ANTI-MAGIC LABELINGS FOR UNICYCLIC AND STAR RELATED GRAPHS.

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ABSTRACT:

A Graph with "q" edges is called Anti-magic, if its edges can be labeled with 1,2,......,q without repetition such that the sum of the labels of the edges incident to each vertex are distinct. In this paper, we discuss about the anti-magic labelings for unicyclic and star related graphs.

KEYWORDS:

Anti-magic Labeling, Unicyclic, Star Graphs.

INTRODUCTION:

A Graph is a collection of nodes and lines that we call vertices and edges respectively. A Graph can be labeled or unlabeled. A Graph Labeling is an assignment of integers to the vertices and edges. Here in this paper we are interested in labeling a special types of graphs. In many labeled graphs, the labels are used for identification only. Labeling can be used not only to identify vertices and edges, but also to signify some additional properties depending on the particular labeling.

It may seem strange to term a graph as having an “Anti-magic” labeling, but the term comes from the connections to magic labelings and magic squares. Magic squares can trace their origin back to ancient china somewhere around the 7th century BC. A magic square is an arrangement of numbers into a square such that the sum of each row, column and diagonal are equal. The term “Anti-magic” then comes from being the opposite of magic or arranging numbers in such a way that no two sums are equal.

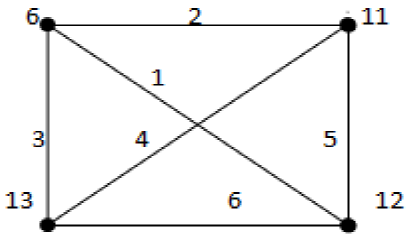
In 1990, Hartsfield and Ringel introduced the concept of anti-magic labeling, which is an assignment of distinct values to different objects in a graph in such a way that when taking certain sums of the labels, the sums will all be different. They had already conjectured that every graph except for K_2 has an anti-magic edge labeling. Then Bodendiek and Walther proved that from some natural number “n” any connected graph other than K_2 will have a weak anti-magic edge labeling if you allow the labels to be natural numbers with an upper bound of “n”. A weak anti-magic labeling is similar to an anti-magic labeling except one does not require bound of "n".

In this paper we discuss about the anti-magic labeling of unicyclic and star related graphs.

Definitions:

Anti-magic labeling:

A Graph with q edges is called Anti-magic, if its edges can be labeled with 1,2,…,q without repetition such that the sums of the labels of the edges incident to each vertex are distinct.



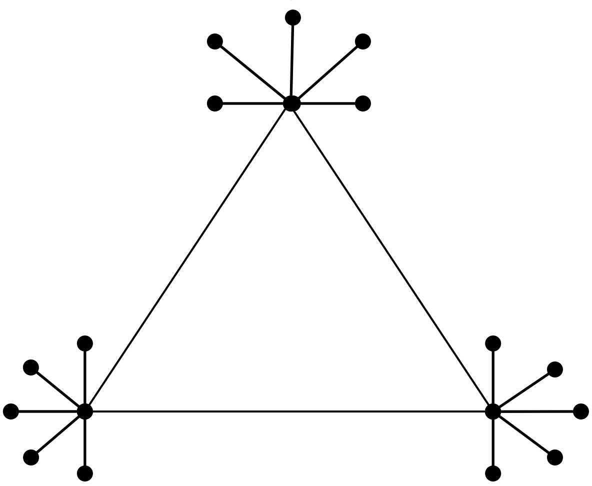
Antimagic labeling for K_4

Crown Graph:

A Crown graph is obtained by joining a pendant edge to each vertex of the cycle C_n.

Thorn Graph:

Thorn Graph is obtained by adding more than two pendant vertex to each vertex of the cycle.



Thorn Graph for K_3

Main Results:

Theorem 1:

The Crown Graph of cycle on “n” vertices is Anti-magic.

Proof:

Let $v_1, v_2, \ldots., v_n$ be the vertices of “n” cycles. Let $e_1, e_2, \ldots., e_n$ be the edges of “n” cycles.

Let us define the labeling function as $f: E(G) \to \{1,2, \ldots 2n\}$ where “n” is the number of Edges.

For (i=1 to q-1)

$u_i v_i$ $u_i$of degree 1, $v_i$of degree 3

$f(e_i) = i$

$u_i v_i$ u_i of degree 3, $v_i$of degree 3

$f(e_{i+1}) = i + 1$

$u_i v_i$ $u_i$of degree n, $v_i$of degree n

$f(e_{i+n}) = i + n$

For (i=1 to n)

$v_i$of degree 3

if i=1

$f(v_i) = f(e_1) + f(e_2) + f(e_{2n})$ for all i.

If i≠1

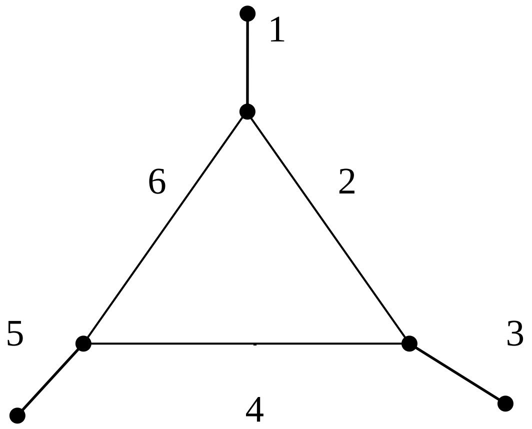
$f(v_i) = f(e_{2n}) + f(e_{2n-1}) + f(e_{2n-2})$ for all i

$v_i$of degree 1

$f(v_i) = f(e_i)$

$f(v_{i+1}) = f(e_{i+1})$

It is verified that, edges can be labeled with $1,2, \ldots, q$ without repetition such that the sums of the labels of the edges incident to each vertex are distinct.



Therefore, then anti-magic labeling for crown graph is verified.

Theorem:2

The Thorn Graph of cycle on n vertices is Anti-magic.

Proof:

Let $v_1, v_2, \dots, v_n$ be the vertices of "n" cycles. Let $e_1, e_2, \dots, e_n$ be the edges of "n" cycles.

Let us define the labeling function as $f: E(G) \to \{1,2, \dots 2n\}$ where "n" is the number of Edges.

For (i=1 to q-1)

$u_i v_i$ $u_i$of degree 1, $v_i$of degree 5

$f(e_i) = i$

$u_i v_i$ u_i of degree 5, $v_i$of degree 5

$f(e_{i+1}) = i + 1$

$u_i v_i$ u_i of degree n, $v_i$of degree n

$f(e_{i+n}) = i + n$

For (i=1to n)

v_i is of degree 5

If i=1

$f(v_i) = f(e_1) + f(e_2) + f(e_3) + f(e_4) + f(e_5)$

If i≠1

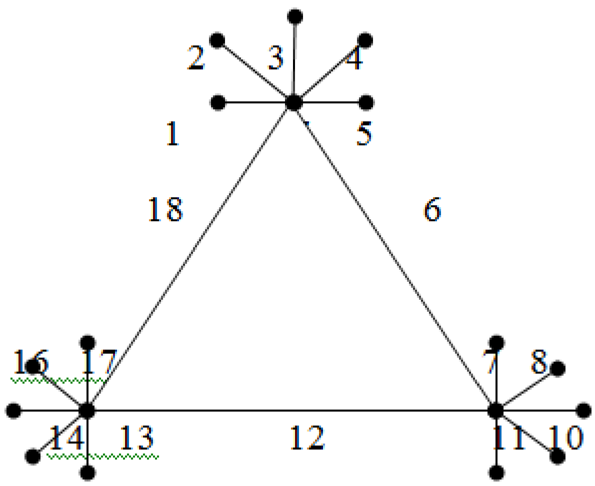
$f(v_i) = f(e_{2n}) + f(e_{2n+1}) + f(e_{2n+2}) + f(e_{2n+3}) + f(e_{2n+4})$

v_i is of degree 1

$f(v_i) = f(e_i)$

$f(v_{i+1}) = f(e_{i+1})$

It is verified that,edges can be labeled with $1,2, \ldots, q$ without repetition such that the sums of the labels of the edges incident to each vertex are distinct.



Therefore, the Anti-magic labeling for Thorn Graph is verified.

Conclusion:

In this paper, we have presented anti-magic labeling for some unicyclic and star related graphs.

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