A STUDY ON NANOFLUID FLOW OVER A ROTATING DISK IN POROUS MEDIUM WITH SLIP CONDITION

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***Abstract*:** In this article both MHD and hydrodynamic frame of viscous nanofluid flow due to rotating disk in a porous medium subject to slip condition in the presence of heat generation/absorption and chemical reaction are examined. The nonlinear ordinary differential equations are obtained and solved numerically using BVP4C Technique. The effects of various thermophysical parameters and porosity of the medium on velocity, temperature and concentration are discussed in the help of graphs through MATLAB.

***Keywords* - MHD, porous medium , heat generation/absorption, chemical reaction, partial slip condition**.

I. INTRODUCTION

Magnetohydrodynamics denotes the study of the motion of highly electrically conducting fluids. It is related with the coupling of electromagnetic fluids and electrically conducting fluids. The role of MHD is important in many fields like nuclear reactors, solar physics etc. Nanofluid is a new kind of heat transfer medium, containing Nanopartiles. It plays an important role in the enhancement of thermal conductivity and can be used for welding equipments, crystal silicon mirror cooling. Nanofluid are developed for bio medical applications such as drug delivery and also cancer therapy, it can also be used for safer surgery by cooling around the surgical region, and reducing the organ damage. It also used for detergency, antibacterial agent , solar insustry etc. Fluid flow evoked by a rotating disk has been a compelling analysis topic since it is relevant during a range of technical applications involving chemical science systems, deposition of coatings on surfaces, rotor-stator system, atmospherical and oceanic circulations, viscometer and varied others. Kumaran[11](2010) studied the transition of MHD boundary layer flow past a strecting sheet. He investigated the transition effects because of applied magntic field were analysed numerically on Newtonian fluid flow. Whereas Noor N G[14](2010) carried out the simple non-pertubative solution of MHD Newtonian fluid flow over a shrinking surface. Turkyilmazoglu M(2012) studied the exact analytical solutions for heat and mass transfer of MHD slip flow in nanofluids. It investigates the influence of slip on the behavior of fluid flow and thermal transport of some electrically conducting nanofluid over a permeable strecting/shrinking sheet. In 2013 Turkyilmazoglu M[22] he studied the heat and mass transfer flow due to rotating rough and porous disk the rotating disk surface is considered with partial slip in the presence of a uniform suction or injection. He analysed the Effects of wall roughness and temperature jump on the heat and mass transfer. Also in 2014[26] he discussed the nanofluid flow and heat transfer properties on rotating disc. It investigates the best performing nanofluids (Cu,Ag,Cuo) over a rotating disk in terms of heat transfer. In 2013 Dandapat and singh[21] has reviewed the unsteadiness effect in two layer film flow on rotating disc. Hayat, T[27](2015) have studied the magnetohydrodynamic (MHD) flow of viscous nanofluid saturating porous medium. He investigates that higher nanoparticle volume fraction decreases the velocity field also an analysis has been carried out for Magnetohydrodynamic three-dimensional flow of viscoelastic nanofluid in the presence of nonlinear. He analysed the MHD 3D flow of nanaofluid in the presence of thermophoresis and Brownian motion effects. The experimental results shows that the thermal boundary layer thickness is an increasing function of radiative effect.

Therefore the objective of the present work is to analyse the effects of MHD viscous flow due to rotating disk in porous medium with the presence of heat generation and chemical reaction. The present work is likely to have bearing on the problem of heat transfer which can be useful in industries for designing rotating machineries and lubrications etc. The impact of the porosity of the medium on the MHD viscous flow and heat transfer is presented and discussed. The objective of this article is to explore the study of viscous nanofluid flow due to rotating rigid disk with slip condition for MHD and hydrodynamic cases. The mutual interaction of thermophoretic and Brownian motion phenomena is acknowledged by considering nano particles. The effects of various themophysical parameters like Magnetic parameter B, Velocity slip parameter λ , suction parameter w_s , Lewis number Le, Prandtl number Pr, Brownian motion parameter N_B and thermophoresis parameter N_T on the velocity, temperature and concentration profiles are examined graphically through MATLAB.

II. MATHEMATICAL FORMULATION

The viscous Nanofluid flow brought by a rotating disk subject to velocity slip condition is considered. In the direction of $\bar{z}$ $-axis$ the uniform magnetic field with strength B_0 is applied. At $\bar{z}=0$, the disk rotates with a constant angular velocity $\overline{\Omega}$. The problem is take into account with low Reynolds number so that the induced magnetic fields is neglected. The effects of heat generation/absorption and chemical reaction are examined. The phenomena of mutual interaction of thermopherosis and brownian motion in heat and mass transfer aspects is considered. $(\bar{u},\bar{v},\bar{w})$ are the velocity components in the direction of $(\bar{r},\bar{\phi},\bar{z})$. The boundary layer approximation which reduces the governing equation to present flow situation as follows:

Continuity Equation

$$\frac{\partial \bar{w}}{\partial \bar{z}}+\frac{\partial \bar{u}}{\partial \bar{r}}+\frac{\bar{u}}{\bar{r}}=0 \quad (1)$$

Momentum Equation

$$\bar{w}\frac{\partial \bar{u}}{\partial \bar{z}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}}-\frac{\bar{v}^2}{\bar{r}}=\nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{u}}{\partial \bar{r}}+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}-\frac{\bar{u}}{\bar{r}^2}\right)-\frac{\sigma B_0^2}{\rho_f}\bar{u} \quad (2)$$

$$\bar{w}\frac{\partial v}{\partial \bar{z}}+\bar{u}\frac{\partial v}{\partial \bar{r}}+\frac{\overline{uv}}{\bar{r}}=\nu\left(\frac{\partial^2 \bar{v}}{\partial \bar{z}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{r}^2}-\frac{\bar{v}}{\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{v}}{\partial \bar{r}}\right)-\frac{\sigma B_0^2}{\rho_f}\bar{v} \quad (3)$$

$$\bar{w}\frac{\partial \bar{w}}{\partial \bar{z}}+\bar{u}\frac{\partial \bar{w}}{\partial \bar{r}}=\nu\left(\frac{\partial^2 \bar{w}}{\partial \bar{z}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{w}}{\partial \bar{r}}+\frac{\partial^2 \bar{w}}{\partial \bar{r}^2}\right) \quad (4)$$

Heat Equation

$$\bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}+\bar{u}\frac{\partial \bar{T}}{\partial \bar{r}}=\alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}}+\frac{\partial^2 \bar{T}}{\partial \bar{r}^2}\right)+\frac{(\rho c)_P}{(\rho c)_f}\left[D_B\left(\frac{\partial \bar{T}}{\partial \bar{z}}\frac{\partial \bar{C}}{\partial \bar{z}}+\frac{\partial \bar{T}}{\partial \bar{r}}\frac{\partial \bar{C}}{\partial \bar{r}}\right)\right]+\frac{(\rho c)_P}{(\rho c)_f}\left[\frac{D_T}{\bar{T}_\infty}\left(\left(\frac{\partial \bar{T}}{\partial \bar{z}}\right)^2+\left(\frac{\partial \bar{T}}{\partial \bar{r}}\right)^2\right)\right]+\frac{Q_0}{c_P\rho}(\bar{T}-\bar{T}_\infty) \quad (5)$$

Mass Equation

$$\bar{w}\frac{\partial \bar{C}}{\partial \bar{z}}+\bar{u}\frac{\partial \bar{C}}{\partial \bar{r}}=D_B\left(\frac{\partial^2 \bar{C}}{\partial \bar{z}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{C}}{\partial \bar{r}}+\frac{\partial^2 \bar{C}}{\partial \bar{r}^2}\right)+\frac{D_T}{\bar{T}_\infty}\left[\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}}+\frac{\partial^2 \bar{T}}{\partial \bar{r}^2}\right]-R_0(\bar{C}-\bar{C}_\infty) \quad (6)$$

The corresponding boundary conditions are:

$$\bar{u}=L\frac{\partial \bar{u}}{\partial \bar{z}}\ ,\ \bar{v}=\bar{r}\overline{\Omega}+L\frac{\partial \bar{v}}{\partial \bar{z}}\ ,\ \bar{w}=w_0\ ,\bar{T}=\bar{T}_W,\ \bar{C}=\bar{C}_W\ \text{ as } \bar{z}=0 \quad (7)$$

$$\bar{u}\to 0\ ,\ \bar{v}\to 0\ ,\ \bar{T}\to \bar{T}_\infty,\ \bar{C}\to \bar{C}_\infty\ as\ \bar{z}\to\infty. \quad (8)$$

Where, $\nu=\frac{\mu}{\rho_f}$ is the kinematic viscosity, μ is the dynamic viscosity, σ is the electrical conductivity of the fluid, ρ is the density, ρ_f is the density of the base fluid, $(\rho c)_P$ is the effective heat capacity of nanoparticles, $\alpha=\frac{k}{(\rho c)_f}$ is the thermal diffusivity, $(\rho c)_f$ is the heat capacity, $\bar{T}$ is temperature, $\bar{C}$ is the concentration, D_T is the thermophoretic diffusion coefficient, D_B is the Brownian diffusion coefficient, L is the velocity slip constant, $\bar{T}_W$ is the surface temperature, $\bar{T}_\infty$ is the ambient temperature, $\bar{C}_W$ is the surface concentration, $\bar{C}_\infty$ is the ambient concentration, Q_0 is the heat generation/absorption coefficient and R_0 is the rate of chemical reaction.

The dimensionless variables that we introduced here is as follow

$$\left.\begin{aligned}&\bar{u}=\bar{r}\overline{\Omega}\frac{dF(\zeta)}{d\zeta}\ ,\quad \bar{v}=\bar{r}\overline{\Omega}G(\zeta)\ ,\quad \bar{w}=-\sqrt{2\overline{\Omega}\nu}F(\zeta)\\&C(\zeta)=\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_W-\bar{C}_\infty}\ ,T(\zeta)=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_W-\bar{T}_\infty}\ ,\zeta=\sqrt{\frac{2\Omega}{\nu}}\ \bar{z}\end{aligned}\right\} \quad (9)$$

By incorporating the Equation 9, The Equation 1- Equation8 is transformed to the non linear ordinary differential equation follows:

The Equation1, satisfied.

The Momentum Equation2, transforms to the ordinary differential equation as follows

$$2\frac{d^3F(\zeta)}{d\zeta^3}+2F(\zeta)\frac{d^2F(\zeta)}{d\zeta^2}-\left(\frac{dF(\zeta)}{d\zeta}\right)^2+\left(G(\zeta)\right)^2-\beta\frac{dF(\zeta)}{d\zeta}=0 \quad (10)$$

The Momentum Equation3, transforms to the ordinary differential equation as follows

$$2\frac{d^2G(\zeta)}{d\zeta^2}+2F(\zeta)\frac{dG(\zeta)}{d\zeta}-2G(\zeta)\frac{dF(\zeta)}{\zeta}-\beta G(\zeta)=0 \quad (11)$$

The Heat Equation5, transforms to the ordinary differential equation as follows

$$\frac{d^2T(\zeta)}{d\zeta^2}+Pr\left(F(\zeta)\frac{dT(\zeta)}{d\zeta}+N_B\frac{dT(\zeta)}{d\zeta}\frac{dC(\zeta)}{d\zeta}+N_T\left(\frac{dT(\zeta)}{d\zeta}\right)^2+\ HT(\zeta)\right)=0 \quad (12)$$

The Mass Equation 6, transforms to the ordinary differential equation as follows

$$\frac{d^2C(\zeta)}{d\zeta^2}+LePrF(\zeta)\frac{dC(\zeta)}{d\zeta}+\frac{N_T}{N_B}\frac{d^2T(\zeta)}{d\zeta^2}-R_PC(\zeta)=0 \quad (13)$$

The boundary conditions Equation7 and Equation8 changes to

$$\left.\begin{aligned}&F(\zeta)=w_s,\frac{dF(\zeta)}{d\zeta}=\lambda\frac{d^2F(\zeta)}{d\zeta^2},G(\zeta)=1+\lambda\frac{dG(\zeta)}{d\zeta},T(\zeta)=1,C(\zeta)=1\ \ at\ \zeta=0\\&\frac{dF(\zeta)}{d\zeta}\to 0,G(\zeta)\to 0,T(\zeta)\to 0,C(\zeta)\to 0\ as\ \zeta\to\infty\end{aligned}\right\} \quad (14)$$

Here, w_s is the suction parameter , $\beta$is the magnetic field parameter, λ is the velocity slip parameter, Pr is the Prandtl number, $N_T$is the thermophoresis parameter, N_B is the Brownian motion parameter, H is the heat generation/absorption parameter, R_P is the chemical reaction parameter, and Le is the Lewis number.

These parameters are defined as follows:

$$\beta = \sqrt{\frac{\sigma \beta_0{}^2}{\rho_f \bar{\Omega}}}, \quad \lambda = L\sqrt{\frac{2\bar{\Omega}}{\nu}}, \quad N_B = \frac{(\rho C)_P}{(\rho C)_f}\frac{(\bar{T}_W-\bar{T}_\infty)D_T}{\bar{T}_\infty}, \quad H = \frac{Q_0}{2\bar{\Omega}\,\rho C_P}, \quad Le = \frac{\alpha}{D_B}, \quad Pr = \frac{\nu}{\alpha}, \; N_T = \frac{(\rho C)_P}{(\rho C)_f}\frac{(\bar{C}_W-\bar{C}_\infty)D_B}{\nu}, \; R_P = \frac{\nu}{2\bar{\Omega}D_B}R_0 \; . \tag{15}$$

the dimensionless forms of skin friction coefficient (SFC),local nusselt number(and local Sherwood number are defined as follows

$$\sqrt{Re_{\bar{r}}}C_F = \frac{d^2F(0)}{d\zeta^2}, \sqrt{Re_{\bar{r}}}C_G = \frac{dG(0)}{d(\zeta)}, \frac{Nu}{\sqrt{Re_{\bar{r}}}} = -\frac{dT(0)}{d\zeta}, \frac{Sh}{\sqrt{Re_{\bar{r}}}} = -\frac{dC(0)}{d\zeta} \tag{16}$$

Where $Re_{\bar{r}} = \frac{(\bar{\Omega}\bar{r})\bar{r}}{2\nu}$ denoted the local rotational Reynolds number.

III. NUMERICAL APPROACH

Mathematical equations are obtained in terms of higher order differential equation. Reduction of partial differential equation to non linear ordinary differential equation by using some suitable similarity transformation.

To solve the Equations10-Equation13 numerically which is subjected to the boundary conditions in Equation14, is executed BVP4C Technique. The system of equations is converted to initial value problem.

The substitutions used here are:

$$q_1 = F(\zeta),\; q_2 = F'(\zeta),\; q_3 = F''(\zeta),\; q_4 = G(\zeta),\; q_5 = G'(\zeta),$$
$$q_6 = T(\zeta),\; q_7 = T'(\zeta),\; q_8 = C(\zeta),\; q_9 = C'(\zeta)$$

and obtained the following system of ordinary differential equations

$$\left.\begin{aligned}
q_1' &= q_2 \\
q_2' &= q_3 \\
q_3' &= \frac{(q_2)^2 - 2q_1q_3 - (q_4)^2 + \beta q_2}{2} \\
q_4' &= q_5 \\
q_5' &= q_2q_4 - q_1q_5 + \tfrac{1}{2}\beta^2 q_4 \\
q_6' &= q_7 \\
q_7' &= -Pr[q_1q_7 + N_Bq_7q_9 + N_T(q_7)^2 + Hq_6] \\
q_8' &= q_9 \\
q_9' &= -LePrq_9 + \frac{N_T}{N_B}q_7' + R_Pq_8
\end{aligned}\right\} \tag{17}$$

With conditions

$$q_1(0) = w_s,\quad q_2(0) = \lambda F''(0),\quad q_3(0) = F''(0),\quad q_4(0) = 1 + \lambda G'(0),$$
$$q_5(0) = G'(0),\quad q_6(0) = 1,\; q_8(0) = 1\,. \tag{18}$$

With additional conditions

$$q_2(\infty) = 0,\; q_4(\infty) = 0,\; q_6(\infty) = 0,\; q_8(\infty) = 0\,. \tag{19}$$

IV. FIGURES

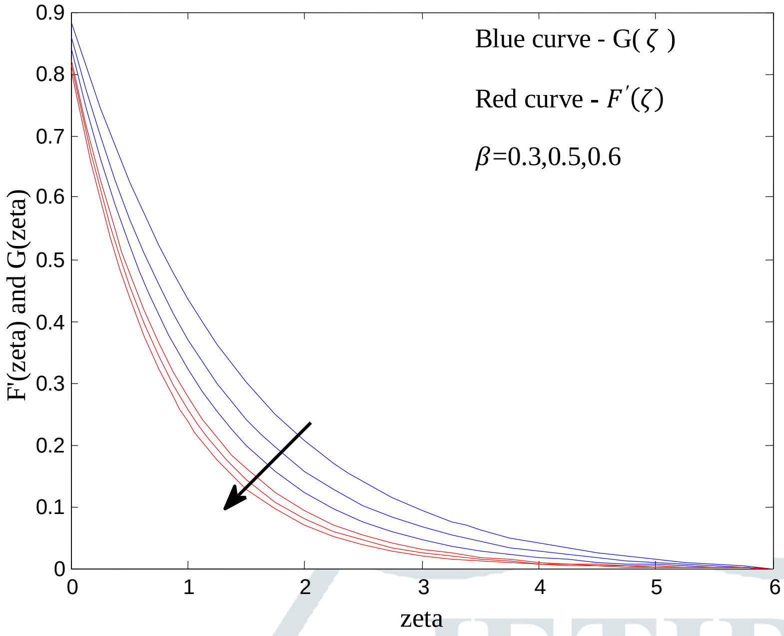

Figure 4.1 Velocity profiles for different values of magnetic field parameter β

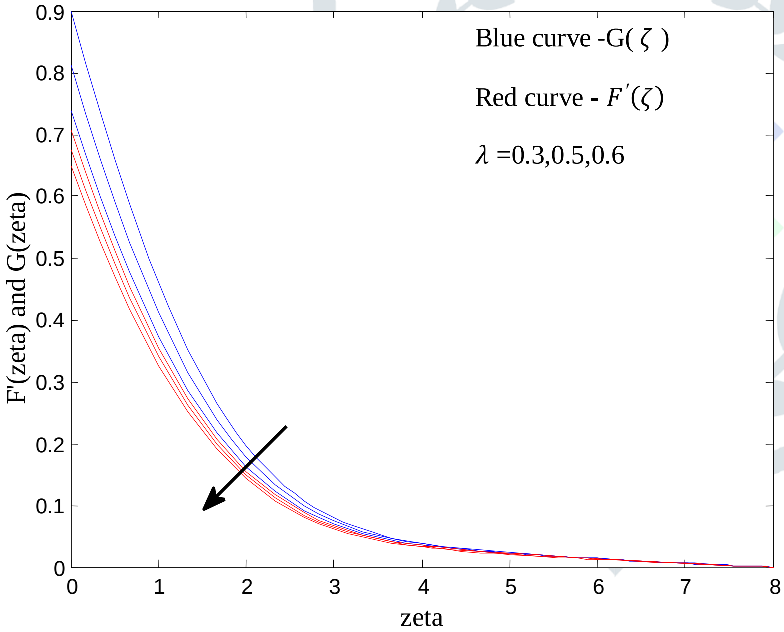

Figure 4.2 Velocity profiles for different values of velocity slip parameter λ

Figure 4.3-Figure 4.6 shows the effects of various parameters on temperature profile for w_s= -1

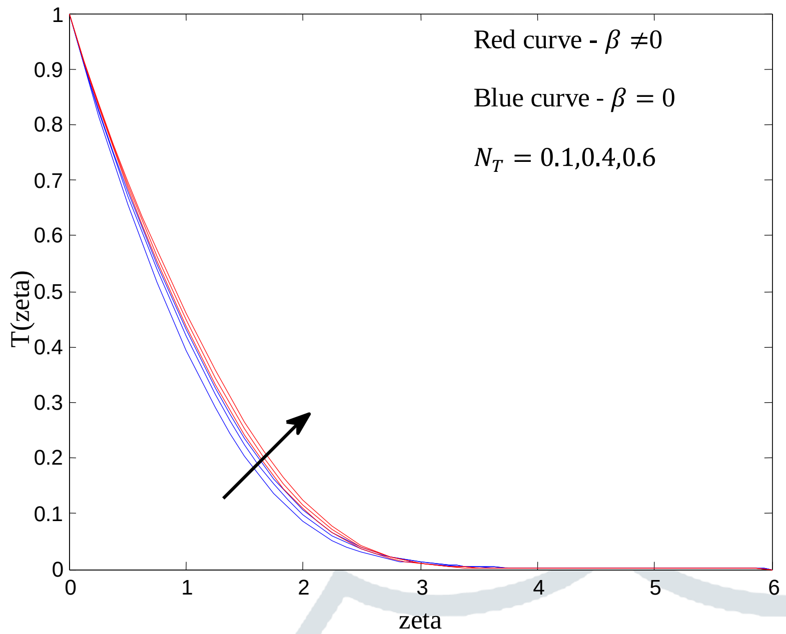

Figure 4.3 Temerature Profile T(ζ) for different values of thermophoresis parameter N_T

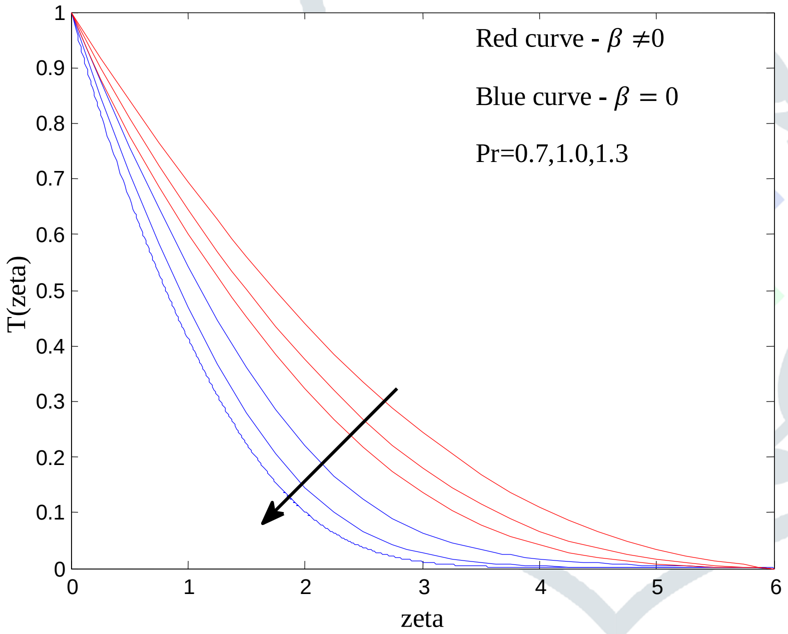

Figure 4.4 Temerature Profile T(ζ) for different values of Prandtl number Pr

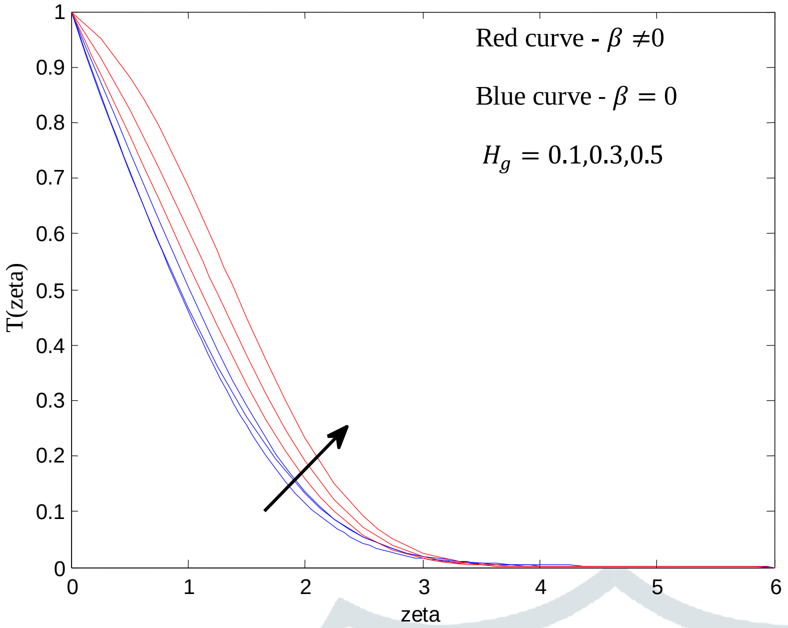

Figure 4.5 Temerature Profile T(ζ) for different values of heat generation parameter H_g

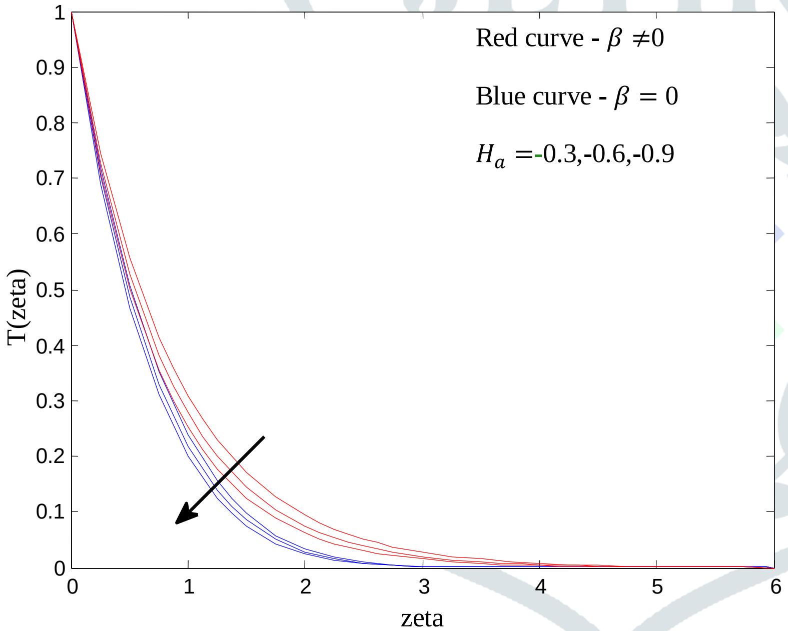

Figure 4.6 Temerature Profile T(ζ) for different values of heat absorption parameter H_a

Figure 4.7-Figure 4.9 shows the effects of various parameters on concentration profile for w_s= -1

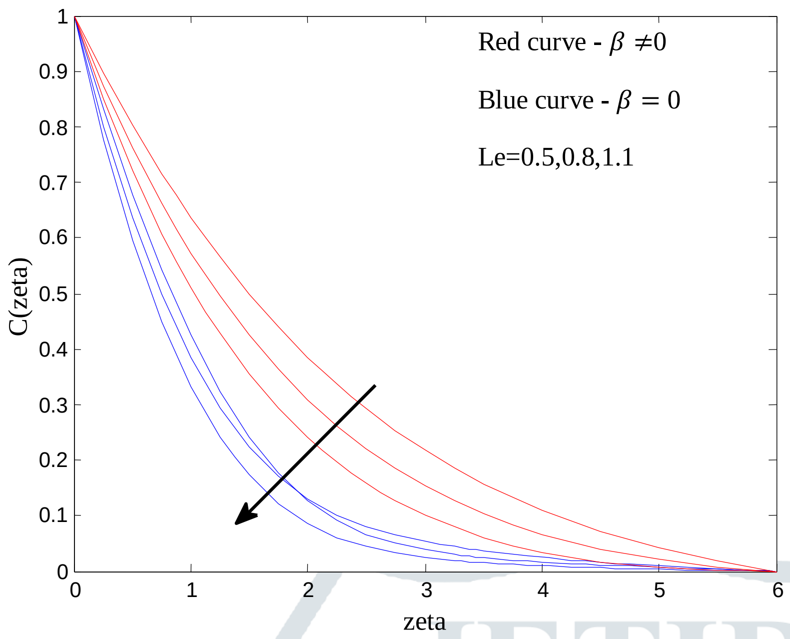

Figure 4.7 Concentration profile C(ζ) for different values of lewis number *Le*

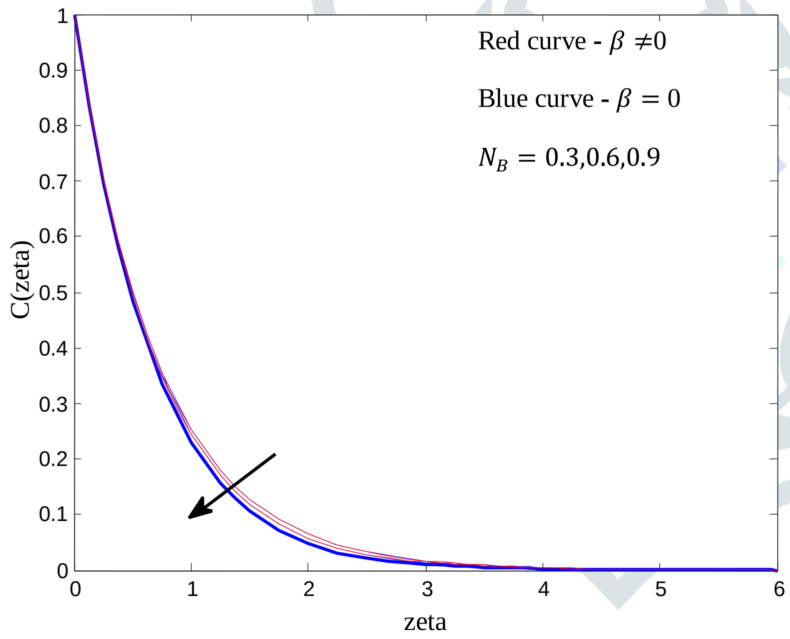

Figure 4.8 Concentration profile C(ζ) for different values of Brownian motion parameter N_B

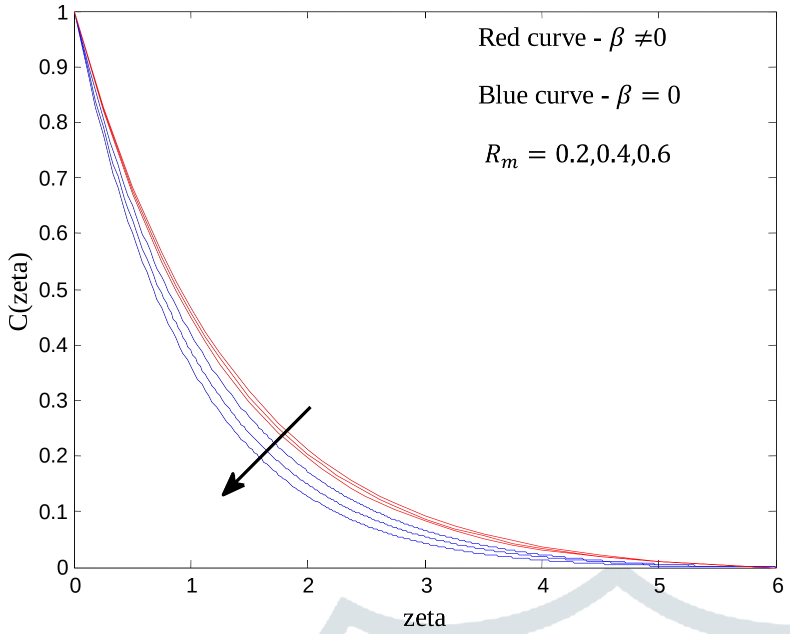

Figure 4.9 Concentration profile C(ζ) for different values of chemical reaction parameter R_m

V. RESULT AND DISCUSSION

A viscous nanofluid flow over a rotating disk in a porous medium subject to velocity slip condition is considered. Heat generation/absorption effect is taken into account by temperature equation while the chemical reaction effect is admitted by concentration equation. The properties of Thermophoresis and Brownian motion taking into account by Nanofluid model.

Velocity Profiles

Figure 4.1 illustrates the viscous fluid velocities F '(ζ) and G(ζ)) for various values of magnetic field parameter. It can be seen that the increase in magnetic field parameter cause a decline effect on both F '(ζ) and G(ζ) Higher the value of β decreases the viscous fluid velocities. Figure 4.2 demonstrate the effect of λ on both F '(ζ) and G(ζ). It shows that the both velocities decreases by increasing the velocity slip parameter λ.

Temperature Profiles

In Figure 4.3 we examined the behavior of parameter N_T on temperature profile T(ζ) in the presence and absence of magnetic field parameter ($\beta \neq 0$ and $\beta = 0$).It shows that the increasing value N_T leads to higher temperature profile T(ζ). Figure 4.4 illustrates the effect of Prandtl number on temperature profile with absence/presence of β . It is noticed that fluid temperature is decreased via increasing value of N_T. Figure 4.5 shows the effects of heat generation on fluid temperature profile for both $\beta \neq 0$ and $\beta = 0$. It is seen that the T(ζ) is increasing function of H_g . Figure 4.6 illustrates the effects of heat absorption H_a of temperature profile on both zero and non zero values of magnetic field parameter β. It depicts that the larger value of H_a leads to a lower temperature profile.

Concentration Profiles

Figures 4.7-4.9 illustrates the influence of Le , N_B, R_m on nanoparticle concentration C(ζ).
The impact of Le on concentration profile is examined in Figure 4.7 . It is seen that for both zero and non zero values of β an increase in Le cause a decay in the concentration profile C(ζ). In Figure 4.8 the influence of Brownian motion parameter N_B on concentration profile C(ζ) is depicted. It is observed that the higher value of N_B brings decrease in fluid concentration for both $\beta \neq 0$ and $\beta = 0$. The remarkable changes in nanoparticle concentration is identified by the way of Figure 4.9 The positive value of chemical reaction parameter brings decline in fluid concentration C(ζ). More precisely for both $\beta \neq 0$ and $\beta = 0$ the concentration profiles shows an inciting nature for the higher values of R_m.

VI. CONCLUSION

In this article both MHD and hydrodynamic frame of viscous nanofluid due to rotating disk with slip condition is examined. The fluid flow situation is narrated with partial slip condition, chemical reaction and heat sink/source. The numerical solution for solving governing non linear ordinary differential equation is executed Runge-kutta 5^{th} order algorithm. The following conclusions are made from the present investigations.

It is observed that the fluid velocities F'(ζ) and G(ζ) are decreasing function of magnetic field parameter β and also it exhibits decline nature towards the higher value of slip parameter λ .The Fluid temperature T(ζ) is an increasing function for high values of thermophoresis parameter N_T whereas it is also observed that a decline nature towards Pr , H_g and H_a for both MHD and hydrodynamic case.

The viscous Nanofluid concentration C(ζ) shows a decline attitude for Lewis number, Brownian motion and chemical reaction parameters in both the cases $\beta \neq 0$ and $\beta = 0$. It is observed that the fluid velocities F'(ζ) and G(ζ) are decreasing function of magnetic field parameter β and also it exhibits decline nature towards the higher value of slip parameter λ .

The Fluid temperature T(ζ) is an increasing function for high values of thermophoresis parameter N_T .It is observed that a decline nature towards Pr , H_g and H_a for both MHD and hydrodynamic case. The viscous Nanofluid concentration C(ζ) shows a decline attitude for Lewis number, Brownian motion and chemical reaction parameters in both the cases $\beta \neq 0$ and $\beta = 0$.

VII. ACKNOWLEDGEMENTS

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