ANALYSES & DESIGN OPTIMIZATION OF PARABOLIC ROCKET NOZZLE USING TAGUCHI METHOD

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Abstract: Convergent-Divergent (C-D) nozzle is a flow device which usually being used in rocket engines which converts chemical energy into kinetic energy in order to propel the rocket. Commonly the C-D nozzle have two major types of contours namely conical and parabolic. The study deals with geometrical optimization of parabolic nozzle using Taguchi Methodology which focuses on improvement in quality and making products and processes less sensitive towards manufacturing and environmental variations. This method derives the optimum conditions by employing two significant approaches known as S/N (Signal-to-Noise) Ratio and Analysis of Variance (ANOVA), which are used to comprehend the significance of process variables and regression models are developed for predicting the appropriate contour of nozzle. This is a 3-Levels and 3-Factors study considering the area ratio (A_{Exit}/A_{Throat}), angle ratio (θ_n/θ_e) and length ratio ($L_{Divergent}/L_{Convergent}$) as study factors which resulted in maximization of Thrust and Mach number of nozzle for optimized geometrical parameters. Software tool for optimization used here is Minitab18.

Index Terms: C-D Nozzle, Optimization, Minitab18, Taguchi Methodology, Orthogonal Arrays, ANOVA, S/N Ratio, Regression Model, ANSYS Fluent, Contour plots, Supersonic velocities.

I. Introduction

Between 1970 and 2000, the cost to launch a kilogram to space remained fairly steady, with an average of US$18,500 per kilogram. When the space shuttle was in operation, it could launch a payload of 27,500 kilograms for $1.5 billion, or $54,500 per kilogram. For a SpaceX Falcon 9, the rocket used to access the ISS, the cost is just $2,720 per kilogram. The cost has been a major hurdle limiting access to space. Since the 1950s, the high cost of a space program has traditionally put it beyond the reach of most countries. Today, state and private actors alike have ready access to space. SpaceX has particularly focused on recovering key parts of the Falcon 9 to enhance reusability and reduce costs[1]. Optimization is another novel approach that primarily aims on cost reduction of these missions. Optimization is indispensable in many different aspects of any space mission. Space missions are continuously becoming more complex requiring the solution of increasingly hard optimization problems. Some of the many areas where optimization is applicable are Mission analysis and trajectory planning, Cargo loading and unloading, System and Sub-system design, Ergonomic aspects, Cost and revenue management etc. For instance, optimization in System and Sub-system design deals with problems associated with structural, thermal, avionic, power, navigation, control design.
Objectives of this paper are:

1. To establish a relationship between the input factors in the study – Area Ratio, Angle Ratio, Length Ratio and response factor – Thrust, Mach number.
2. To find out the optimum nozzle parameters in order to achieve better Thrust and an improvised Exit Mach Number using Conventional Taguchi Robust Design Method.
3. Re-verification of the flow properties of Thrust Optimized Parabolic Nozzle geometry using CFD Analysis.

1.1 Basics of Rocket Propulsion and C-D Nozzle

A rocket can be considered as a large body carrying small units of propellant and travelling with a velocity V. The reaction due to expelling the propellant from the rocket exhaust causes the velocity of the rocket to increase.

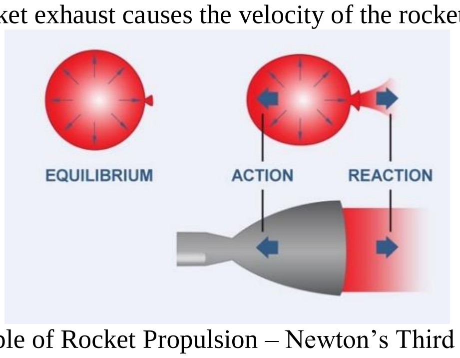

Figure 1: Principle of Rocket Propulsion – Newton's Third Law of Motion.

$$V_f - V_i = V_e \ \ ln\left(\frac{M_i}{M_f}\right) \tag{1}$$

where
V_i = The initial velocity of the rocket
V_f = The final velocity of the rocket
ln = The natural logarithmic function
M_i= The initial mass of the rocket including its payload all its propellant
M_f= The final mass of the rocket and its payload including its remaining propellant
$\frac{M_i}{M_f}$ = The rocket's Mass Ratio
This is known as Tsiolkovsky's Equation or Rocket Equation [4].

A nozzle is a device designed to control the direction or characteristics of a fluid flow (especially to increase velocity) as it exits (or enters) an enclosed chamber or pipe. A nozzle is often a pipe or tube of varying cross-sectional area, and it can be used to direct or modify the flow of a fluid (liquid or gas). Nozzles are frequently used to control the rate of flow, speed, direction, mass, shape, and/or the pressure of the stream that emerges from them. In a nozzle, the velocity of fluid increases at the expense of its pressure energy. Frequently, the goal of a nozzle is to increase the kinetic energy of the flowing medium at the expense of its pressure and internal energy [5].

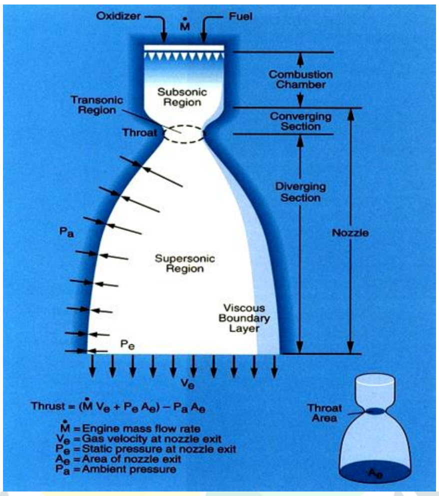

Figure 2: Principle of CD Nozzle

II. Research Methodology

2.1 Computational Fluid Dynamics (CFD)

CFD is dedicated to the study of fluids in motion and how the fluid flow behavior influences processes that may include heat transfer and possibly chemical reactions in combusting flows. A complete analysis which appears in CFD codes, including ANSYS FLUENT, consists of three main elements: pre-processor, solver and post-processor.

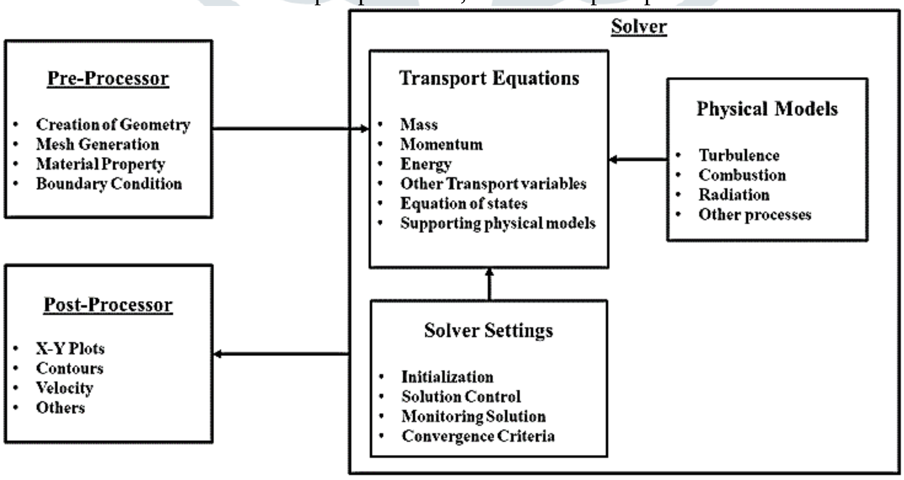

Figure 3: The inter-connectivity functions of these three main elements within a CFD analysis framework

The cornerstone of computational fluid dynamics is the fundamental governing equations of fluid dynamics—the continuity, momentum and energy equations [2].

1. mass is conserved;

2. F = ma (Newton's second law);
3. energy is conserved.

The x-, y- and z-components respectively of the momentum Equation

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho f_x \quad (2)$$

$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \rho f_y \quad (3)$$

$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho f_z \quad (4)$$

Continuity equation in non-conservation form.

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{V} = 0 \quad (5)$$

The conservation form of the energy equation, written in terms of the total energy, $(e+V^2/2)$.

$$\begin{aligned}\frac{\partial}{\partial t}\left[\rho\left(e+\frac{V^2}{2}\right)\right] + \nabla\cdot\left[\rho\left(e+\frac{V^2}{2}\vec{V}\right)\right] &= \rho\dot{q} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) - \frac{\partial(up)}{\partial x} - \frac{\partial(wp)}{\partial z} + \frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} \\ &+ \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial(v\tau_{xy})}{\partial x} + \frac{\partial(v\tau_{yy})}{\partial y} + \frac{\partial(v\tau_{zy})}{\partial z} + \frac{\partial(w\tau_{xz})}{\partial x} + \frac{\partial(w\tau_{yz})}{\partial y} + \frac{\partial(w\tau_{zz})}{\partial z} + \rho\vec{f}\cdot\vec{V} \quad (6)\end{aligned}$$

2.2 Design Optimization

An optimization problem is a problem in which certain parameters (design variables) needed to be determined to achieve the best measurable performance (objective function) under given constraints. Design Optimization - determining design parameters that lead to the best "performance" of a mechanical structure, device, or system.

2.2.1 Conventional Taguchi Robust Design Method

Engineers conduct experiments to improve the quality of the products or processes. Experiments are generally performed to explore, estimate or confirm. Design of Experiments (DOE), or statistically designed experiments (SDE), is a scientific approach that allows the experimenter to understand a process and to determine how the input variables (factors) affect the output characteristics. Dr. Taguchi of Nippon Telephones and Telegraph Company, Japan has developed a method based on "orthogonal array" experiments which gives much reduced "variance" for the experiment with "optimum settings" of control parameters. "Orthogonal Arrays" (OA) provide a set of well balanced (minimum) experiments and Dr. Taguchi's Signal-to-Noise ratios (S/N), which are log functions of desired output, serve as objective functions for optimization, help in data analysis and prediction of optimum results. An orthogonal array (OA) is a matrix of numbers arranged in rows and columns. Each row represents the levels (or states) of the selected factor in a given experiment, and each column represents a specific factor whose effects on the output (or response) are of interest to the experimenters. To minimize financial resources, and due to time constraints, the experimenter usually attempts to employ the smallest size OA, which will meet the objective of the experiment. In the Taguchi method, the term 'signal' represents the desirable value (mean) for the output characteristic and the term 'noise' represents the undesirable value (standard deviation) for the output characteristic. Therefore, the SNR is the ratio of the mean to the standard deviation. Taguchi uses the SNR to measure the quality characteristic deviating from the desired value [6]. In an orthogonal array experiment, number of parameters and levels are selected according to the respective study and an orthogonal array based on them is chosen from the standard table given below.

Table 1: Standard Table

		Parameters										
		2	3	4	5	6	7	8	9	10	11	12
Levels	2	L4	L4	L8	L8	L8	L8	L12	L12	L12	L12	L16
	3	L9	L9	L9	L18	L18	L18	L18	L27	L27	L27	L27
	4	L16	L16	L16	L16	L32	L32	L32	L32	L32		
	5	L25	L25	L25	L25	L25	L50	L50	L50	L50	L50	L50

The current study is focused on 3 geometrical parameters or factors namely area ratio (A_{Exit}/A_{Throat}), angle ratio (θ_n/θ_e) and length ratio ($L_{Divergent}/L_{Convergent}$) and each with 3 levels. The figure 4 is the schematic representation of nozzle geometry. Table 2 represents the values of these parameters for each level considered for study. From standard table (Table 1), L9 is chosen as the orthogonal array for the 3-parameters and 3-level study. Thus, the selected array L9 has the matrix representation as shown in Table 3(a). Once the OA is selected, the experiments are selected as per the level combinations. The performance parameter is noted for each experimental run for analysis. Table 3(b) shows the 9 experiments based on L9.

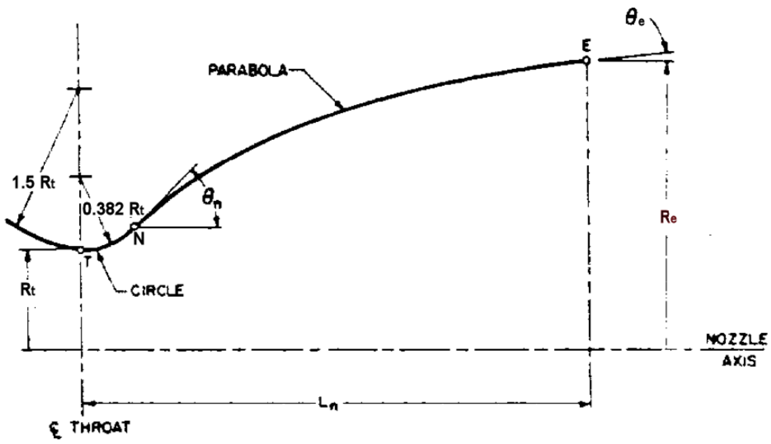

Figure 4: Geometrical specifications of Parabolic Nozzle[9]

Table 2: Values of factors at 3 levels

Factors	Level 1	Level 2	Level 3
Area Ratio	13.9214	10.4675	17.3807
Angle Ratio	2.8000	2.6638	3.9275
Length Ratio	7.6797	6.2923	8.9205

Table 3(a): L9 Orthogonal Array

Experiment Run	Parameter 1	Parameter 2	Parameter 3
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

Table 3(b): L9 Orthogonal Array

S. No.	Area Ratio	Angle Ratio	Length Ratio
1	13.9214	2.8000	7.6797
2	13.9214	2.6638	6.2923
3	13.9214	3.9275	8.9205
4	10.4675	2.8000	6.2923
5	10.4675	2.6638	8.9205
6	10.4675	3.9275	7.6797
7	17.3807	2.8000	8.9205
8	17.3807	2.6638	7.6797
9	17.3807	3.9275	6.2923

In the Taguchi method, the term 'signal' represents the desirable value (mean) for the output characteristic and the term 'noise' represents the undesirable value (standard deviation) for the output characteristic. Therefore, the SNR is the ratio of the mean to the standard deviation. Taguchi uses the SNR to measure the quality characteristic deviating from the desired value [7]. Their mathematical expressions are formulated as follows:

Table 4: Different S/N Ratios

Case 1: ''Smaller-the-better'': aiming to minimize the performance.	Case 2: ''Larger-the-better'': aiming to maximize the performance.	Case 3: ''Nominal-is-best'': aiming to target the predetermined nominal value.
$SNR = -10log_{10}\left(\frac{\sum_{i=1}^{N} y_i^2}{N}\right)$ (7) where the y denotes the performance indicator, subscript i experiment number, N number of replicates of experiment 'i'.	$SNR = -10log_{10}\left(\sum_{i=1}^{N}\frac{1}{y_i^2}\right)$ (8) (To obtain optimal Thrust and Mach No. for a parabolic nozzle, the Larger-the-better case is considered.)	$SNR = 10log_{10}\left[\left(\frac{\bar{y}}{s}\right)^2\right]$ (9a) $\bar{y} = \frac{y_1 + y_2 + y_3 \ldots + y_N}{N}$ (9b) $S = \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N-1}$ (9c)

The performance parameters, Thrust and Mach No. are derived from CFD based experimentations of C-D Nozzle. The velocity contours are obtained and Thrust and Mach No. are then calculated based on the following relations[3]:

$$\frac{A}{A_t} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (10)$$

$$F = mv_e \quad (11)$$

Here, "F" = Thrust Force (kN) and M = Mach number at exit

III. Results and Discussion

The results for first experiment from the orthogonal array are obtained from CFD analysis in which the Area Ratio, Angle Ratio and Length Ratio is 13.9214, 2.8000, and 7.6797 respectively. Boundary conditions applied for experiments are as follows[8]:

Table 5: Boundary Conditions

Boundary Name	Parameters
Pressure-inlet (chamber)	P_{total} = 2100000 Pa, T_{total} = 400K, Air as ideal gas
Pressure-inlet (ambient)	P_{total} = 100000 Pa, T_{total} = 300K, Air as ideal gas
Free-slip wall	Adiabatic, Zero shear
Wall	Adiabatic, No Slip
Pressure-Outlet	P_{total}= 100000 Pa
Mass flow-rate	15 kg/sec

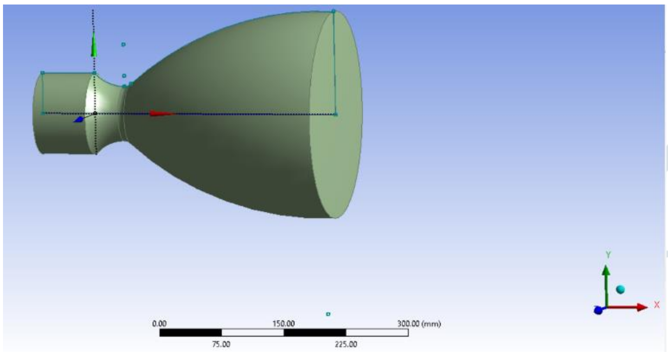

Figure 5: 3-D view of Rocket Nozzle Design

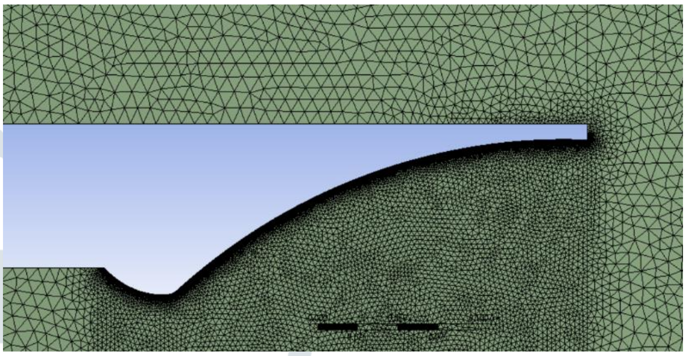

Figure 6: Condensed mesh near the nozzle wall

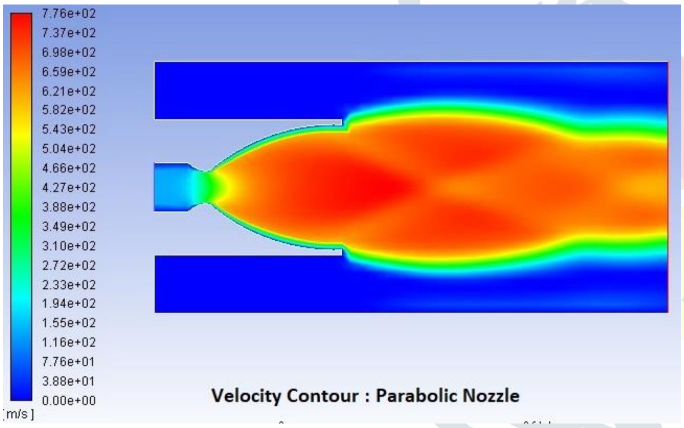

Figure 7: Velocity magnitude contour

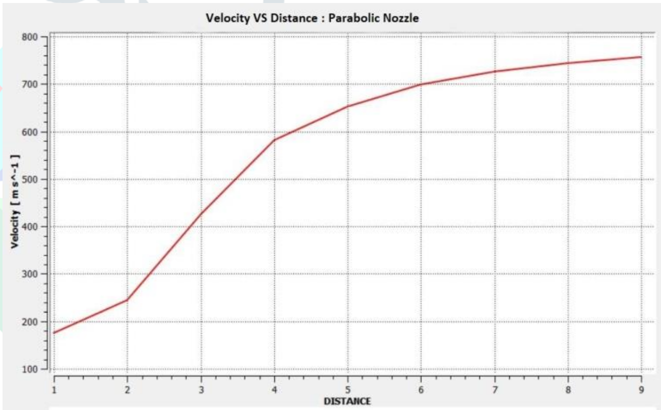

Figure 8: Velocity(m/s) vs distance(mm) plot

The velocity obtained at the exit section of nozzle is used to derive Thrust and Mach No. as per the equations (10) & (11). The similar procedure is followed for remaining sets of experiments. The response values for each experiment is recorded. For the purpose of optimization, the software tool used is MINITAB 18. The general linear modelling (GLM) assists in performing analysis of variance (ANOVA). ANOVA is a useful tool to sub-divide the total variation into variation due to main effect, variations due to interaction effects and variations due to error.

Taguchi Analysis: Thrust versus Area Ratio, Angle Ratio, Length Ratio

Table 6: Response Table for Signal to Noise Ratios (Larger is better)

Level	Area Ratio	Angle Ratio	Length Ratio
1	23.88	24.18	24.30
2	24.18	24.39	24.24
3	24.81	24.29	24.31
Delta	0.93	0.20	0.07
Rank	1	2	3

Taguchi Analysis: Mach No. versus Area Ratio, Angle Ratio, Length Ratio

Table 7: Response Table for Signal to Noise Ratios (Larger is better)

Level	Area Ratio	Angle Ratio	Length Ratio
1	12.07	12.41	12.51
2	12.50	12.67	12.55
3	13.05	12.54	12.55
Delta	0.98	0.25	0.04
Rank	1	2	3

Table 8: Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Area Ratio	2	4.9078	2.4539	126.36	0.008
Angle Ratio	2	0.2250	0.1125	5.79	0.147
Length Ratio	2	0.0388	0.0194	1.00	0.500
Error	2	0.0388	0.0194		
Total	8	5.2106			

Table 9: Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Area Ratio	2	0.3459	0.1729	416.33	0.002
Angle Ratio	2	0.0231	0.0115	27.84	0.035
Length Ratio	2	0.0008	0.0004	1.00	0.500
Error	2	0.0008	0.0004		
Total	8	0.3707			

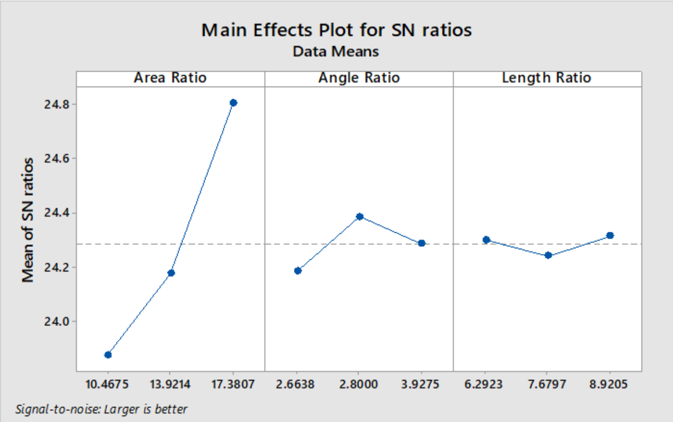

Figure 9: Main Effect Plot for SN Ratio (Thrust Analysis)

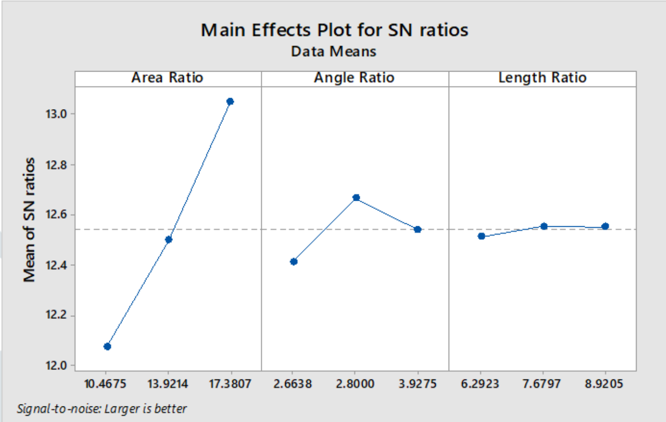

Figure 10: Main Effect Plot for SN Ratio (Mach No. Analysis)

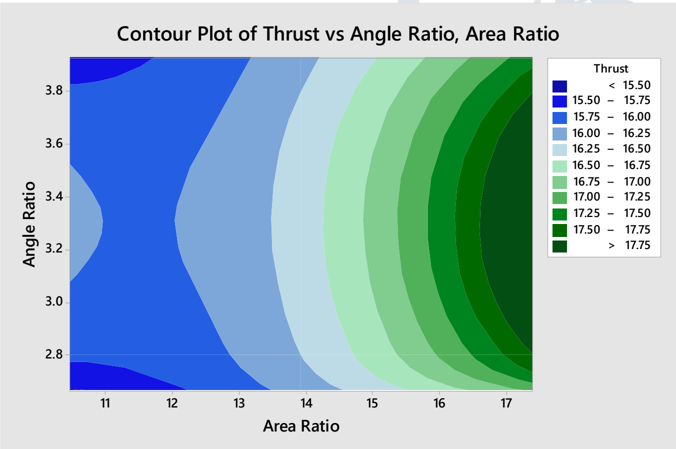

Figure 11: Contour Plot of Thrust vs Angle Ratio, Area Ratio

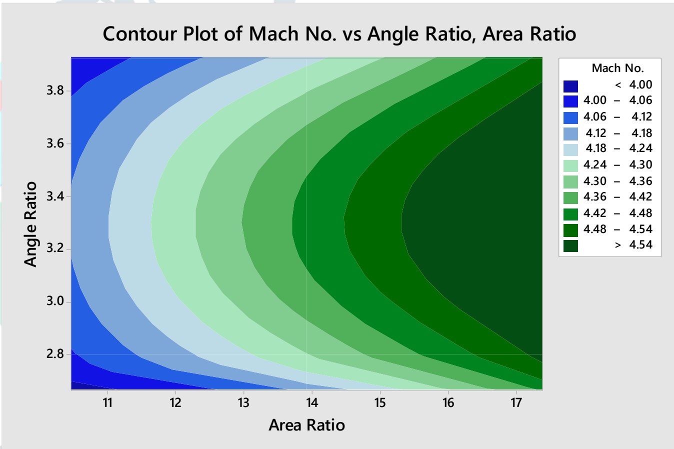

Figure 12: Contour Plot of Mach No. vs Angle Ratio, Area Ratio

Contour plots (also called Level Plots) are a method to show a three-dimensional surface on a two-dimensional plane. The plots comprise of two predictor variables X and Y and a response variable Z as contours. Basically a contour plot is appropriate if an experimenter wishes to see how some value Z changes as a function of two inputs, X and Y i.e.z = f(x,y). Figure 11 and 12 shows contour plots between two responses (Thrust and Mach No.) and the first two statistically significant parameters i.e. Rank 1 (Area Ratio) and Rank 2 (Angle Ratio). These plots help to recognize feasible region for manufacturer to work in while trading off between response and factors.

After the Taguchi Analysis is carried out for the given problem, the three statistically significant parameters were obtained which are as follows:

Table 10: Optimal values of parameters

Parameter	Optimal Value
Area Ratio	17.3807
Angle Ratio	2.80
Length Ratio	8.9205

The new “Thrust Optimized Parabolic” Nozzle is designed in accordance with the respective optimal value of the constraints. A CFD analysis using ANSYS Fluent is then performed on TOP Nozzle. The new exit velocity (as per analysis) is found to be increased in comparison to the previous nozzle geometry. Thus, the nozzle geometry is successfully optimized to obtain relatively higher Thrust which is the desired objective of the project being achieved.

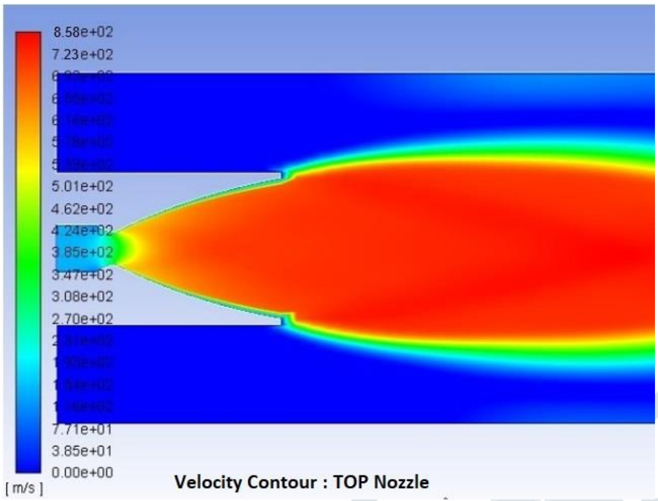

Figure 14: Velocity magnitude: TOP nozzle

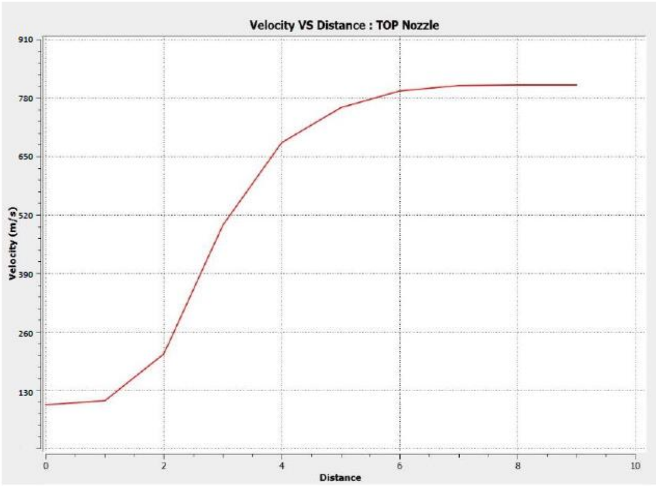

Figure 15: Velocity vs distance: TOP Nozzle

IV. Conclusions

1. In the current work, Taguchi Robust Design Method was efficiently use to obtain the optimal control factors of parabolic nozzle.
2. In this work, it was found that the parameters Area Ratio, Angle Ratio and Length Ratio have the optimal values as 17.3807, 2.80 and 8.9205 respectively.
3. According to Response Table for Signal to Noise Ratios, the parameter Area Ratio is found to be most statistically significant (Rank 1) followed by Angle Ratio and Length Ratio.
4. The confirmatory experiment done results in increase of thrust from 11.4 kN to 12.87 kN which is 12.89% increment.

V. References

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