NEW PROPERTIES OF 3x3 MAGIC SQUARE WITH COMPLEX NUMBERS

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Abstract : In this paper, we established some new results in the area of square magic having entries as complex numbers and proved some new properties, in which one is M(A+α)+ M(A-α)= 2X M(A). Also, some new properties of Magic Constant of a Magic Square with complex numbers(MSCN) and it's derived matrix in the view of Eigenvalues, Eigen vectors have been established. This paper is organized as introduction, definition & preliminaries, methodology and results in relation to 3X3 square magic (MSCN).

Keywords: Magic Square, Magic Constant, Matrix, Eigen Value and Eigen Vector.

MSC Classification: 97A20, 97A40, 97A80.

1. INTRODUCTION :

The development of Magic Squares (MSCN) in the world reached to the conclusion that the Magic Square in all forms are very interesting and having potential of wide application in balancing of wheels and other area of scientific applications like finding center of gravity of lamina. Interested readers may visit [1, 2, 3, 4, 5, and 6].

In this paper, we established some properties of 3X3 square magic (MSCN) having virtue in association with its magic constant, Eigen values and Eigen vectors.

2. Definition & Preliminaries:

A Magic Square of order n is a square matrix having array of n^2 numbers such that the sum of each row and column as well as the main diagonal and main back diagonal [1], is the same number called Magic Constant (Magic Sum or Lowest Required Sum). If X is a Magic Square and each element of another same order square matrix by addition, subtraction, multiplication or division to the corresponding element of X by the same number (not 0 for multiplication or division), then new square matrix will be a Magic Square.
An upper bound for the number of normal Magic Squares of order n can be given by $n^2!/8(2n+1)$. There is only one distinct third order normal Magic Square with lowest sum.

3. Methodology:

Example 3.1: Let us consider a third-order Magic Square.

	6+6i	5+5i	10+10i	
$A=$	11+11i	7+7i	3+3i	
	4+4i	9+9i	8+8i	
				21+21i

The Magic Constant of the Magic Square is M (A) =21+21i.
Taking a combination of the two numbers 1 and 20 of the Magic Constant 21+21i such that 2+16. Now, adding 2+2i to each element of the Magic Square A, we get a Magic Square A_{2+2i} (Say).

	8+8i	7+7i	12+12i	
A_{2+2i} =	13+13i	9+9i	5+5i	
	6+6i	11+11i	10+10i	
				27+27i

$M(A_{2+2i}) = 27 + 27\text{i}$

Similarly,

	22+22i	21+21i	26+26i	
A_{16+16i} =	27+27i	23+23i	19+19i	
	20+20i	25+25i	24+24i	
				69+69i

$M(A_{16+16i}) = 69+69\text{i}$.

	4+4i	3+3i	8+8i	
A_{-2-2i} =	9+9i	5+5i	1+i	
	2+2i	7+7i	6+6i	
				15+15i

$M(A_{-2-2i}) = 15+15\text{i}$.

	-10-10i	-11-11i	-6-6i	
$A_{-16-16i}$ =	-5-5i	-9-9i	-13-13i	
	-12-12i	-7-7i	-8-8i	
				-27-27i

$M(A_{-16-16i}) = -27 - 27i.$

Now Combining

$M(A_{16+16i}) + M(A_{-16-16i}) = 69 + 69i + (-27) - 27i = 2*(21+21i) = 2M(A)$

$M(A_{2+2i}) + M(A_{-2-2i}) = 27 + 27i + 15 + 15i = 2*(21+21i) = 2M(A)$

4. Results and discussion:

A square matrix X has an Eigen vector w, then $(X + wY^T)$ is a new matrix, still have the Eigen vector w, where Y is any compatible vector of choice and Eigen vector w is associated with the Eigen value β of A. After Eigen decomposition process, the Eigen values and Eigen vectors of these obtained matrices are also linearly related together. Eigen vector of one matrix are the basis of an invariant subspace within the range of the corresponding linear map.

Hence, it is possible to find a large number of essentially different matrices. But it is a difficult and time consuming task.

Adding matrices together will seriously change the geometry of their corresponding linear maps and linear subspaces. The places where they overlap will stay the same but everywhere they differ gets thrown out.

But in our methodology they always overlap.

Discussion on Example 3.1

M(A)=21+21i, multiplicity =1

β_1 =21+21i, β_2 =-4.9+4.9i, β_3 = 4.9-4.9i

w_1 =(1,1,1), w_2 =(-0.2-0.979795i, 0.8+0.979795i,1) w_3 =(-0.2+0.979795i, -0.8- 0.979795i,1)

M(A_{16+16i})=69+69i, multiplicity =1

β_1 =69+69i, β_2 =-4.9+4.9i, β_3 = 4.9-4.9i

w_1 =(1,1,1), w_2 =(-0.2-0.979795i, 0.8+0.979795i,1) w_3 =(-0.2+0.979795i, -0.8- 0.979795i,1)

M(A_{2+2i})=27+27i

β_1 =24, β_2 =-4.9+4.9i, β_3 = 4.9-4.9i

w_1 =(1,1,1), w_2 =(-0.2-0.979795i, 0.8+0.979795i,1) w_3 =(-0.2+0.979795i, -0.8- 0.979795i,1)

M($A_{-16-16i}$)=-27-27i

β_1 =-27-27i, β_2 =-4.9+4.9i, β_3 = 4.9-4.9i

w_1 =(1,1,1), w_2 =(-0.2-0.979795i, 0.8+0.979795i,1) w_3 =(-0.2+0.979795i, -0.8- 0.979795i,1)

M(A_{-2-2i})= 15+15i

β_1 =15+15i, β_2 =-4.9+4.9i, β_3 = 4.9-4.9i

w_1 =(1,1,1), w_2 =(-0.2-0.979795i, 0.8+0.979795i,1) w_3 =(-0.2+0.979795i, -0.8- 0.979795i,1)

Conclusion:

Adding matrices together will seriously change the geometry of their corresponding linear maps and linear subspaces. The places where they overlap will stay the same but everywhere they differ gets thrown out.

But in our methodology they always overlap.

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