Some Definitions of Ideals In Ternary Semigroup

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Abstract: A ternary semigroup is a nonempty set together with a ternary multiplication which is associative. Any semigroup can be reduced to a ternary semigroup but a ternary semigroup does not necessarily reduce to a semigroup. In this paper, we observed some different types of ideals in ternary semigroups.

Key Words: ternary semigroups,Ideals,Co-Ideals, Pseudo Symmetric Ideals.

Introduction and Preliminaries

Algebraic structures play a prominent role in mathematics with wide applications in many disciplines such as computer sciences, information sciences, engineering, physics etc. The theory of ternary algebraic system was introduced by Lehmer [1] in 1932, but earlier such structures was studied by Kasner [2] who give the idea of n-ary algebras. Lehmer investigated certain algebraic systems called triplexes which turn out to be commutative ternary groups. Ternary semigroups are universal algebras with one associative ternary operation. The notion of ternary semigroup was known to Banach (cf. Los [3]) who is credited with example of a ternary semigroup which can not reduce to a semigroup.

A nonempty set T with a ternary operation $\mathrm{T} \times \mathrm{T} \times \mathrm{T} \rightarrow \mathrm{T}$, written as $(x_1, x_2, x_3) \rightarrow [x_1x_2x_3]$ is called a ternary semigroup if it satisfies the following associative law holds:

$$\big[[x_1x_2x_3]x_4x_5\big] = [x_1[x_2x_3x_4]x_5] = [x_1x_2[x_3x_4x_5]]$$

for any $x_1, x_2, x_3, x_{4,}x_5 \in T$. In this paper, we denote $(x_1, x_2, x_3)\ by\ [x_1x_2x_3]$. We can see that any semigroup can be reduced to a ternary semigroup. Banach showed that a ternary semigroup does not necessarily reduce to a semigroup by this example.[4]. SIOSON [5] introduced the ideal theory in ternary semigroups. He also introduced the notion of regular ternary semigroup and characterized by using the properties of quasi-ideals.

Example 1.1. T = {−i, 0, i} is a ternary semigroup while T is not a semigroup under the multiplication.

Let T be a ternary semigroup. For nonempty subsets A, B and C of T, let ABC := {abc | a ∈ A, b ∈ B and c ∈ C}. A nonempty subset S of T is called a ternary subsemigroup if SSS ⊆ S

A nonempty subset A of a ternary semigroup T is called a left ideal of T if TTA ⊆ A, a right ideal of T if ATT ⊆ A and a lateral ideal of T if TAT ⊆ A. If A is a left, right and lateral ideal of T, A is called an ideal of T. A ternary subsemigroup B of T is called a bi-ideal of T if BTBTB ⊆ B. A ternary subsemigroup Q of T is called a quasi-ideal of T if QTT ∩ (TTQTT ∪ TQT) ∩ TTQ ⊆ Q. Every right, left and lateral ideal of T is a quasi-ideal of T. But the converse is not true in general. Moreover, the intersection of left ideal, right ideal and lateral ideal of T is a quasi-ideal of T and every quasi-ideal of T can be obtained in this way. In [6], Dixit and Dewan proved that every quasi-ideal of T is a bi-ideal of T but the converse is not true in general by giving example. Moreover, Dixit and Dewan studied minimal quasi-ideals in ternary semigroups in [7].

Definition 1.1. [8] A ternary subsemigroup I of a ternary semigroup S is called

(1) a left ideal of S if SSI ⊆ I,

(2) a right ideal of S if ISS ⊆ I,

(3) a lateral ideal of S if SIS ⊆ I,

(4) a two-sided ideal of S if I is both left and right ideal of S,

(5) an ideal of S if I is a left, a right, a lateral ideal of S. An ideal I of a ternary semigroup S is called a proper ideal if $I \neq S$.

2. Co-Ideals in Ternary Semigroup

Definition 2.1.[9] A non-empty subset I of a ternary semigroup S is called a co-ideal

if (1) a, b, c ∈ I implies abc ∈ I,

(2) a ∈ I, s ∈ S implies as ∈ I.

Theorem 2.1. Every ideal is a co-ideal.

Proof. Let I is an ideal of ternary semigroup of S and a, b, c ∈ I. Then abc ∈ I. Let a ∈ I and s ∈ S then as ∈ I, since I is an ideal of S. Thus I is a co-ideal of S.[10]

3. LATERAL TERNARY IDEALS

Definition 3.1: A nonempty subset of a ternary semigroup T is said to be a lateral ternary ideal or simply lateral ideal of T if b, c ∈ T , a ∈ A implies bac ∈ A.

Theorem 3.1: The nonempty intersection of any two lateral ideals of a ternary semigroup T is a lateral ideal of T.

Proof : Let A, B be two lateral ideals of T. Let a ∈ A∩B and b, c ∈ T.

a ∈ A∩B ⇒ a ∈ A and a ∈ B.

a ∈A ; b, c ∈ T , A is a lateral ideal of T ⇒ bac∈ A.

a ∈ B ; b, c ∈ T , B is a lateral ideal of T ⇒ bac∈ B. bac∈ A, bac∈ B ⇒ bac∈ A∩B . Therefore A∩B is a lateral ideal of T.

4. . Different Ideals In Ternary semigroup[11]

Definition 4.1: A nonempty subset A of a ternary semigroup T is said to be left ideal of T if b, c∈ T, a∈ A implies bca ∈ A.

Definition 4.2: A nonempty subset A of a ternary semigroup T is a right ideal of T if b, c ∈T , a ∈A implies abc ∈A.

Definition 4.3: A nonempty subset A of a ternary semigroup T is a two sided ideal of T if b, c∈ T , a∈ A implies bca∈ A, abc ∈A.

NOTE : A nonempty subset A of a ternary semigroup T is a two sided ideal of T if and only if it is both a left ideal and a right ideal of T .

Definition 4.4: A nonempty subset A of a ternary semigroup T is said to be ternary ideal or simply an ideal of T if b, c∈ T , a∈ A implies bca ∈A, bac A, abc∈ A.

NOTE : A nonempty subset A of a ternary semigroup T is an ideal of T if and only if it is left ideal, lateral ideal and right ideal of T .

Definition 4.5: An ideal A of a ternary semigroup T is said to be a proper ideal of T if A is different from T.

Definition 4.6: An ideal A of a ternary semigroup T is said to be a trivial ideal provided T\ A is singleton. **Definition 4.7:** An ideal A of a ternary semigroup T is said to be a maximal left ideal provided A is a proper left ideal of T and is not properly contained in any proper left ideal of T.

Definition 4.8: An ideal A of a ternary semigroup T is said to be a maximal lateral ideal provided A is a proper lateral ideal of T and is not properly contained in any proper lateral ideal of T. **Definition 4.9:** An ideal A of a ternary semigroup T is said to be a maximal right ideal provided A is a proper right ideal of T and is not properly contained in any proper right ideal of T. **Definition 4.10:** An ideal A of a ternary semigroup T is said to be a maximal two sided ideal provided A is a proper two sided ideal of T and is not properly contained in any proper two sided ideal of T.

Definition 4.11: An ideal A of a ternary semigroup T is said to be a maximal ideal provided A is a proper ideal of T and is not properly contained in any proper ideal of T.

Definition 4.12:[11] A ternary semigroup T is said to be left pseudo commutative provided abcde = bcade = cabde = bacde = cbade = acbde ∀ a,b,c,d,e ∈ T.

Definition 4.13: A ternary semigroup T is said to be a lateral pseudo commutative ternary semigroup provide abcde = acdbe = adbce = acbde = adcbe = abdce for all a,b,c,d,e ∈ T. **Definition 4.14:** A ternary semigroup T is said to be right pseudo commutative provided

abcde = abdec = abecd = abdce = abedc = abced ∀ a,b,c,d,e ∈ T.

5. PSEUDO SYMMETRIC IDEALS[11]

Definition 5.1: An ideal A of a ternary semigroup T is said to be pseudo symmetric provided x, y, z ∈ T, xyz∈ A implies xsytz ∈ A for all s, t ∈ T.

Definition 5.2: An ideal A in a semigroup T is said to be semi pseudo symmetric provided for any odd natural number n, x ∈ T, x^n∈ A ⇒ $<x>^n$ ⊆ A.

6. Other Definition

Definition 6.1: An element a of a ternary semigroup T is said to be zero of T provided

abc = bac = bca = a ∀ b,c∈ T.

THEOREM 6.1: Any ternary semigroup has at most one zero element.

Proof : Let a,b,c be three zeros of a ternary semigroup T. Now a can be considered as a left zero , b can be considered as a lateral zero and c can be considered as a right zero of T. By theorem 1.2.26, a = b = c. Then T has at most one zero.

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