Role of External Magnetic Field and Slip Velocity on Pulsatile Flow of Blood Through Stenosed Artery

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Abstract

The pulsatile motion of blood through a constricted artery has been studied theoretically. The arterial vessel has been assumed to be a cylindrical tube of circular cross-section and there is a non-uniform suspension of viscosity of blood and a prescribed volume flux, Infinite series solutions are obtained for the distributions of axial velocity and pressure gradient. Effects of hematocrit and Womersley parameters on the flow have been discussed. A mathematical model has been developed to study the influence of externally applied magnetic field on the blood flow through a mammalian blood vessel with slip velocity in the wall in the presence of a stenosis. Using the momentum integral technique, analytical expressions for the velocity profile, pressure gradient and skin-friction are obtained. The condition for an adverse pressure gradient is also deduced. It is observed that the slip velocity as well as the magnetic field bear the potential to influence the velocity distribution of blood to a considerable extent and to reduce remarkably the pressure gradient as well as the skin-friction.

Keywords : Blood flow, constricted artery, stenosis,hematocrit, Womersley parameter.

Introduction.

Localised narrowing in a blood vessel iscommonly known as stenosis in medical science. Many cardiovasculardiseases, particularly in mammalian arteries, are closely relatedto the nature of blood movement and the dynamic behaviour of bloodvessel. The disease in its severe form may lead to morbidity andfatality. Although the exact mechanism for the development ofstenosis in the lumen of artery is not clearly known, various investigators [15, 16] emphasized that some of the major factorsfor the initiation and development of this vascular disease aredue to the formation of intravascular plagues and the impingementof ligaments and spurs on wall of the blood vessel. It has beenobserved that the blood flow characteristics are significantlyaltered in the vicinity of stenotic constrictions and manyabnormalities arise in the flow pattern. Some experimentalinvestigations on models of arterial stenosis have been carriedout by *Young* and *Tsai* [6] and it was noted that the changedcharacteristics of the blood flow may have a coupling effect onthe further development of the vascular disease. Various investigators [5, 8, 21] pointed out that the study ofdifferent hydrodynamic factors such as skin-friction and pressure under normal physiological conditions and in pathological states provide useful informations for better understandingof the pathogenesis and a proper treatment of variousarterialsdiseases like myocardial infarction, stroke etc.

Different mathematical models studied by several researchers [7, 10, 14, 18, 19, 23] were investigatedto consider blood flow through stenosed blood vessels of which*Young's* [23] work may be considered as one of the earliestworks of prime importance. Lee and Fung [10] employed numericaltechniques to study the blood flow through astenosed tube.

It may be pointed out that although blood isa non-Newtonian suspension of cells in plasma, McDonald [11]remarked that for vessels at radius greater than 0.25 mm, blood maybe considered as a homogeneous Newtonian fluid. At lower shearrates blood exhibits non-Newtonian behaviour [12], but in largerarteries where the shear rate is high, blood may be considered as Newtonian [20].

It is worthwhile to mention that most of theaforementioned studies are based on the usual assumption of theno-slip condition at the vessel wall. But *Benneth* [3], on thebasis of his in-vitro experiments to study the behaviour of redcells during blood flow, suggested that there might exist thepossibility of the red cells to have a slip-velocity at the wallunder certain conditions. Subsequently, several investigators [2, 4, 13, 15] also indicated the possibility of slip-velocityat the inner surface of the wall.

On the other hand, *Barnothy* [1] reportedthat biological systems, in general, are effected by theapplication of an external magnetic field. In a recent paper,*Halder* and *Ghosh* [9] investigated the effect of magnetic fieldon blood flow through an indented tube in the presence of erythrocytes.

In the present investigation, a mathematicalmodel has been developed to study the effect of externally applieduniform magnetic field on the characteristics of blood flowthrough*stenosed* vessels, by accounting for the slip velocity attheendothelium of the blood vessel. The analytical expressionsare computed numerically in order to quantitate of the extent towhich the slip velocity and the magnetic field can influence theblood flow pattern of a given stenosed blood vessel in a specificsituation. Momentum integral technique has been employed to solvethe problem. The effects of an external magnetic field may havesome consequences in these type of situations, for example, duringMRI scanning.

The Stenosis Model.

Let us consider an axially symmetric steady,laminar flow of blood through an artery in which a mild stenosishas been developed and the fluid is acted on by an externallyapplied uniform magnetic fieldB_0. The geometry of thestenosis is shown Fig.-1 and is described as [13].

$$\frac{R(z)}{R_0} = 1 - \frac{\delta}{R_0}\exp\left(-\frac{m^2\epsilon^2 z^2}{R_0^2}\right) \qquad (1)$$

in which R(z) is the radius of the artery in the stenosed portion;R_0 denotes the radius of the artery outside the stenosis;δ and m are the height and slope of the stenosis where itinterests the vessel wall; $\epsilon = \frac{R_0}{L_0}$is therelative length of the stenosed portion; z represents the axialdistance and $2L_0$ is the length of the stenosed segment.Stenosis geometry described by equation (1) can bewritten alternatively in the form

$$\frac{R(z)}{R_0} = 1 - \alpha\exp\left(-\frac{m^2x^2}{m_0^2}\right) \qquad (2)$$

where $\alpha = \frac{\delta}{R_0}$, $x = \frac{z}{L}$, $m_0 = \frac{L_0}{L}$ and 2L is length of artery.

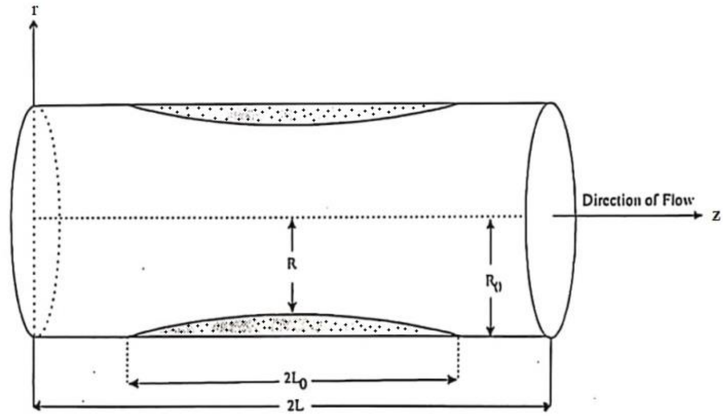

Figure 1: Geometry of the stenosis

In biological systems and particularly incase of problems of blood flow through artery, the condition ofsteady flow in general may not be valid. But the consideration ofa steady laminar flowis meaningful in certain situations as discussed below:

Blood flow in large arteries is pulsatile innature, the frequency parameter β being given by $\beta = R_0\sqrt{\frac{2\pi f}{\nu}}$, where R_0 is the radius ofthe artery, f is the frequency of the pulsation and ν is thecoefficient of kinematic viscosity of blood. The flow may betreated as quasi-steady for $\beta > 0$ in smaller arteries.*McDonald* [11] pointed out that for several blood vessels, e.g.the human femoral artery for which 2.5 <β< 3.5, thequasi-steady condition remains valid and it is also likely to bevalid in arteries much smaller than the human femoral artery. Itmay also be possible that such a quasi-steady flow exists in somelarger arteries due to an acquired constriction in a major artery [7, 5]. Thus, the assumption of steady laminar flow isjustified in that part of the arterial tree where the flow isnearly steady.

Moreover, when a stenosis develops in anartery, an immediate effect is hardening of the walls due tocomplex physiological changes. For this reason, the stenosedportion of the arterial wall may also be treated as rigid.

Governing Equations.

Let us take theartery to be a long cylindrical tube with the axis coinciding withz-axis and the motion is axially symmetric. Assuming quasi-steadycondition and the azimuthal dependence because of the rotationalsymmetry of the stenosis, the basic equations of motion in the cylindrical co-ordinate system (r, θ, z) are givenby.

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma B_0^2 u}{\rho} \quad (3)$$

$$u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}\right) \quad (4)$$

The continuity equation is

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(vr) = 0 \quad (5)$$

In the above equations, u and v represent the axial and radialvelocity components respectively; ρ the density; p, thepressure; ν the kinematic viscosity coefficient of blood; σ, the conductivity of the fluid and B_0 is theapplied external uniform transverse magnetic field.

Due to the presence of the nonlinear termsrepresenting convective acceleration, an analytical solution ofthe above system of equations seems to be difficult and hence anattempt has been made to consider an approximate solutions of the problem, by preserving theprincipal considerations regarding thestenosis geometry.

For a mild stenosis $\frac{\delta}{L_0}$is considerably small compared to unity and the normal stressgradient $\frac{\partial^2 u}{\partial z^2}$ is negligiblecompared to the shear stress $\frac{\partial^2 u}{\partial r^2}$. Also if $\frac{\delta}{L_0}$is sufficiently smallcompared to unity, the radial variation of pressure, i.e.$\frac{\partial p}{\partial r}$ may be neglected. Thus thedifferential equations determining the flow past a mild stenosismay be approximated as

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{\sigma B_0{}^2 u}{\rho} \quad (6)$$

and
$$\frac{\partial p}{\partial r} = 0 \quad (7)$$

Now integrating equation (6) over thecross-section of the vessel and using the continuity equation(5), we obtain the momentum integral equation as

$$\frac{\partial}{\partial z}\int_0^R ru^2 dr = -\frac{1}{\rho}\frac{R^2}{2}\frac{dp}{dz} + \boldsymbol{\nu} \mathrm{R}\left(\frac{\partial u}{\partial r}\right)_{r=R} - \left(\frac{\sigma B_0{}^2}{\rho}\right)\int_0^R rudr \quad (8)$$

where we have used the boundary conditions u = W (the velocity slip condition) and v = 0 at r = R.

Integrating the continuity equation (5),the volume flux Q is obtained as

$$\mathrm{Q} = \pi R^2 \bar{U} = 2\pi \int_0^R rudr \quad (9)$$

where $\bar{U}$ is the mean velocity at any givencross-section with radius R.

In the present analysis, wetake the velocity constraints as

$$u = U \text{ at } r = 0 \quad (10a)$$

$$u = W \text{ at } r = R \quad (10b)$$

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \quad (10c)$$

$$\frac{\partial^2 u}{\partial r^2} = -\frac{2U}{R^2} \text{ at } r = 0 \quad (10d)$$

and
$$\frac{dp}{dz} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{\sigma B_0{}^2 u}{\rho} \text{ at } r = R \quad (10e)$$

In the above, the first condition defines the centre linevelocity, the second is the condition of slip velocity on theartery wall, the third is the regularity condition and is deducedby considering the forces on a cylindrical fluid element in thefollowing way :

If the pressure and the inertial forces are to beinfinite as the radius of the element tends to zero, the viscousforce that is proportional to $\frac{\partial u}{\partial r}$must tend to zero. Assuming the velocity profile to be nearlyparabolic at the axis, as represented by the Poiseulle's profile$\frac{u}{\bar{U}} = 1 - \left(\frac{r}{R}\right)^2$, the secondradial derivative of u at r = 0 may be approximated by the fourth condition. Finally, the fifthcondition represents the validity of equation (6) at r = R.

Solutions.

We choose the velocity profile in thedimensionless form as

$$\hat{u} = \frac{u}{U} = \mathrm{A} + \mathrm{B}\,\eta + \mathrm{C}\eta^2 + \mathrm{D}\,\eta^3 + \mathrm{E}\,\eta^4 \quad (11)$$

where
$$\eta = \frac{R-r}{R} \quad (11a)$$

U being the centre line velocity and A, B, C, D, E are constantsto be determined from the velocity constraints. Using equations(11) and (11a) the volume flux given in (9) may bere-written as

$$\mathrm{Q} = 2\pi R^2 U \int_0^1 (1-\eta)\hat{u}\,d\eta \quad (11b)$$

The velocity constraints in terms of η are given by

$$\hat{u} = 1 \text{ at } \eta = 1 \quad (12a)$$

$$\hat{u} = \frac{W}{U} \text{ at } \eta = 0 \quad (12b)$$

$$\frac{\partial \hat{u}}{\partial \eta} = 0 \text{ at } \eta = 1 \quad (12c)$$

$$\frac{\partial^2 \hat{u}}{\partial \eta^2} = -2 \text{ at } \eta = 1 \quad (12d)$$

and

$$\frac{dp}{dz} = \frac{\mu U}{R^2(1-\eta)}\left[(1-\eta)\frac{\partial^2 \hat{u}}{\partial \eta^2} - \frac{\partial \hat{u}}{\partial \eta}\right] - \sigma B_0^2 U \hat{u} \text{at } =0 \quad (12e)$$

Applying the conditions (12a) to (12e)the velocity profile $\hat{u}$ is evaluated in the form

$$u = A + \frac{1}{7}(-\lambda + 10 - 12A)\eta + \frac{1}{7}(3\lambda + 5 - 6A)\eta^2 + \frac{1}{7}(-3\lambda - 12 + 20A)\eta^3 + \frac{1}{7}(\lambda + 4 - 9A)\eta^4 (13) \quad (13)$$

In which $\lambda = \frac{R^2}{\mu U}\left[\frac{dp}{dz} + \sigma B_0^2 W\right]$, $A = \frac{W}{U}$

(14)

From (13), it is clear that when A is known, the velocityprofile becomes a function of a single parameter $\boldsymbol{\lambda}$ whichis a function of the pressure gradient $\frac{dp}{dz}$ and themagnetic field strength B_0.

Substituting (13) into the equation(11b) and then integrating we obtain

$$U = \frac{210}{97}\frac{Q}{\pi R^2} + \frac{2}{97}\frac{R^2}{\mu}\frac{dp}{dz} + \frac{2}{97}\frac{R^2}{\mu}\sigma B_0^2 W - \frac{102}{97}W \quad (15)$$

The parameter λ can bedetermined from the integral equation (8) as

$$\lambda = \frac{4}{5}(6A - 5) + \frac{7R^2}{5\mu}\sigma B_0^2 \frac{W}{U} - \frac{14}{5\nu U}\left[\frac{\partial}{\partial z}\left\{U^2 R^2 \int_0^1 (1-\eta)\hat{u}^2 \, d\eta\right\}\right] - \frac{14R^2}{5\mu}\sigma B_0^2 \int_0^1 (1-\eta)\hat{u} \, d\eta \quad (16)$$

The subsequent part of the analysis will be carried out byneglecting the terms higher than two in the velocity profile andretaining only the Poiseuille profile [13].

$$u = 2\bar{U}\left[1 - \left(\frac{r}{R}\right)^2\right] \quad (17)$$

$$\bar{U} = -\left(\frac{R^2}{8\mu}\right)\frac{dp}{dz} \quad (17a)$$

is the average velocity at any given cross - section and $\frac{\partial p}{\partial z} < 0$.

Now substituting the value of u obtainedfrom equation (17) into the momentum integral equation (8) wehave

$$\frac{d}{dz}\left(\frac{2}{3}R^2\bar{U}^2\right) = -\frac{1}{\rho}\frac{R^2}{2}\frac{dp}{dz} + \frac{\nu}{7}(\lambda U - 10U + 12AU) - \frac{\sigma B_0^2 R^2}{\rho}\left(-\frac{\lambda U}{210} + \frac{97U}{420} + \frac{17AU}{70}\right). \quad (18)$$

In this equation if we substitute $\bar{U} = \frac{Q}{\pi R^2}$and combine the resulting equation with the (15), thepressure gradient is obtained in the form

$$\frac{dp}{dz} = \frac{776}{225}\left(\frac{\rho Q^2}{\pi^2 R^2}\right)\frac{dR}{dz} - \frac{8\mu Q}{\pi R^4} + \frac{624}{75}\frac{W\mu}{R^2} + \frac{22}{75}\sigma B_0^2 W - \frac{97}{75}\frac{Q}{\pi R^2}\sigma B_0^2 \quad (19)$$

The first term on the right hand side of equation (19) is due tothe inertia of blood, the second term is due to the viscousshearing stress, the third term is due to the slip velocity, thefourth and fifth terms represent the influence of magnetic field on the pressure gradient.

In non-dimensional form, theequation (19) is reduced to

$$\left(\frac{R_0}{\rho\bar{U}_0^2}\right)\frac{dp}{dz}=\frac{776}{255}\left(\frac{R_0}{R}\right)^5\frac{dR}{dz}-\frac{16}{R_e}\left(\frac{R_0}{R}\right)^4+\frac{1248}{75}\frac{1}{R_e}\left(\frac{R_0}{R}\right)^2\left(\frac{W}{\bar{U}_0}\right)$$
$$+\frac{22}{75}\left(\frac{\sigma B_0^2}{\rho}\right)\left(\frac{W}{\bar{U}_0}\right)\left(\frac{R_0}{\bar{U}_0}\right)-\frac{97}{75}\left(\frac{\sigma B_0^2}{\rho}\right)\left(\frac{R_0}{R}\right)^2\left(\frac{R_0}{U_0}\right) \quad (20)$$

where $R_e=\frac{2\rho R_0\bar{U}_0}{\mu}$ (20a)

is the Reynolds number upstream from the stenosis,$\bar{U}_0$ being the average velocity at a cross-section of the normal artery.

The condition for an average pressure gradient to develop (when $\frac{dp}{dz}>0$) is

$$R_e\left(\frac{R_0}{R}\right)\frac{dR}{dz}\geq 4.64-4.82\left(\frac{W}{\bar{U}_0}\right)\left(\frac{R_0}{R}\right)^{-2}-0.09\left(\frac{\sigma B_0^2}{\rho}\right)\left(\frac{W}{\bar{U}_0}\right)\left(\frac{R_0}{\bar{U}_0}\right)\left(\frac{R_0}{R}\right)^{-5}R_e$$
$$+0.38\left(\frac{\sigma B_0^2}{\rho}\right)\left(\frac{R_0}{R}\right)^{-3}\left(\frac{R_0}{\bar{U}_0}\right)R_e \quad (21)$$

Using equations (15) and (18), the velocity distribution u isobtained from (13) as a function of r and z in the form

$$\frac{u}{\bar{U}_0}=R_e\left(\frac{R_0}{R}\right)^3\frac{dR}{dz}f(\eta)+2\left(\frac{R_0}{R}\right)^2(2\eta-\eta^2)+\frac{W}{\bar{U}_0}g(\eta)$$
$$-\left(\frac{R_0}{R}\right)^2M^2\frac{W}{\bar{U}_0}\varphi(\eta)+M^2\varphi(\eta) \quad (22)$$

where $f(\eta)=-0.2\eta+0.76\eta^2-0.8\eta^3+0.24\eta^4(23a)$ (23a)

$g(\eta)=1-4.16\eta+2.08\eta^2+0.8\eta^3-0.6\eta^4$ (23b)

$\varphi(\eta)=0.15\eta-0.57\eta^2+0.6\eta^3-0.2\eta^4$

and $M=B_0R_0\sqrt{\frac{\sigma}{\mu}}$ = Hartmann number (23d)

The skin-friction τ_w is given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial r}\right)_{r=R} \quad (24)$$

which in non-dimensional form is obtained with the help of (22) as

$$\frac{\tau_w}{\rho\bar{U}_0^2}=0.4\left(\frac{R_0}{R}\right)^4\frac{dR}{dz}-\frac{8}{R_e}\left(\frac{R_0}{R}\right)^3+\frac{8.32}{R_e}\left(\frac{R_0}{R}\right)\left(\frac{W}{\bar{U}_0}\right)+\frac{0.3}{R_e}\left(\frac{R_0}{R}\right)M^2\left(\frac{W}{\bar{U}_0}\right)$$
$$-\frac{0.3}{R_e}\left(\frac{R_0}{R}\right)M^2 \quad (25)$$

In the case of incipient separation for which the Reynolds numberis just enough to cause separation, the separation location in thediverging section of the stenosis is given by the condition that $\frac{1}{R}\frac{dp}{dz}$ is maximum, which demands that

$$R\left(\frac{d^2R}{dz^2}\right)=\left(\frac{dR}{dz}\right)^2 \quad (27)$$

For the stenosis geometry defined by equation (1), the location$\frac{z}{L_0}$ of the initial point of separation is given bythe relation

$$e^{\frac{m^2z^2}{L_0^2}}\left(1-\frac{2m^2z^2}{L_0^2}\right)=\frac{\delta}{R_0}\frac{2m^2z^2}{L_0^2} \quad (28)$$

$$\left(\frac{z}{L_0}\right)^2\approx\frac{1}{4m^2}\left[\sqrt{\left(9-4\frac{\delta}{R_0}\right)}-\left(1+2\frac{\delta}{R_0}\right)\right] \quad (29)$$

Results and Discussions.

The analytical expressions derived in the previous section havebeen computed numerically for different Reynolds and Hartmannnumbers. The aim of the computational work is to quantify the influence of the magnetic field and the slip velocity at the wallon the velocity distribution. The computation has been carried outat the location defined by z=0.06 for three different values of Reynolds number Re=100, 300, 500 and Hartmann number M given byM^2 = 0, 9, 18. The slip velocity has been taken to be equalto 10% of the average velocity of blood in a normal artery[13]. The length of the stenosis has been taken to be 20 mmwhile the maximum depth of the stenosis is assumed to be 0.2 mm.Figures 2, 3 and 4 illustrate the variation of thenon-dimensional axial velocity of blood flow in the stenosedarterial segment for different Hartmann number. It may be observedthat the magnetic field increasesthe blood velocity near the wall but decreases it near the central axisof the artery. Figures 5, 6 and 7 predict the same behaviour of bloodwithout slip velocity. The variation of blood flow with and withoutslip velocity has been shown in figure 8 for Re= 500 andM^2= 9. It is noted that the slip velocity increases the flowvery near to the wall but decreases it as we pass on to thecentre. In figure 9, the effect of Reynolds number on the bloodvelocity has been shown and the influence is to reduce thevelocity with increasing Reynolds number near the wall and then toincrease.

Conclusions.

Although the present investigation of themathematical model of blood flow through a stenosed segment of theartery is based on some approximations, it bears the potential toreveal some characteristics of the problem. The model firmlyestablishes the fact that the velocity slip at the wall of thearterial segment as well as the magnetic field enhance the axialvelocity of the blood.

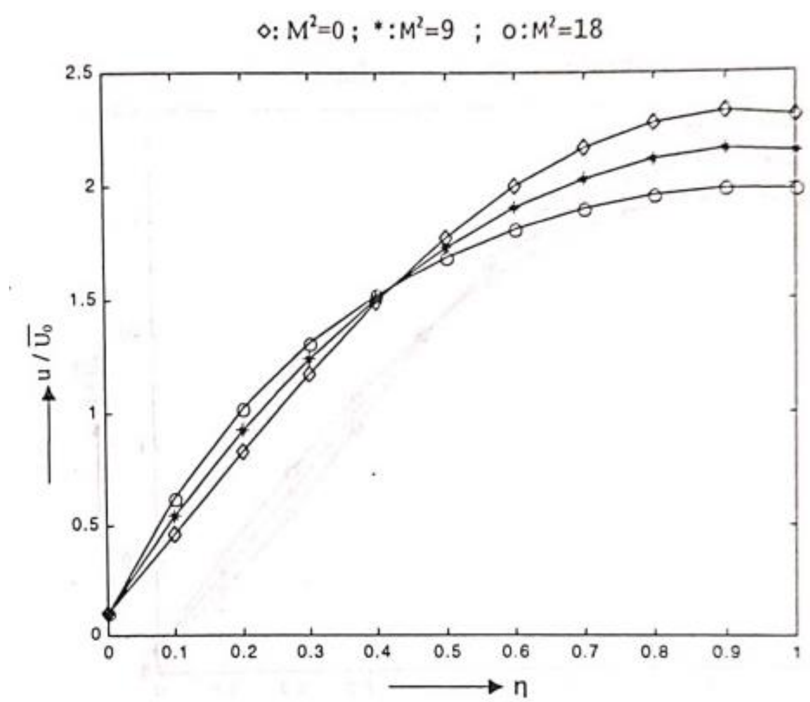

Figure 2: Variation of velocity for Re = 100 considering slip velocity.

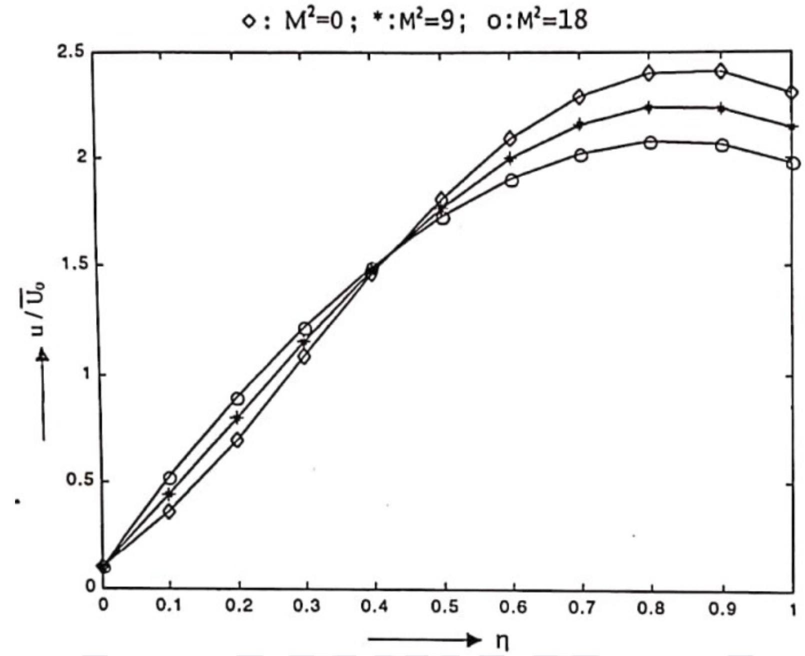

Figure 3: Variation of velocity for Re = 300 considering slip velocity.

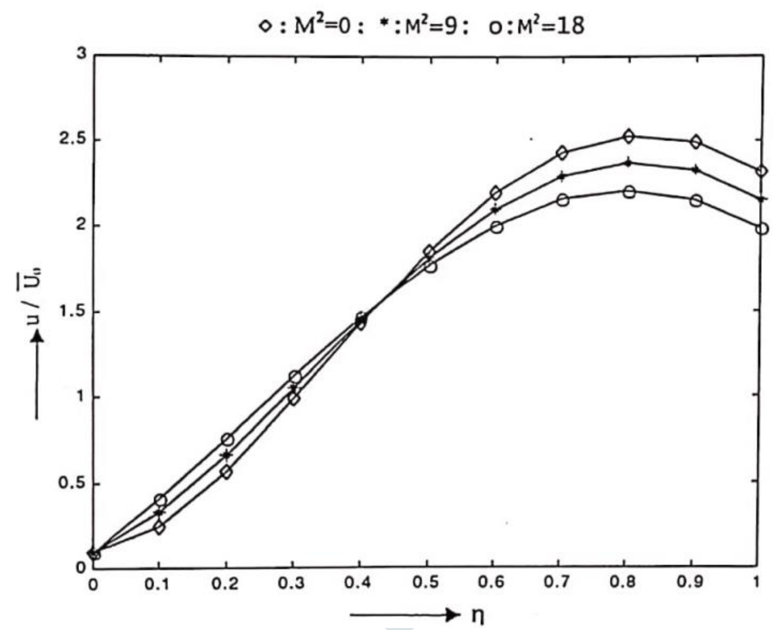

Figure 4: Variation of velocity for Re = 600 considering slip velocity.

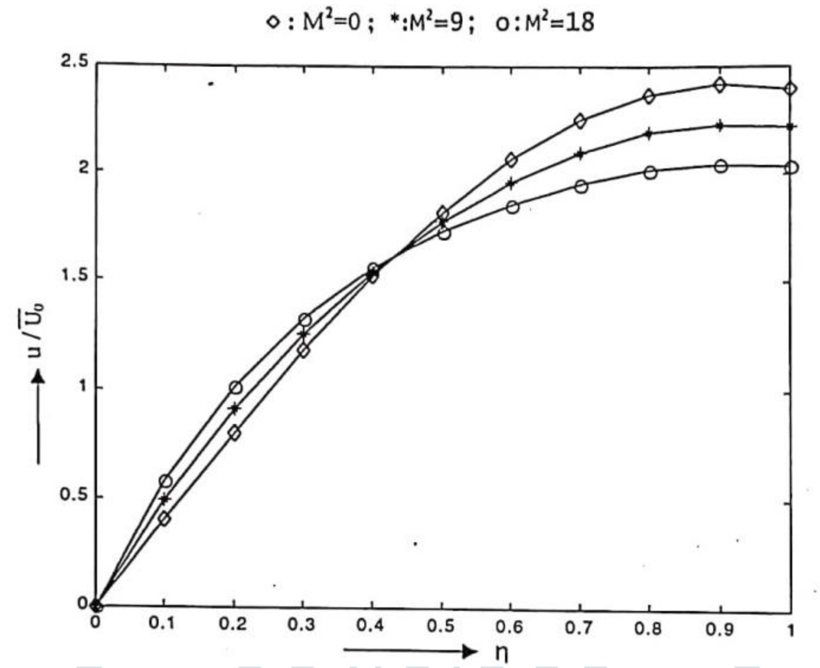

Figure 5: Variation of velocity for Re = 100 ignoring slip velocity.

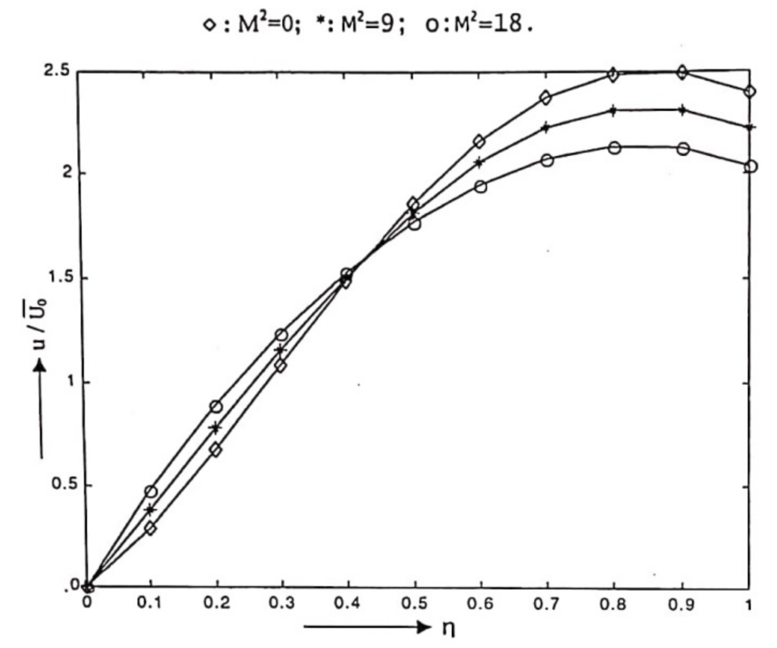

Figure 6: Variation of velocity for Re = 300 ignoring slip velocity.

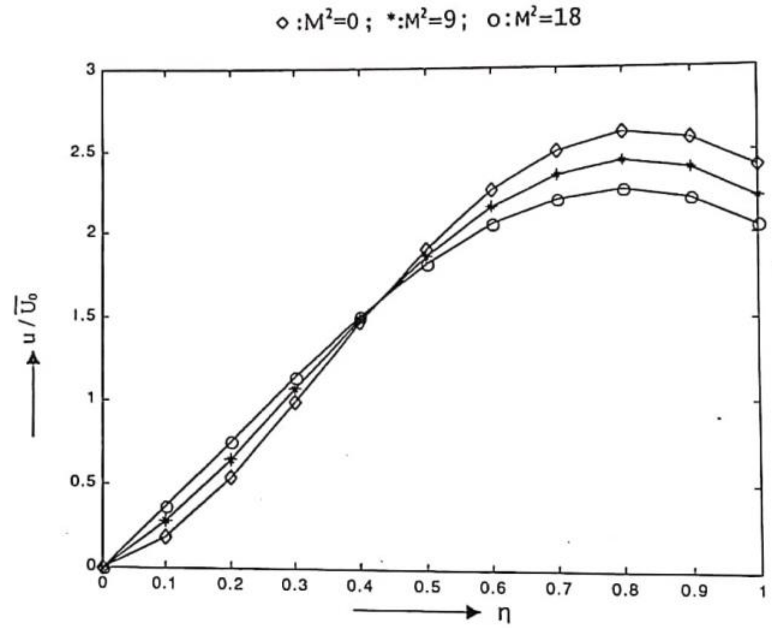

Figure 7: Variation of velocity for Re = 600 ignoring slip velocity.

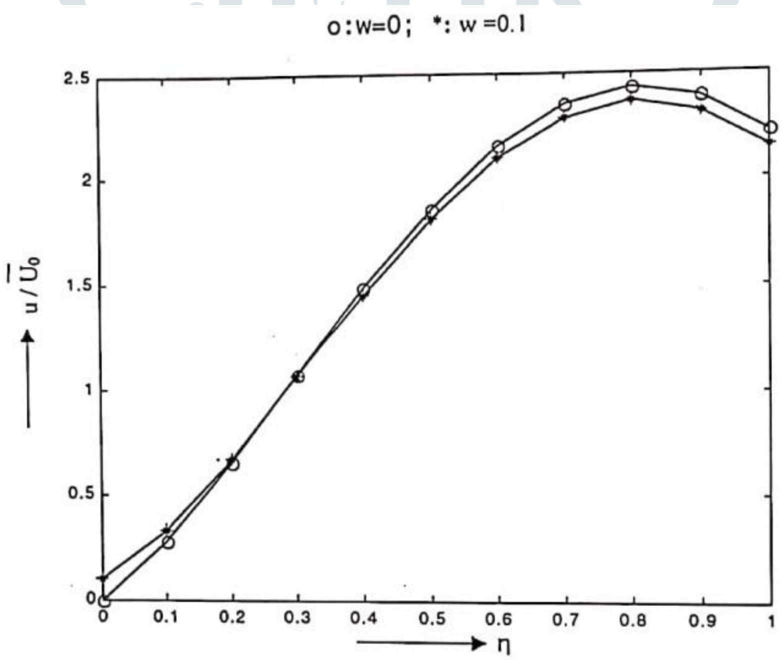

Figure 8: Variation of velocity for Re = 600 with $M^2 = 9$

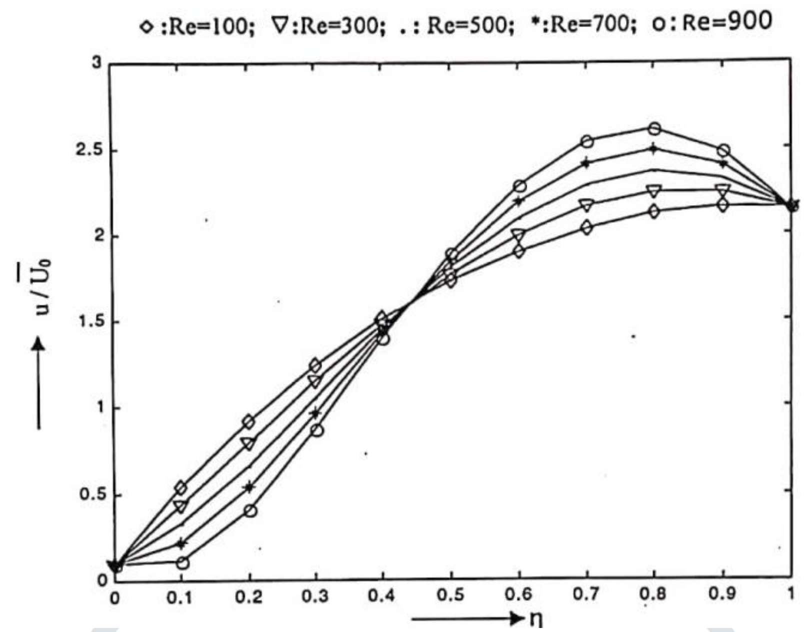

Figure 9: Variation of velocity for $M^2 = 9$and W = 0.1.

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