Neutrosophic α-Baire Spaces

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Abstract: In this paper the concepts of neutrosophic α-Baire spaces are introduced and characterizations of neutrosophic α-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: *Neutrosophic α-open set, Neutrosophic α-nowhere dense set, Neutrosophic α-first category, Neutrosophic α-second category, Neutrosophic α-Baire spaces,*

1. Introduction

The fuzzy set was introduced by L.A. Zadeh [15] in 1965, where each element had a degree of membership. The Intuitionstic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [2, 3,4] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. The fuzzy topological space was introduced by C.L.Chang in 1968[6]. The idea of "neutrosophic set" was first given by Smarandache [9,10]. Neutrosophic operations have been investigated by A.A.Salama at el. [1]. A.A.Salama and S.A.Alblowi presented the concept of Neutrosophic Topological Spaces[12].The concept of Neutrosophic α-open sets was given by I. Arokiarani and R. Dhavaseelan [5].The concept of Baire space in fuzzy setting was introduced and studied by G.Thangaraj and S. Anjalmose [14].The idea of neutrosophic Baire spaces are introduced by R. Dhavaseelan, S. Jafari ,R. Narmada Devi, Md. Hanif [8].

2. Preliminaries

Now we introduce some basic notions and results that are used in the sequel.
In this work by a neutrosophic topological space we shall mean a non-empty set X together with a neutrosophic topology T (in the sense of Chang) and denote it by (X, T). The interior, closure and then complement of a neutrosophic set A will be denoted by int(A), cl(A) and 1-A (or $\bar{A}$) respectively.

Definition 1.1. [5] A neutrosophic topology (NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N , 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$.

(iii)$\cup G_i$ for arbitrary family $\{G_i | \, i \in \wedge \}$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic Topological Space (briefly NTS) and each Neutrosophic set in T is called a neutrosophic open set (briefly NOS). The complement A of a NOS A in X is called a neutrosophic closed set (briefly NCS) in X.

Definition 1.2. [5] Let A be a neutrosophic set in a neutrosophic topological space X. Then

Nint(A) = $\cup \{G \mid G \text{ is neutrosophic open set in } X \text{ and } G \subseteq A\}$ is called the neutrosophic interior of A;

Ncl(A) = $\cap \{G \mid G \text{ is neutrosophic closed set in } X \text{ and } G \supseteq A\}$ is called the neutrosophic closure of A.

Definition 1.3:[5] A neutrosophic set A in a neutrosophic topological space X is said to a Neutrosophic α-Open set(NSOS) if$A \subseteq Nint(Ncl(Nint(A)))$ and Neutrosophic α-Closed set (NSCS) if $Ncl(Nint(Ncl(A))) \supseteq A$

Definition 1.4:[5] Let A be a neutrosophic set in a neutrosophic topological space X. Then

Nαint(A) = $\cup\{G \mid G\ is\ neutrosophic\ \alpha - open\ set\ in\ X\ and\ G \subseteq A\}$ is called the neutrosophic interior of A;

Nαcl(A) = $\cap\{G \mid G\ is\ neutrosophic\ \alpha - closed\ set\ in\ X\ and\ G \supseteq A\}$ is called the neutrosophic closure of A;

Result: 1.5 Let A be a neutrosophic set in a neutrosophic topological space X. Then

Nαcl(A) = $A \cup Ncl(Nint(Ncl(A)))$

Nαint(A) = $A \cap Nint(Ncl(Nint(A)))$

2. Neutrosophic α-nowhere dense sets

Definition:2.1: A neutrosophic set A in neutrosophic topological space (X, T) is called neutrosophic α-dense if there exists no neutrosophic α-Closed set B in (X, T) such that $A \subset B \subset 1_N$.

That is $N\alpha cl(A) = 1_N$

Example 2.1: Let X = {a, b}. Define the Neutrosophic set A, B and C on X as follows:

$$A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}\right) \right\rangle$$

$$B = \left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.2}\right), \left(\frac{a}{0.6}, \frac{b}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}\right) \right\rangle$$

$$C = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right) \right\rangle$$

Then the families $T = \{0_N, 1_N, A, B, A \cup B\}$ is neutrosophic topology on X. Thus (X, T) is a Neutrosophic topological space. Now the sets $B, A \cup B$ & C are neutrosophic α-dense set in (X, T).

Definition 2.2 A neutrosophic set A in neutrosophic topological space (X, T) is called neutrosophic α-nowhere dense if there exists no non-zero neutrosophic α -open set B in (X, T) such that $B \subset N\alpha cl(A)$. That is $N\alpha\, \text{int}(N\alpha cl(A)) = 0_N$

In **Example 2.1:** The sets $\overline{B}\ and\ \overline{A \cup B}$ are neutrosophic α-nowhere dense set in (X, T).

Proposition 2.1: If A is a Neutrosophic α-nowhere dense set in (X, T), then $\overline{A}$ is a Neutrosophic α-dense set in (X, T)

Proposition 2.2: If A is be a Neutrosophic α - closed set in (X, T), then A is a neutrosophic α-nowhere dense set in (X, T) if and only if $N\alpha\, \text{int}(A) = 0_N$

Proof: If A is non zero neutrosophic α - closed set in (X, T) then $N\alpha cl(A) = A$ .If $N\alpha\, \text{int}(A) = 0_N$ then $N\alpha\, \text{int}(N\alpha cl(A)) = N\alpha\, \text{int}(A) = 0_N$. So A is a neutrosophic α-nowhere dense set in (X, T). Conversely let A is a neutrosophic α-nowhere dense set in (X, T). Then $N\alpha\, \text{int}(N\alpha cl(A)) = 0_N$.

Which implies that $N\alpha\, \text{int}(A) = N\alpha\, \text{int}(N\alpha cl(A) = 0_N$, since A is a Neutrosophic α-closed, $N\alpha cl(A) = A$.

Proposition 2.3: Let (X, T) be a neutrosophic topological space in (X, T), then every neutrosophic nowhere dense set is neutrosophic α-nowhere dense set in (X, T).

Proof: Let A be a neutrosophic nowhere dense and non zero closed set in (X, T), then $Nint(Ncl(A) = 0_N$ $\Rightarrow Ncl(N\,\text{int}(Ncl(A))) = 0_N \Rightarrow A \cup Ncl(N\,\text{int}(Ncl(A))) = A \cup 0_N = A$. So $N\alpha cl(A) = A$ which implies A

is neutrosophic α-closed set. Now from $N\alpha cl(A) = A \Rightarrow N\,\text{int}(N\alpha cl(A)) = N\,\text{int}(A) = 0_N \Rightarrow$ $Ncl(N\,\text{int}(N\alpha cl(A))) = Ncl(0_N) = 0_N \Rightarrow N\,\text{int}(Ncl(N\,\text{int}(N\alpha cl(A)))) = N\,\text{int}(0_N) = 0_N \Rightarrow$ $N\alpha cl(A) \cap N\,\text{int}(Ncl(N\,\text{int}(N\alpha cl(A)))) = N\alpha cl(A) \cap 0_N = A \cap 0_N = 0_N \Rightarrow N\alpha\,\text{int}(N\alpha cl(A)) = 0_N$.
Hence A is neutrosophic α-nowhere dense set in (X, T).

Definition 2.3: Let (X, T) be a neutrosophic topological space. A neutrosophic set A in (X, T) is called Neutrosophic α-first category if A = $\bigcup_{i=1}^{\infty} A_i$ where A_i's are neutrosophic α-nowhere dense sets in (X, T).

Example 2.2: Let X = {a, b}. Define the neutrosophic set A, B ,C and D on X as follows:

$$A = \left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right)\right\rangle$$

$$B = \left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right)\right\rangle$$

$$C = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.5}\right)\right\rangle$$

$$D = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}\right)\right\rangle$$

Then the families $T = \{0_N, 1_N, A, B\}$ is neutrosophic topology on X. Thus (X, T) is a neutrosophic topological space. Now the sets $\overline{A}, \overline{B}, C, D$ are neutrosophic α-nowhere dense set and $\left[\overline{A} \cup \overline{B} \cup C \cup D\right] = \overline{B}$ is neutrosophic α-first category set in (X, T)

Proposition 2.4: If A be a neutrosophic first category set in (X; T), then $\overline{A} = \bigcap_{i=1}^{\infty} B_i$ where $N\alpha cl(B_i) = 1_N$:

Proof: Let A be a neutrosophic α-first category set in (X,T). Then A = $\bigcup_{i=1}^{\infty} A_i$, where Ai's are neutrosophic α-nowhere dense sets in (X, T).Now $\overline{A} = \overline{\bigcup_{i=1}^{\infty} A_i} = \bigcap_{i=1}^{\infty} \overline{A_i}$. Now A_i is a Neutrosophic α -nowhere dense set in (X, T). Then, by **Proposition 2.1**, we have $\overline{A_i}$ is a neutrosophic dense set in (X, T). Let us put $B_i = \overline{A_i}$. then $\overline{A} = \bigcap_{i=1}^{\infty} B_i$ where $N\alpha cl(B_i) = 1_N$.

3. Neutrosophic α-Baire space

Motivated by the concept of neutrosophic Baire space introduced in [9] we shall now define:

Definition 3.1. Let (X, T) be a neutrosophic topological space. Then (X, T) is called a Neutrosophic α-Baire space if $N\alpha\,\text{int}\left(\bigcup_{i=1}^{\infty} A_i\right) = 0_N$, where A_i 's are Neutrosophic α- nowhere dense sets in (X,T).

In **Example 2.2**: The sets $\overline{A}, \overline{B}, C, D$ are neutrosophic α-nowhere dense set and Nαint$\left[\overline{A} \cup \overline{B} \cup C \cup D\right]$= N$\alpha$int ($\overline{B}$) = 0_N is neutrosophic α-Baire space.

Definition 3.2: Let A be a Neutrosophic α-first category set in a Neutrosophic topological space (X, T). Then 1 - A is called a Neutrosophic α-residual set in (X, T).

Proposition 3.1: Let (X, T) be a neutrosophic topological space. The$\bigcup_{i=1}^{\infty} A_i$ n the following are equivalent:

(1). (X, T) is a neutrosophic α-Baire space.
(2). Nαint (A) = 0_N for every neutrosophic α first category set A in (X, T).
(3). Nαcl (B) = 1_N for every neutrosophic α -residual set in (X, T).

Proof: (1) → (2). Let λ be a Neutrosophic α -first category set in (X, T). Then A =$\bigcup_{i=1}^{\infty} A_i$, where A_i's are neutrosophic α -nowhere dense sets in (X, T). Now Nαint (A) = Nαint ($\bigcup_{i=1}^{\infty} A_i$) = 0_N (since (X, T) is a neutrosophic α -Baire space). Therefore Nαint(A) = 0_N.

(2) → (3). Let B be a neutrosophic semi-residual set in (X, T). Then 1-B is a neutrosophic α -first category set in (X, T). By hypothesis, Nαint (1-B) = 0_N which implies that 1- Nαcl (B) = 0_N.
Hence we have Nαcl (B) = 1_N.

(3)→ (1). Let A be a neutrosophic α-first category set in (X, T). Then A = $\bigcup_{i=1}^{\infty} A_i$ where A_i's are neutrosophic α -nowhere dense sets in (X, T). 1- A is a neutrosophic α -residual set in (X, T). Since A is a Neutrosophic α -first category set in (X, T), By hypothesis, we have Nαcl (1- A) = 1_N. Then 1- Nαint (A) = 1_N, which implies that Nαint(A) = 0_N. Hence Nαint ($\bigcup_{i=1}^{\infty} A_i$) = 0_N where A_i's are neutrosophic α -nowhere dense sets in (X,T). Hence (X, T) is a neutrosophic α-Baire space.

Proposition 3.2 Every neutrosophic α-Baire space need not to be a Baire space.

Example 3.1: Let X = {a, b}. Define the Neutrosophic set A, B,C and D on X as follows:

$$A=\left\langle x,\left(\frac{a}{0.5},\frac{b}{0.2}\right),\left(\frac{a}{0.5},\frac{b}{0.2}\right),\left(\frac{a}{0.3},\frac{b}{0.4}\right)\right\rangle, \quad B=\left\langle x,\left(\frac{a}{0.3},\frac{b}{0.4}\right),\left(\frac{a}{0.3},\frac{b}{0.4}\right),\left(\frac{a}{0.6},\frac{b}{0.5}\right)\right\rangle$$

$$C=\left\langle x,\left(\frac{a}{0.3},\frac{b}{0.4}\right),\left(\frac{a}{0.3},\frac{b}{0.4}\right),\left(\frac{a}{0.2},\frac{b}{0.5}\right)\right\rangle, \quad D=\left\langle x,\left(\frac{a}{0.3},\frac{b}{0.4}\right),\left(\frac{a}{0.5},\frac{b}{0.4}\right),\left(\frac{a}{0.5},\frac{b}{0.2}\right)\right\rangle$$

Then the family $T=\{0_N,1_N,A,B,A\cup B\}$ is neutrosophic topology on X. Thus (X, T) is a neutrosophic topological space. The sets $\overline{A}, \overline{A\cup B}$ are Neutrosophic semi α-nowhere dense and also nowhere dense sets in (X, T). Here $N\alpha\,\mathrm{int}[(\overline{A})\cup(\overline{A\cup B})]=N\alpha\,\mathrm{int}(D)=0_N$, and $N\,\mathrm{int}[(\overline{A})\cup(\overline{A\cup B})]=N\,\mathrm{int}(D)\neq 0_N$ Hence the neutrosophic topological space (X, T) is neutrosophic α-Baire space but it is not neutrosophic Baire space.

Proposition 3.3: If A = $\bigcup_{i=1}^{\infty} A_i$ where A_i's are Neutrosophic nowhere dense sets in (X, T) then neutrosophic α-Baire space will be a neutrosophic Baire space.
Proof:
In **Example 2.2** the sets $\overline{A},\overline{B},C,D$ are neutrosophic nowhere dense set in (X, T) and $\left[\overline{A}\cup\overline{B}\cup C\cup D\right]=\overline{B}$.
Here Nint$\left[\overline{A}\cup\overline{B}\cup C\cup D\right]$=Nint$(\overline{B})=0_N$ and also N αint$\left[\overline{A}\cup\overline{B}\cup C\cup D\right]$=N$\alpha$int$(\overline{B})=0_N$

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