RESEARCH ARTICLE ON THE NON-HOMOGENEOUS HEPTIC EQUATION WITH FIVE UNKNOWNS

$$(x^3-y^3)- (x^2+y^2) + z^3-w^3 = 2+11(x-y)(z-w)^2p^4$$

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Abstract

The non-homogeneous heptic equation with five unknowns given by $(x^3-y^3)- (x^2+y^2)+ z^3-w^3 = 2+11(x-y)(z-w)^2p^4$ is analyzed for its non-zero distinct integer solutions. In particular, four different patterns and some properties are presented.

Keywords

Non-homogeneous heptic equation, integer solutions.

Introduction

The theory of diophantine equation offers a rich variety of fascinating problems. In [4-7], non-homogeneous heptic equation with five unknowns are analyzed. In [8&9], non-homogeneous heptic equation with four unknowns are discussed. Particularly in [10], non-homogeneous heptic equation with three unknowns are mentioned. Here we discuss a non-homogeneous heptic equation with five unknowns given by $(x^3-y^3) - (x^2+y^2)+ z^3-w^3 = 2+11(x-y)(z-w)^2p^4$ for finding its non-zero distinct integer solutions.

Notations Used

t_{mn}: Polygonal number of rank 'n' with size 'm'.
p_n^m: Pyramidal number of rank 'n' with size 'm'.
CP_n^m: Centered pyramidal number of rank 'n' with size 'm'.
Pr_n: Pronic number of rank 'n'.
FN_n^4: Four dimensional figurate number of rank 'n'.
Nex_n: Nexus number of rank 'n'.
CH_n: Centered Hexagonal number of rank 'n'.

Method of Analysis

The non-homogeneous heptic equation with five unknowns to be solved for its distinct non-zero integer solutions is

$(x^3-y^3)- (x^2-y^2) + z^3-w^3 = 2+11(x-y)(z-w)^2p^4$ (1)

Assign x=u+1, y=u-1, z=v+1, w=v-1 (2)

(1) becomes $4u^2+6v^2 = 88p^4$ (3)

To find integer solutions different patterns are listed below:

Pattern 1:

Let $p = 4a^2+6b^2$ (4)

88 can be written as

$$88 = (8+i2\sqrt{6})\,(8-i2\sqrt{6}) \quad (5)$$

Substituting (4) and (5) in (3) and applying the method of factorization, define

$$2u+i\sqrt{6}v = (8+i2\sqrt{6})\,(2a+i\sqrt{6}b)^4 \quad (6)$$

By equating real and imaginary parts we get

$$u = 64a^4-576a^2b^2+144b^4-192a^3b+288ab^3 \quad (7)$$
$$v = 32a^4+256a^3b-288a^2b^2-384ab^3+72b^4 \quad (8)$$

Using (7), (8) and (2) we get the integer solutions of (1) as

$$x = 64a^4-192a^3b-576a^2b^2+288ab^3+144b^4+1$$
$$y = 64a^4-192a^3b-576a^2b^2+288ab^3+144b^4-1$$
$$z = 32a^4+256a^3b-288a^2b^2-384ab^3+72b^4+1$$
$$w = 32a^4+256a^3b-288a^2b^2-384ab^3+72b^4-1$$
$$p = 4a^2+6b^2$$

Properties:

1. $w(1,b)-864FN_b^4+256CP_b^9+128Obl_b+128Pr_b \equiv 31 \pmod{384}$
2. $z(a,1)-Nex_a-324FN_a^4-492P_a^5+t_{1002,a}+t_{36,a} \equiv 72 \pmod{904}$
3. $x(a,1)-768FN_a^4+144CP_a^{14}-96CP_a^9-CH_a+t_{1002,a}+t_{32,a} \equiv 144 \pmod{660}$
4. $p(a+1,a)-t_{14,a}-4Pr_a \equiv 4 \pmod 9$
5. $p(2a,2)-24$ is a perfect square.

Pattern 2:

Instead of (5), 88 can also be written as

$$88=\frac{(16+i4\sqrt{6})(16-i4\sqrt{6})}{2^2} \quad (9)$$

Using (4) and (9) in (3) and applying the same procedure mentioned in pattern 1, we get the same integer solutions to (1).

Pattern 3:

(3) can be re-written as

$$4u^2+6v^2 = 88p^4*1 \quad (10)$$

Write 1 as

$$1 = \frac{(5+i2\sqrt{6})(5-i2\sqrt{6})}{7^2} \quad (11)$$

Substituting (4), (5) and (11) in (10) and using method of factorization, define

$$2u+i\sqrt{6}v = \frac{1}{7}(8+i2\sqrt{6})(5+i2\sqrt{6})(2a+i\sqrt{6}b)^4 \quad (12)$$

By equating real and imaginary parts we have

$$u = \frac{1}{7}(128a^4 - 2496a^3b - 1152a^2b^2 + 3744ab^3 + 288b^4)$$
$$v = \frac{1}{7}(416a^4 + 512a^3b - 3744a^2b^2 - 768ab^3 + 936b^4)$$

To get integer solutions take $a = 7A$ and $b = 7B$ (13)

$$u = 7^3(128A^4 - 2496A^3B - 1152A^2B^2 + 3744AB^3 + 288B^4) \quad (14)$$
$$v = 7^3(416A^4 + 512A^3B - 3744A^2B^2 - 768AB^3 + + 936B^4) \quad (15)$$

Using (14), (15) in (2) and (13) in (4) we obtain the corresponding integer solutions to (1) as

$x = 7^3 (128A^4 – 2496 A^3B – 1152A^2B^2 + 3744 AB^3 + 288B^4) + 1$
$y = 7^3 (128A^4 – 2496 A^3B – 1152A^2B^2 + 3744 AB^3 + 288B^4) – 1$
$z = 7^3 (416A^4 + 512 A^3B – 3744A^2B^2 – 768 AB^3 + 936B^4) + 1$
$w =7^3 (416A^4 + 512 A^3B – 3744A^2B^2 – 768 AB^3 + 936B^4) - 1$
$p = 196A^2 + 294B^2$

Pattern 4:

Re-write 1 as

$$1 = \frac{(10+i4\sqrt{6})\,(10-i4\sqrt{6})}{14^2} \qquad (16)$$

Substituting (4), (5) and (16) in (10) and applying the same procedure mentioned in the previous pattern, we get the following non-zero distinct integer solutions,

$x = 14^3 (256A^4 – 4992 A^3B – 2304A^2B^2 + 7488AB^3 + 576B^4) + 1$
$y = 14^3 (256A^4 – 4992 A^3B – 2304A^2B^2 + 7488AB^3 + 576B^4) – 1$
$z = 14^3 (832A^4 + 1024 A^3B – 7488A^2B^2 – 1536AB^3 + 1872B^4) + 1$
$w = 14^3 (832A^4 + 1024 A^3B – 7488A^2B^2 – 1536AB^3 + 1872B^4) - 1$
$p = 784A^2 + 1176B^2$

Conclusion

In this paper, we have made an attempt to find four different patterns of non-zero distinct integer solutions to the homogenous haptic equation with five unknowns. To conclude, one may search for some other patterns or some other forms of haptic equation with five variables or more than five variables.

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