FUZZY SIMPLY $\ddot{g}$-LINDELÖF SPACE AND FUZZY $\ddot{g}$-BAIRE SPACE

Dr. S. ANJALMOSE[1]
Assistant Professor, PG & Research Department of Mathematics,
St. Joseph's College of Arts and Science (Autonomous),
Cuddalore, Tamil Nadu, India.

A. NITHYA[2]
Research Scholar, PG & Research Department of Mathematics,
St. Joseph's College of Arts and Science (Autonomous),
Cuddalore, Tamil Nadu, India.

ABSTRACT:

In this paper the concept of fuzzy simply $\ddot{g}$ - Lindelöf space are introduced. The relations between simply fuzzy simply $\ddot{g}$ - Lindelöf space and fuzzy $\ddot{g}$ - Baire space are discussed. Several examples are given to illustrate in this concept.

KEY WORDS:

Fuzzy simply open, fuzzy dense, fuzzy nowhere dense, fuzzy first category, fuzzy $\ddot{g}$ - Lindelöf space and fuzzy $\ddot{g}$ - Baire space.

1. INTRODUCTION:

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by L.A.ZADEH [9]. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. In this paper we introduce the concept of fuzzy simply $\ddot{g}$ - Lindelöf space are introduced. The relations between fuzzy simply $\ddot{g}$ - Lindelöf space and fuzzy $\ddot{g}$ - Baire space are discussed.

2. PRELIMINARIES:

In this section, we recall the basic definitions.

Definition 2.1 [8] If λ is a fuzzy open and fuzzy dense set in a fuzzy topological space (X, T), then λ is a fuzzy simply open set in (X, T).

Definition 2.2 [5] A fuzzy set λ in a fuzzy topological space (X, T), is called fuzzy dense if there exist no non-zero fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. (i.e) $cl(\lambda) = 1$

Definition 2.3 [2] A fuzzy set λ in a fuzzy topological space (X, T)is called fuzzy g¨-dense if there exists no fuzzy g¨-closed set μ in (X, T)such that λ < μ < 1 That is g¨-cl(λ) = 1.

Definition 2.4 [6] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exist no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$, i.e. $\operatorname{int} cl(\lambda) = 0$ in (X, T).

Definition 2.5 [2] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy g¨-nowhere dense if there exists no non-zero fuzzy g¨-open set μ in (X, T) such that μ < g¨-cl(λ) That is, g¨-i nt g¨-cl(λ) = 0.

Definition 2.6 [8] A fuzzy topological space (X,T) is said to be fuzzy simply Lindelöf if each cover of X by fuzzy simply open sets has a countable subcover. That is, (X,T) is a fuzzy simply Lindelöf space if α ϵ Δ {λα}= 1 where intcl [bd(λα)] = 0 in (X,T), then n ϵ N { λ_{α_n} }= 1 in (X,T).

Definition 2.7 [7] Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if, $\text{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ where λ_i's are fuzzy nowhere dense sets in (X,T).

Definition 2.8 [2] Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy g¨-Baire space if g¨-i nt $\ddot{g}-\text{int}\cup_{i=1}^{\infty}(\lambda_i) = 0$, where λ_i's are fuzzy g¨-nowhere dense sets in (X, T).

3. FUZZY SIMPLY $\ddot{g}$ - LINDELÖF SPACE:

Motivated by the concept of fuzzy Lindelöf space introduced in [6] we shall now define:

Definition 3.1. A fuzzy topological space (X,T) is said to be fuzzy simply $\ddot{g}$ -Lindelöf if each cover of X by fuzzy simply $\ddot{g}$ open sets has a countable sub cover. That is, (X,T) is a fuzzy simply $\ddot{g}$ -Lindelöf space if α ϵ Δ {λα}= 1 where int cl [bd(λα)] = 0 in (X,T), then n ϵ N { λ_{α_n} }= 1 in (X,T).

Example 3.2. Let X={a,b,c}. The fuzzy sets $\lambda, \mu, \delta, \alpha, \beta,$ and γ is defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.4; λ(b) = 0.5; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.6; μ(b) = 1 ; μ(c) = 0.7;
$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.5; δ(c) = 0.6;
$\alpha : X \to [0, 1]$ is defined as α(a) = 0.4; α(b) = 0.6;α(c) = 1
$\beta : X \to [0,1]$ is defined as β(a) = 0.7;β(b) = 1;β(c) = 0.8;
$\gamma : X \to [0, 1]$ is defined as γ(a) = 1;γ(b) = 0.5;γ(c) = 0.5

Then T = $\{0, \lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu), 1\}$ is fuzzy topology on X. On computation, we see that the fuzzy set $\{\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu)\}$ are fuzzy $\ddot{g}$ - open and fuzzy $\ddot{g}$ - dense sets in (X,T). Now $\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu)$ are fuzzy simply $\ddot{g}$ - open sets in (X,T). Also $\ddot{g}$ cl(α) = $\ddot{g}$ cl(β) = $\ddot{g}$ cl(γ) = $\ddot{g}$ cl(1-β) = 1; $\ddot{g}$ cl(1-γ) = 1- δ ; $\ddot{g}$ cl(1-α) = 1-λ; $\ddot{g}$ int(α) = λ; $\ddot{g}$ int(β) = 0; $\ddot{g}$ int(γ) = $\delta \vee (\lambda \wedge \mu)$; $\ddot{g}$ int(1-α) = $\ddot{g}$ int(1-β) = $\ddot{g}$ int(1-γ) = 0. Now $\ddot{g}$ int $\ddot{g}$ cl[$\ddot{g}$ cl(α) ∧ $\ddot{g}$ cl(1-α)] = 0, $\ddot{g}$ int $\ddot{g}$ cl[$\ddot{g}$ cl(β) ∧ $\ddot{g}$ cl(1-β)] = 1 $\neq$ 0, $\ddot{g}$ int $\ddot{g}$ cl[$\ddot{g}$ cl(γ) ∧ $\ddot{g}$ cl(1-γ)] = 0 and hence α, γ are fuzzy simply $\ddot{g}$ open sets in (X,T) and then β is not fuzzy $\ddot{g}$ simply open sets in (X,T). Now for the cover $\{\lambda, \delta, \lambda \vee \mu, \mu \vee \delta, \lambda \vee (\mu \wedge \delta)\}$ of X by simply $\ddot{g}$ open sets then $\{(\lambda \vee \delta) \vee (\mu \vee \delta)\}$ = 1 and hence for the cover $\{\lambda, \gamma, \lambda \vee \mu, \mu \vee \delta\}$ of X by simply $\ddot{g}$ open sets $\{\lambda \vee (\lambda \vee \mu) \vee \gamma\} = 1 =$ 1implies that (X,T) is a fuzzy simply $\ddot{g}$ - Lindelöf space.

4. FUZZY SIMPLY $\ddot{g}$ -LINDELÖF SPACE AND FUZZY $\ddot{g}$ -BAIRE SPACE:

A fuzzy simply $\ddot{g}$ -Lindelöf space is a fuzzy $\ddot{g}$ -Baire space. Consider the following example,

Example 4.1. Let X={a, b, c}. Consider the fuzzy sets λ ,μ and δ defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.4; λ(b) = 0.5; λ(c) =1;

$\mu : X \to [0, 1]$ is defined as μ(a) = 0.6; μ(b) = 1 ; μ(c) = 0.7;
$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.5; δ(c) = 0.6;
$\alpha : X \to [0, 1]$ is defined as α(a) = 0.4; α(b) = 0.6;α(c) = 1
$\beta : X \to [0,1]$ is defined as β(a) = 0.7;β(b) = 1;β(c) = 0.8;
$\gamma : X \to [0, 1]$ is defined as γ(a) = 1;γ(b) = 0.5;γ(c) = 0.5

$T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \mu \vee \delta, \lambda \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu), 1\}$. Now $1-\lambda, 1-\mu, 1-\delta, 1-\lambda \vee \mu, 1-\mu \vee \delta, 1-\lambda \vee \delta, 1-\lambda \wedge \mu, 1-\lambda \wedge \delta, 1-\mu \wedge \delta, 1-\lambda \vee (\mu \wedge \delta), 1-\mu \wedge (\lambda \vee \delta), 1-\delta \vee (\lambda \vee \mu)$ are fuzzy $\ddot{g}$ -nowhere dense sets in (X,T). Now $\text{int}[(1-\lambda) \vee (1-\mu) \vee (1-\delta) \vee (1-(\lambda \vee \mu)) \vee (1-(\mu \vee \delta)) \vee (1-(\lambda \vee \delta)) \vee (1-(\lambda \wedge \mu)) \vee (1-(\lambda \wedge \delta)) \vee (1-(\mu \wedge \delta)) \vee (1-(\lambda \vee (\mu \wedge \delta))) \vee (1-(\mu \wedge (\lambda \vee \delta))) \vee (1-(\delta \vee (\lambda \vee \mu)))] = \text{int}[1-(\lambda \wedge \delta)] = 0$. Hence (X,T) is a fuzzy $\ddot{g}$ -Baire space and by example 3.2 (X,T) is a fuzzy simply $\ddot{g}$ -Lindelöf space.

Therefore a fuzzy simply $\ddot{g}$ -Lindelöf space is a fuzzy $\ddot{g}$ -Baire space.

A fuzzy simply Lindelöf space need not be fuzzy Baire space. Consider the following example,

Example 4.2. Let X={a, b, c}. Consider the fuzzy sets λ ,μ and γ defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.4; λ(b) = 0.5; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.6; μ(b) = 1 ; μ(c) = 0.7;
$\gamma : X \to [0, 1]$ is defined as γ (a) = 1; γ (b) = 0.5; γ (c) = 0.6;

Let X = {a, b, c} and λ, μ, γ be the fuzzy sets defined on X as follows:
λ : X ! [0, 1] defined as λ (a) = 1; λ (b) = 0.2; λ (c) = 0.7.
μ : X ! [0, 1] defined as μ(a) = 0.3; μ(b) = 1; μ(c) = 0.2.
γ : X ! [0, 1] defined as γ (a) = 0.7; γ (b) = 0.4; γ (c) = 1.
Then T = {0, $\mu, (\mu \vee \gamma), (\mu \wedge \gamma), 1$ } is a fuzzy topology on X. Now
$cl(\lambda) = cl(\mu) = cl(\gamma) = cl(\lambda \vee \mu) = cl(\lambda \vee \gamma) = cl(\mu \vee \gamma) = 1$ therefore $\lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma$ are fuzzy open and fuzzy dense sets in (X,T) and
$\text{int}\, cl(\lambda \wedge 1-\lambda) = 0, \text{int}\, cl(\mu \wedge 1-\mu) = 0, \text{int}\, cl(\gamma \wedge 1-\gamma) = 0, \text{int}\, cl((\lambda \vee \mu) \wedge 1-(\lambda \vee \mu)) = 0,$
$\text{int}\, cl((\lambda \vee \gamma) \wedge 1-(\lambda \vee \gamma)) = 0, \text{int}\, cl((\mu \vee \gamma) \wedge 1-(\mu \vee \gamma)) = 0$
The cover (X,T) is $\{\lambda, \gamma, \lambda \vee \mu, \mu \vee \gamma\}$ then $\{\lambda \vee (\lambda \vee \mu) \vee (\mu \vee \gamma)\} = 1$ and the cover of (X,T) is $\{\mu \vee (\mu \vee \gamma) \vee \gamma\} = 1$. Hence (X,T) is fuzzy simply Lindelöf space.
Now $1-\mu, 1-(\mu \vee \gamma), 1-\gamma$ are fuzzy nowhere dense sets in (X,T), then
$\text{int}[(1-\mu) \vee 1-(\mu \vee \gamma) \vee (1-\gamma)] = \mu \wedge \gamma \neq 0$. Therefore (X,T) is not a fuzzy Baire space.

Therefore a fuzzy simply Lindelöf space is need not be fuzzy Baire space.

CONCLUSION:

In this paper we first defined fuzzy simply $\ddot{g}$ – Lindelöf space. Inter relations between fuzzy simply $\ddot{g}$ - Lindelöf space and fuzzy $\ddot{g}$ - Baire space. Several examples are given to illustrate the concepts introduced in this paper.

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