta) = \frac{\mu_0}{Sin\,\mu_0} Cos(\mu_0) \tag{53}$$

Then we find f_1 K_1, f_2 *and* K_2 . For asymptotically long times, i.e., $\tau \to \infty,$ equation (30) and (32) give $K_i's$ *and* $f_k's$ as

$$K_i(\tau) = \frac{\delta_{ij}}{Pe^2} - \beta f_i(\infty,1) - \int_{-1}^{1} U'\, f_{i-1}(\infty,\eta)\, d\eta \qquad (i = 1,2,3,\ldots.) \tag{54}$$

$$\frac{\partial^2 f_k}{\partial \eta^2} + \mu_0^2 f_k = (U'+K_1) f_{k-1} - \left(\frac{1}{Pe^2} - K_{2I}\right) f_{k-2}, \qquad (k = 1,2,\ldots) \tag{55}$$

The $f_k's$ must satisfy the conditions (26) and this permits the eigen function expansion in the form of

$$f_k(\infty,\eta) = \sum_{j=0}^{9} B_{j,k} Cos(\mu_j \eta), \quad k = 1,23,.. \tag{56}$$

Substituting (56) in (55) and multiplying the resulting equation by $\cos(\mu_j \eta)$ and integrating with respect to η from -1 to 1, gives

$$B_{j,k}\, Cos(\mu_j \eta) = \frac{1}{\mu_j^2 - \mu_0^2}\left[\frac{1}{Pe^2}\sum_{j=0}^{9} B_{j,k-2} Cos(\mu_j \eta) - U'\sum_{j=0}^{9} B_{j,k-1} Cos(\mu_j \eta) - \sum_{j=0}^{9} K_{iI} B_{j,k-i} Cos(\mu_j \eta)\right]$$

Multiplying by $Cos(\mu_j \eta)$ and integrating with respect to η , we get

$$B_{j,k} = \frac{1}{\mu_j^2 - \mu_0^2}\left[\frac{1}{Pe^2}\sum_{j=0}^{9} B_{j,k-2} - U'\sum_{j=0}^{9} B_{j,k-1} - \left(1 + \frac{Sin\,\mu_j}{2\mu_j}\right)^{-1} \sum_{j=0}^{9} B_{j,k-i}\, I(j,l)\right] \quad k = (1,2) \tag{57}$$

Where

$$I(j,l) = \int_{-1}^{1} U' Cos(\mu_j \eta) Cos(\mu_l \eta) d\eta = I(l,j) \tag{58}$$

$$B_{j,-1} = 0, B_{j,0} = 0 \qquad for \;\; j = 1 \;\; to \;\; 9 \tag{59}$$

The first expansion coefficient $B_{0,k}$ in equation (56) using conditions(33) to (36) can be expressed in terms of $B_{j,k}$ (j = 1 to 9) as, (Using the boundary condition $\int_{-1}^{1} f_k(\tau,\eta) d\eta = \delta_{k0} = 0$)

$$B_{0,k} = -\left(\frac{\mu_0}{\sin\,\mu_0}\right)\sum_{j=0}^{9} B_{j,k}\, \frac{Sin\,\mu_j}{2\mu_j} \quad k = (1,2,3,\ldots) \tag{60}$$

Further, from (52) and (56) we find that

$$B_{0,0} = \frac{\mu_0}{Sin\,\mu_0} \tag{61}$$

Substituting i = 1 in (54) and using (58), (59) and (61) in the resulting equation, we get

$$K_1(\infty) = -\frac{I(0,0)}{\left[1 + \frac{Sin\,2\mu_0}{2\mu_0}\right]} \tag{62}$$

Substituting i = 2 in (54) and using (57), (58) and (61) in the resulting equation, we get

$$K_2 = \frac{1}{Pe^2} - \frac{\sin\mu_0}{\mu_0\left(1+\frac{Sin\,2\mu_0}{2\mu_0}\right)}\sum_{j=0}^{9} B_{j,k-i}\, I_{j,0} \tag{63}$$

where $B_{j,1} = -\frac{1}{\mu_j^2-\mu_0^2}\left(1+\frac{Sin\,\mu_j}{2\mu_j}\right)^{-1}\frac{\mu_0}{Sin\,\mu_0} I(j,0)$

Using the asymptotic coefficients $K_0(\infty), K_1(\infty), and\ K_2(\infty)$, in (29), we determine the mean concentration distribution as a function of ξ, τ and the parameters a and β.

The initial condition for solving(29) can be obtained from (19) by taking the cross-sectional average. Making long time evaluations of the coefficients, its effect is independent of _m on the initial concentration distribution the solution of (29) with asymptotic coefficients can be written as

$$\theta_m(\tau,\xi) = \frac{1}{2Pe\sqrt{\pi K_2(\infty)\tau}}\exp\left[K_0(\infty)\tau - \frac{[\xi+K_1(\infty)\tau]^2}{4K_2(\infty)\tau}\right] \tag{64}$$

where $\theta_m(\tau,\infty)=0,\ \frac{\partial\theta_m}{\partial\xi}(\tau,\infty)=0$

IV Results and Discussion

Dispersion of solute in a couple stress fluid(blood) flow through a porous medium in rectangular channel bounded by porous beds with effects of magnetic field and heterogeneous chemical reaction are discussed. The walls of the channel act as catalysts to the reaction.

The most dominant dispersion coefficient $K_2(\tau) - Pe^{-2}$, convection coefficient $-K_1$ and mean concentration θ_m are computed for various values of Hartmann number (M = 0.5, 1, 1.5) , couple stress parameter a = (5, 10, 20), and porous parameter $\sigma = (100, 200, 300)$ and reaction rate parameter $(\beta = 10^{-2}, 1, 10^2)$ for fixed values $\xi_s = 0.019, \xi = 0.1, Pe = 100, \alpha = 0.10$ using MATHEMATICA 8.0 are displayed graphically in Figures 2 to 12 . The expression for absorption coefficient $-K_0(\infty)$ are numerically evaluated using equation (52) and absorption coefficient $-K_0(\infty)$ with β is shown in Figure 2 . It is evident that the $-K_0(\infty)$ increases with an increase in the wall reaction parameter β but it is without affected by Hartmann number, porous parameter and the couple stress parameter. If the absorption parameter takes very large values $(\beta = 100)$ the reaction at the wall consumes very rapidly than it can be supplied by molecular diffusion. Thus, there is more absorption of solutes at the wall in an annulus compared to the tubular flow.

Figure 3 to 8 illustrates that the variation of convection and dispersion coefficient drop with increasing the range of wall reaction parameter β. The expression for convection coefficient $K_1(\infty)$ are numerically evaluated using equation (62) and are shown in Figures 3 and 5 for various values of the Hartmann number(M) and porous parameter σ with wall reaction parameter β.From both the figures, it is observed that the increase in Hartmann number and porous parameter decreases the convection dispersion coefficient. Figure 4 shows that convection coefficient increases with an increase in couple stress parameter.

The expression for dispersion coefficient $K_2(\tau) - Pe^{-2}$ are numerically evaluated using equation (63) and are shown in Figures 6 and 7 for different values of the Hartmann number and couple stress parameter with wall reaction parameter. From both the figures, it is observed that the increase in Hartmann number and couple stress parameter decreases the axial dispersion coefficient. Figure 8 shows that dispersion coefficient increases with an increase in couple stress parameter. This result is useful in understanding the causes for haemolysis which in turn useful in the design of an artificial organ. In this figure, the results $\sigma \to \infty$ and $a \to \infty$ corresponds to those given by Rudraiah et al.,(1986). This is advantageous in maintaining the laminar flow.

Figures 9 to 11 depict the mean concentration θ_m with θ_m for different values of M, a and _ . Figures 9 and 10 observe that θ_m increases with increasing Hartmann number and couple stress parameter. Figure 11 shows that decrease in θ_m with increasing the value of σ for wall catalyzed reaction. This information is also useful in understanding the control of haemolysis. In Figure 12, we have plotted the mean concentration distribution θ_m against ξ for two values of the reaction rate parameter β. The variation clearly show that the peak of the distribution decreases as the wall reaction parameter β increases, which is mainly due to the increase of transverse transport of the solute. It is also observed that in all the cases the spread of the distribution starts at the same time but ends at an earlier time as β increases. This is because of the decrease in the magnitude of the mean concentration with the increased _ (Shashikala and Ranganatha(2008)).

The above results are relevant to understand several physiological processes such as dispersion of drugs and nutrients in the human circulatory system. This also has applications in artificial blood such as blood oxygenators. It is known that the blood flow in the human circulatory system is affected by several complexities that arise due to the elastic properties of the arterial wall, branching and curvature pulsatile flow, etc. Besides the influence of these complexities on the transfer of any passive species in blood stream, the non Newtonian character of the blood also plays a vital role on the transport (Ramana and Sarojamma(2012b)).

V Conclusion

In bioengineering problems, particularly in the mechanism of controlling haemolysis , the assumption of capillary bounded by rigid walls is unrealistic. There is transport of oxygen, proteins and other nutrients from capillaries to the permeable tissue. Therefore, the study involving the control of haemolysis it is important that the effect of couple stress, magnetic field and slip at the porous layer with interphase mass transfer have to be considered. Generalized model considering the solute dispersion in a non-Newtonian fluid with interphase mass transfer; as interfacial transport tends to zero $(\beta = 0)$, it reduce to that of no wall reaction.

The dispersion coefficient decreases with wall catalyzed. Apart from the above biomechanical applications of this study, certain general conclusion of a mathematical nature can also be made. They are

(i) The couple stresses are valid only for small value of ′a′ and the present results reduce to Newtonian fluid.

(ii) Taylor's dispersion model form a particular case of the generalized dispersion model for asymptotic values of τ . In other words, the generalized dispersion model reduces to Taylor's dispersion model asymptotically.

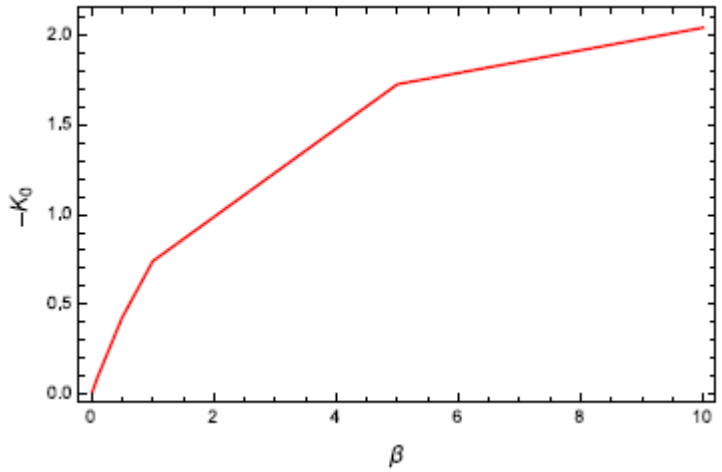

Figure 2: Variation of absorption coefficient $-K_0$ versus reaction rate parameter β

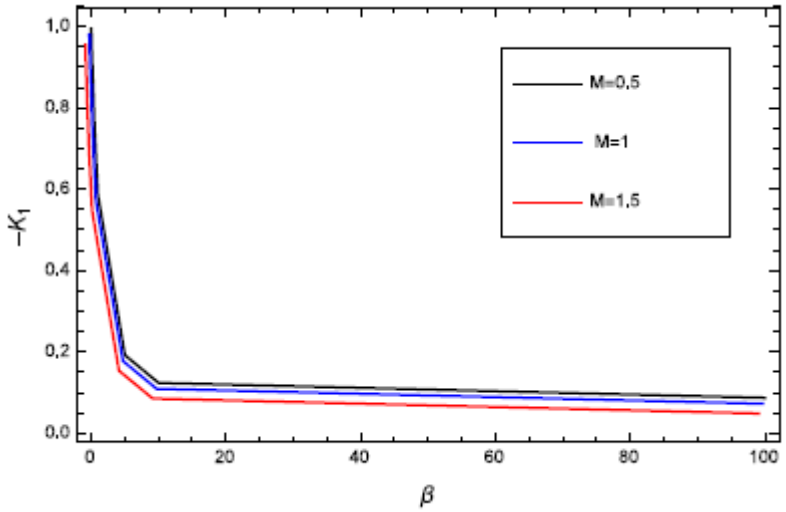

Figure 3: Variation of convection coefficient $-K_1$ versus β for different values of M

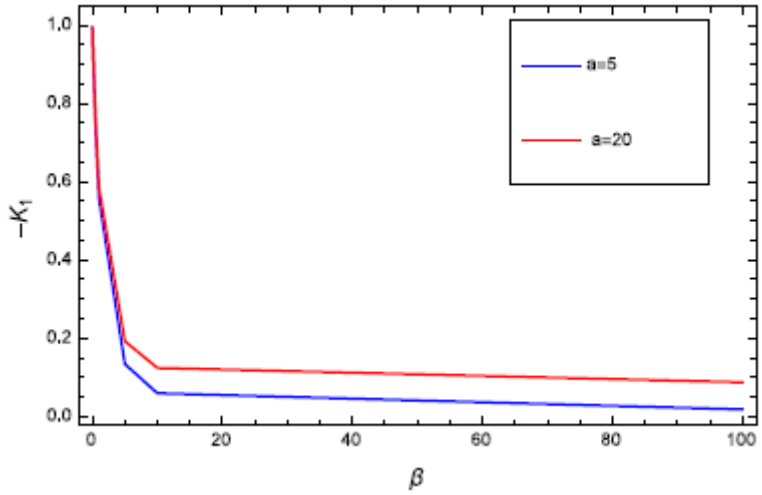

Figure 4: Variation of convection coefficient $-K_1$ versus β for different values of a

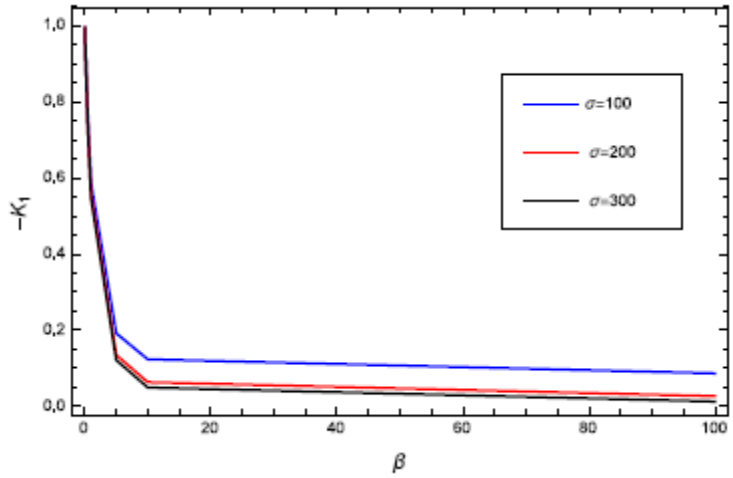

Figure 5: Variation of convection coefficient $-K_1$ versus β for different values of σ

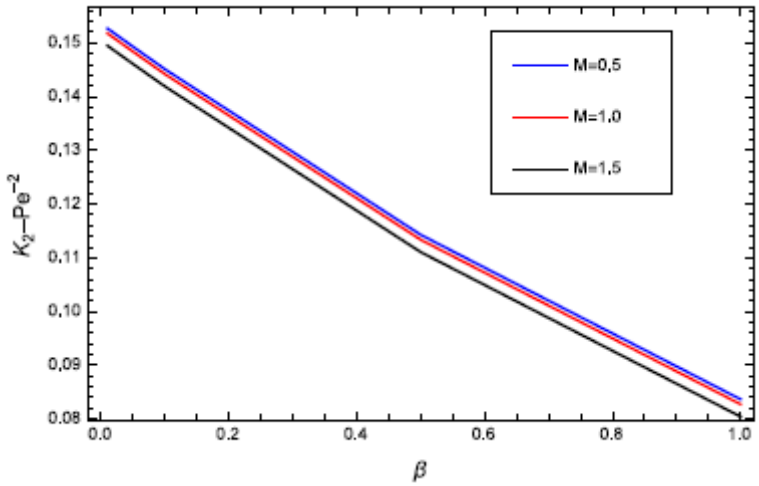

Figure 6: Variation of $K_2(\tau)-Pe^{-2}$ versus β for different values of M

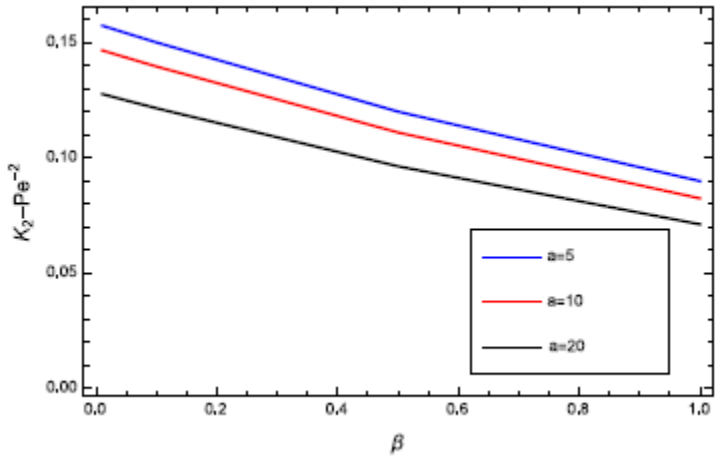

Figure 7: Variation of $K_2(\tau) - Pe^{-2}$ versus β for different values of a

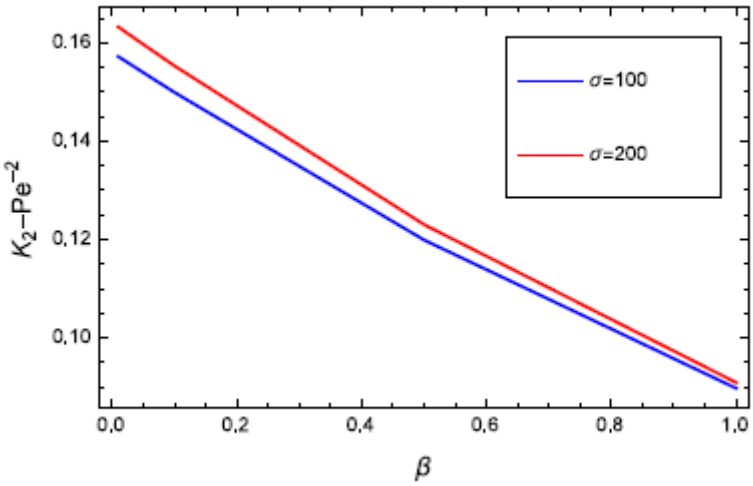

Figure 8: Variation of $K_2(\tau) - Pe^{-2}$ versus β for different values of σ

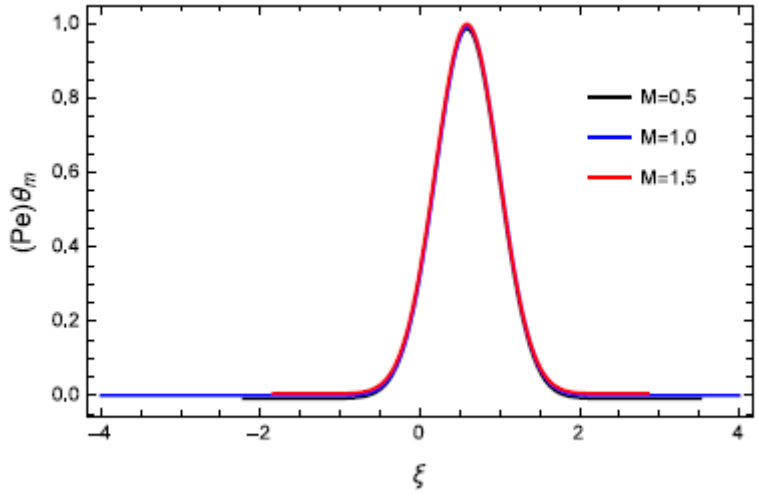

Figure 9: Variation of mean concentration θ_m versus ξ for different values of M

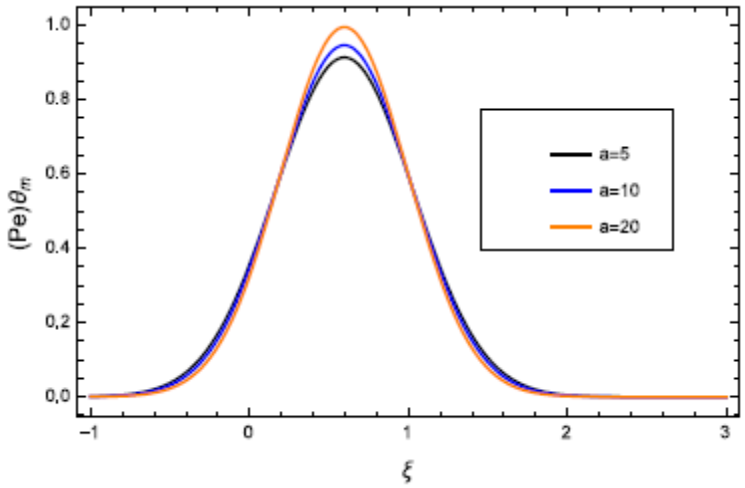

Figure 10: Variation of mean concentration θ_m versus ξ for different values of a

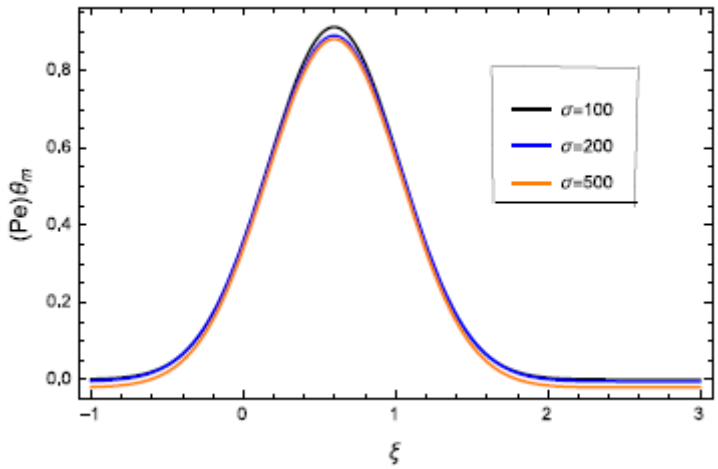

Figure 11: Variation of mean concentration θ_m versus ξ for different values of σ

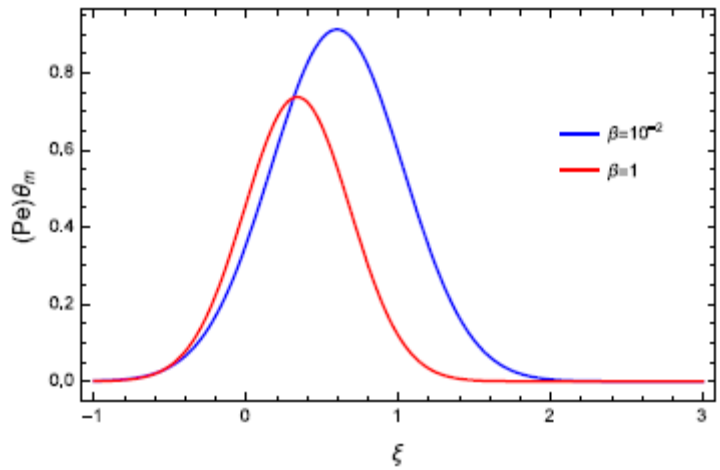

Figure 12: Variation of mean concentration θ_m versus ξ for different values of β

Table 1 Roots of the equation $\mu_n \tan \mu_n = \beta$

β	μ_0	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8	μ_9
10^{-2}	0.099834	3.14477	6.28478	9.42584	12.5672	15.7086	18.8501	21.9916	25.1331	28.2747
0.05	0.22176	3.15743	6.29113	9.43008	12.5703	15.7111	18.8522	21.9934	25.1347	28.2761
10^{-1}	0.311053	3.1731	6.29906	9.43538	12.5743	15.7143	18.8549	21.9957	25.1367	28.2779
0.5	0.653271	3.29231	6.36162	9.47749	12.606	15.7397	18.876	22.0139	25.1526	28.292
1.0	0.860334	3.42562	6.4373	9.52933	12.6453	15.7713	18.9024	22.2126	25.1724	28.3096
5.0	1.31384	4.03357	6.9096	9.89275	12.9352	16.0107	19.1055	22.2126	25.3276	28.4483
10.0	1.42887	4.3058	7.22811	10.2003	13.2142	16.2594	19.327	22.4108	25.5064	28.6106
100.0	1.55525	4.66577	7.77637	10.8871	13.9981	17.1093	20.2208	23.3327	26.445	29.5577

Appendix 1

$$m_1 = \frac{\sqrt{a^2 - \sqrt{a^4 - 4a^2(M^2 + \sigma^2)}}}{\sqrt{2}}$$

$$m_1 = \frac{\sqrt{a^2 - \sqrt{a^4 - 4a^2(M^2 + \sigma^2)}}}{\sqrt{2}}$$

$$a_3 = (m_1 + \alpha\sigma)e^{m_1}$$

$$a_4 = (m_1 - \alpha\sigma)e^{-m_1}$$

$$a_5 = (m_3 + \alpha\sigma)e^{m_3}$$

$$a_6 = (m_3 - \alpha\sigma)e^{-m_3}$$

$$a_7 = \left(\frac{P}{(M^2 + \sigma^2)} - \frac{P}{\sigma^2(1 + \beta_1)}\right)$$

$$a_8 = m_1^2 e^{m_1}$$

$$a_9 = m_1^2 e^{-m_1}$$

$$a_{10} = m_3^2 e^{m_3}$$

$$a_{11} = m_3^2 e^{-m_3}$$

$$A_1 = \frac{2C_1 \, Sinh \, m_1}{m_1} + \frac{2C_3 \, Sinh \, m_3}{m_3} + \frac{P}{(M^2 + \sigma^2)}$$

$$A_2 = \frac{C_1 \, Sinh \, m_1}{m_1} + \frac{C_3 \, Sinh \, m_3}{m_3}$$

$$A_3 = \frac{C_1 \, Sinh \, m_1}{m_1^3} + \frac{C_3 \, Sinh \, m_3}{m_3^3}$$

$$C_1 = C_2 = \frac{-a_7 a_{10} - a_7 a_{11}}{a_5 a_8 - a_6 a_8 + a_5 a_9 - a_6 a_9 - a_3 a_{10} + a_4 a_{10} - a_3 a_{11} + a_4 a_{11}}$$

$$C_3 = C_4 = \frac{a_7 a_8 + a_7 a_9}{a_5 a_8 - a_6 a_8 + a_5 a_9 - a_6 a_9 - a_3 a_{10} + a_4 a_{10} - a_3 a_{11} + a_4 a_{11}}$$

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