Degree Splitting of Contra Harmonic Mean Graphs

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ABSTRACT

In this paper we contribute some new results on Contra Harmonic Mean Labeling of graphs. We investigate on some standard graphs that accept Contra Harmonic Mean Labeling and we proved that the Degree Splitting of these Contra Harmonic Mean Labeling graphs are also Contra Harmonic Mean graphs.

Keyword: Contra Harmonic Mean Graph, DS(P_3), DS(P_4), DS($P_3 \odot K_1$), DS($K_{1,3}$), etc.

1. Introduction

By a graph we mean a finite undirected graph without self-loops or parallel edges. The vertex set is denoted by V (G) and the edge set is denoted by E (G). A path of length 'n' is denoted by P_n and the cycle of length 'n' is denoted by C_n. For all other standard terminology and notations we follow Harary [2]. For a detailed survey of graph labeling we refer to J.A.Gallian [1]. The concept of Harmonic Mean Labeling has been introduced by S.Somosundaram and S.S Sandhya in 2012 [3]. The concept of Super Geometric Mean Labeling has been introduced by S.S.Sandhya, E.Ebin Raja Merly and B.Shiny in 2015 [4]. The Concept of Contra Harmonic Mean Labeling has been introduced by S.S Sandhya, S.Somosundaram and J.Rajeshni Golda in 2017. [5] Silviya Francies, V. Balaji, 2017, 'Mean Labeling on Degree Splitting graph of Star graph', International journal of Advances in applied Mathematics and Mechanics [6].

In this paper we investigate Degree Splitting of some standard Contra Harmonic Mean graphs. We will provide a brief summary of definitions and other information which are necessary for our present investigation.

Definition 1.1

A Graph G = (V, E) with *p* vertices and *q* edges is called a Harmonic Mean graph if it is possible to label the vertices $x \in V$ with distinct labels *f (x)* from 1,2…..*q*+1 in such a way that when each edge *e=uv* is labeled with $f(uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the edge labels are distinct,. In this case f is called **Harmonic Mean Labeling** of *G*.

Definition 1.2

A Graph *G* (V, E) is called a Contra Harmonic Mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct element *f(x)* from 0,1,…..*q* in such a way that when each edge *e=uv* is labeled with $f(e = uv) = \left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels. Here *f* is called a **Contra Harmonic Mean Labeling** of G.

Definition 1.3

Let G = (V, E) be a graph with $V = S_1 \cup S_2 \cup \ldots \cup S_t \cup T$, Where each Si is a set of vertices having atleast two vertices and $T = V - \cup S_i$. The **degree splitting graph** of G is denoted by DS (G) and is obtained from G by adding vertices $w_1, w_2, \ldots\ldots w_t$ and joining w_i to each vertex of S_i ($1 \le i \le t$). The graph G and its degree splitting graph DS (G) are given in Figure 1.

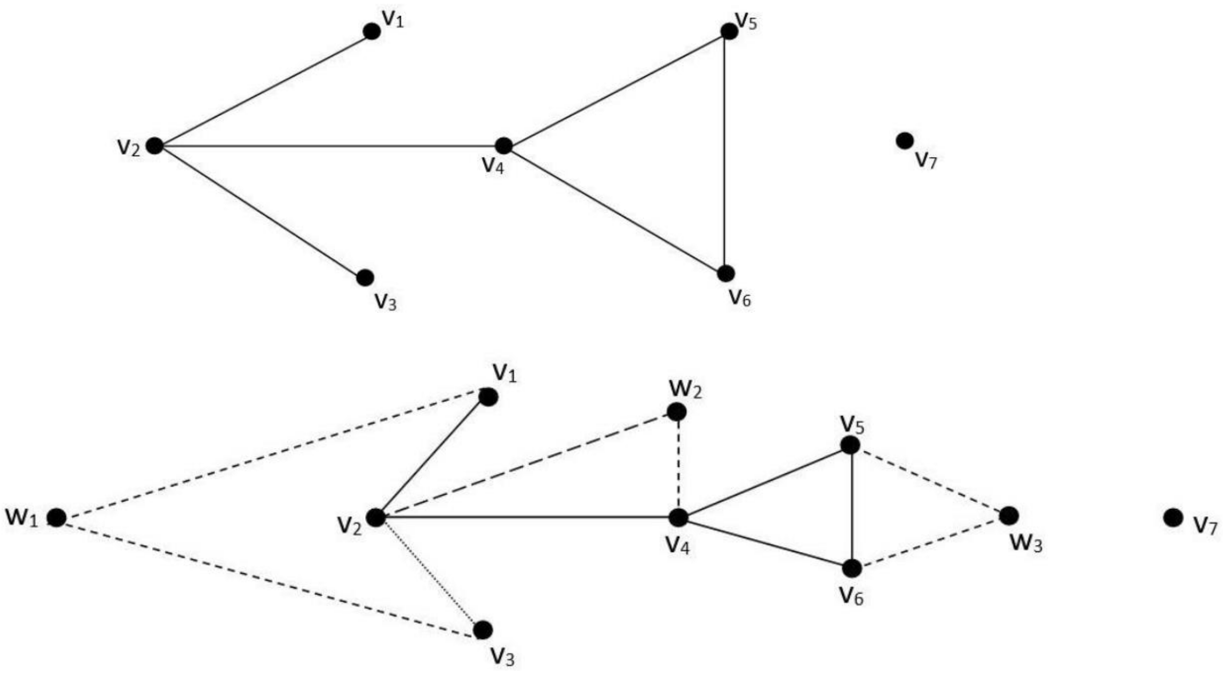

Figure: 1

Definition 1.4

A **Path** P_n is a walk in which all the vertices are distinct.

Definition 1.5

The graph obtained by joining a single pendent edge to each vertex of a path P_n is called **$P_n \odot K_1$ graph**.

Definition 1.6

The graph obtained by joining K_2 to each vertex of a path P_n is called **$P_n \odot K_2$ graph**.

Definition 1.7

A **Complete Bipartite graph** $K_{m,n}$ is a bipartite graph with bipartition (V_1, V_2) such that every vertex of V_1 is joined to all the vertices of V_2, where $|V_1| = m$ and $|V_2| = n$. A **star graph** is the complete bipartite graph $K_{1,n}$.

Definition 1.8

A **Cycle** C_n is a closed path.

Theorem 1.9: Any path P_n is a Contra Harmonic Mean graph.

Theorem 1.10: Any cycle C_n $n \geq 3$ is a Contra Harmonic Mean graph.

Theorem 1.11: Star $K_{1,n}$ is a Contra Harmonic Mean graph.

Theorem 1.12: $P_n \odot K_1$ is a Contra Harmonic Mean graph.

2. Main Results

Theorem: 2.1

The graph nDS (P_3) is a Contra Harmonic Mean graph.

Proof:-

Let G =nDS(P_3). Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$.

Let $V = \{v_1^i, v_2^i, v_3^i, w_1^i / 1 \leq i \leq n\}$ be the vertex set of i^{th} copy of DS (P_3)

The graph DS (P_3) is given below,

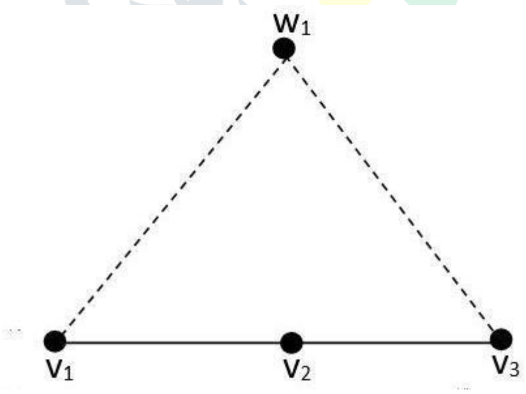

Figure: 2

Define a function $f: V(G) \rightarrow \{0,1, \ldots\ldots.q\}$ by

$f(v_1^1) = 0$

$f(v_1^i) = 4i - 3, 2 \le i \le n$

$f(v_2^1) = 1$

$f(v_2^i) = 4i - 2, 2 \le i \le n$

$f(v_3^1) = 2$

$f(v_3^i) = 4i - 1, 2 \le i \le n$

$f(w_i) = 4i, 1 \le i \le n.$

$f(v_1^i v_2^i) = 4i - 3, 1 \le i \le n.$

$f(v_2^i v_3^i) = 4i - 2, 1 \le i \le n.$

$f(v_1^1 w_1^1) = 4$

$f(v_1^i w_1^i) = 4i - 1, 2 \le i \le n.$

$f(v_3^1 w_1^1) = 3,$

$f(v_3^i w_1^i) = 4i - 1, 2 \le i \le n.$

Thus nDS(P_3) admits Contra Harmonic Mean Labeling. Hence nDS (P_3) is a Contra Harmonic Mean graph.

Example 2.2:

Contra Harmonic Mean Labeling of 4DS(P_3) is shown in Figure 3.

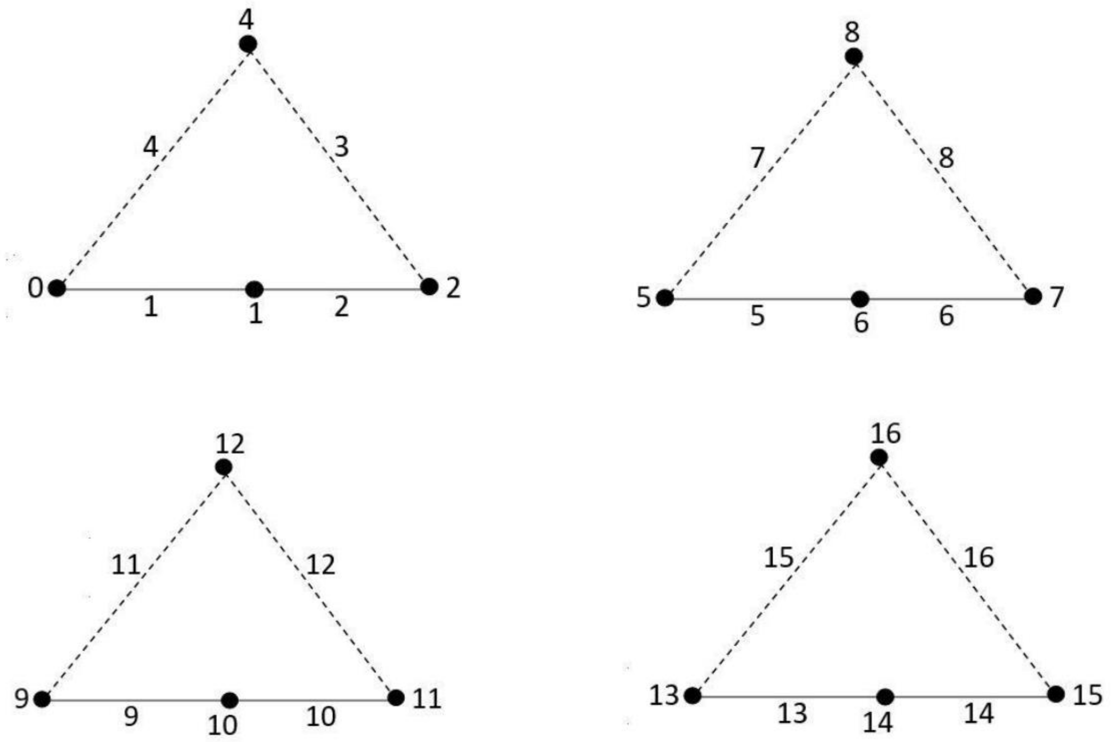

Figure: 3

Theorem: 2.3

The graph nDS (P_4) is a Contra Harmonic Mean graph.

Proof:-

Let G = nDS (P_4). Let the vertex set of G be V= $V_1 \cup V_2 \cup \ldots \cup V_n$. Let V= { v_1^i, v_2^i, v_3^i, v_4^i, w_1^i, w_2^i / $1 \leq i \leq n$ } be the vertex set of i^{th}copy of DS (P_4)

The graph DS (P_4) is shown in Figure 4

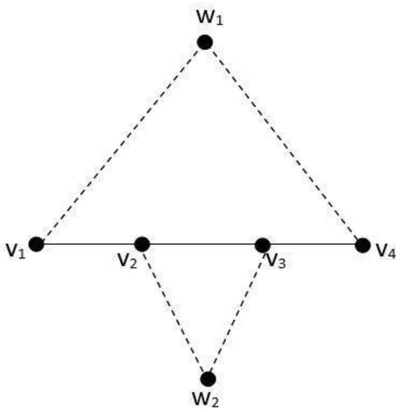

Figure: 4

Define a function $f : V(G) \rightarrow \{0, 1, \ldots.. q\}$ by

$f(V_1^{1)}) = 0$
$f(v_1^i) = 7i - 5, 2 \leq i \leq n$
$f(v_2^{1)}) = 1$
$f(v_2^i) = 7i - 3, 2 \leq i \leq n$
$f(v_3^1) = 2$
$f(v_3^i) = 7i - 1, 2 \leq i \leq n$
$f(v_4^1) = 5$
$f(v_4^i) = 7i - 4, 2 \leq i \leq n$
$f(w_1^1) = 3$
$f(w_1^i) = 7i - 6, 2 \leq i \leq n$
$f(w_2^i) = 7i, 1 \leq i \leq n$

The edges are labeled with

$f(v_1^1 v_2^1) = 1$
$f(v_1^i v_2^i) = 7i - 4, 2 \leq i \leq n$

$f(v_2^1 v_3^1) = 2$

$f(v_2^i v_3^i) = 7i - 2,\ 2 \leq i \leq n$

$f(v_3^i v_4^i) = 7i - 3,\ 1 \leq i \leq n$

$f(v_1^1 w_1^1) = 3$

$f(v_1^i w_1^i) = 7i - 6,\ 2 \leq i \leq n$

$f(v_2^1 w_2^1) = 7$

$f(v_2^i w_2^i) = 7i - 1,\ 2 \leq i \leq n$

$f(v_4^1 w_1^1) = 5$

$f(v_4^i w_1^i) = 7i - 5,\ 2 \leq i \leq n$

$f(v_4^1 w_2^1) = 6$

$f(v_4^i w_2^i) = 7i,\ 2 \leq i \leq n$

Thus both vertices and edges get distinct labels. Hence nDS (P_4) is a Contra Harmonic Mean graph.

Example: 2.4

Contra Harmonic Mean labeling of 4DS(P_4) is given in Figure:5

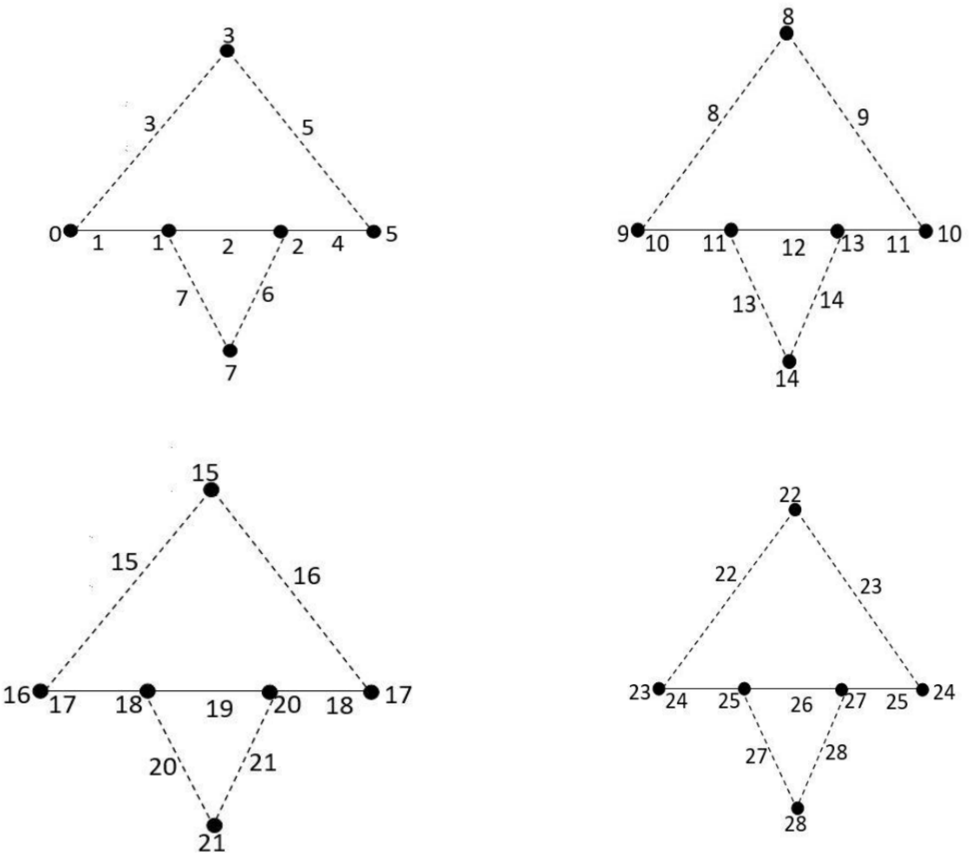

Figure:5

Theorem:- 2.5

The graph nDS ($P_2 \odot K_1$) is a Contra Harmonic Mean graph.

Proof:-

Let G = nDS ($P_2 \odot K_1$). Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots\ldots \cup V_n$. Let

$V = \{v_1^i, v_2^i, v_3^i, v_4^i, w_1^i, w_2^i \,/\, 1 \le i \le n\}$ be the vertex set of i^{th} copy of DS ($P_2 \odot K_1$).

The graph DS ($P_2 \odot K_1$) is shown in Figure 6.

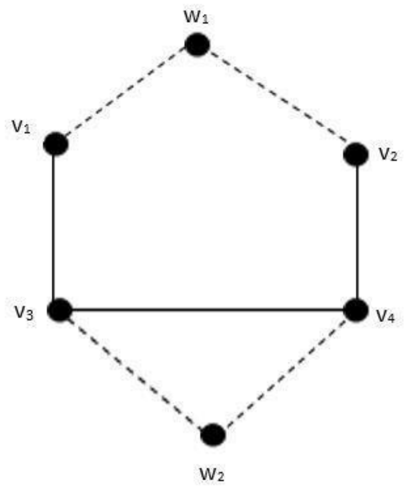

Figure:6

Define a function $f : V(G) \to \{0, 1, 2, \ldots\ldots q\}$ by

$f(v_1^1) = 1$

$f(v_1^i) = 7i - 5,\ 2 \le i \le n$

$f(v_2^1) = 2$

$f(v_2^i) = 7i - 4,\ 2 \le i \le n$

$f(v_3^1) = 4$

$f(v_3^i) = 7i - 2,\ 2 \le i \le n$

$f(v_4^1) = 5$

$f(v_4^i) = 7i - 1,\ 2 \le i \le n$

$f(w_1^1) = 0$

$f(w_1^i) = 7i - 6,\ 2 \le i \le n$

$f(w_2^i) = 7i,\ 1 \le i \le n$

The edges are labeled with

$f(v_1^i v_3^i) = 7i - 4,\ 1 \le i \le n$

$f(v_3^i v_4^i) = 7i - 2,\ 1 \le i \le n$

$f\,(v_4^i v_2^i) = 7i - 3,\ 1 \leq i \leq n$

$f\,(v_1^i w_1^i) = 7i - 6,\ 1 \leq i \leq n$

$f\,(v_2^i w_1^i) = 7i - 5,\ 1 \leq i \leq n$

$f\,(v_3^i w_2^i) = 7i - 1,\ 1 \leq i \leq n$

$f\,(v_4^i w_3^i) = 7i\ ,\ 1 \leq i \leq n$

Thus “f” admits Contra Harmonic Mean Labeling of G. Hence $nDS(P_2 \odot K_1)$ is a Contra Harmonic Mean graph.

Example: 2.6

Contra Harmonic Mean Labeling of $4DS(P_2 \odot K_1)$ is displayed in Figure 7.

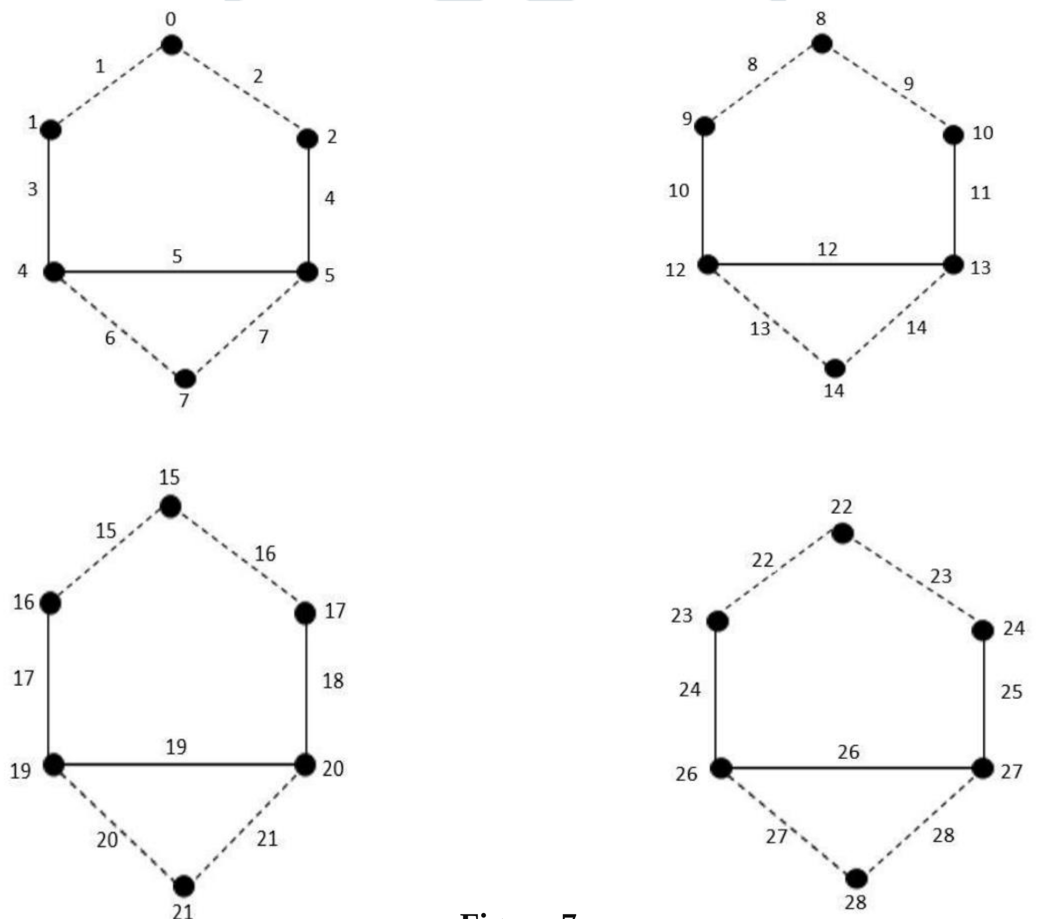

Figure:7

Theorem: 2.7

The graph $nDS(P_3 \odot K_1)$ is a Contra Harmonic Mean graph.

Proof:-

Let G = $nDS(P_3 \odot K_1)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots\ldots \cup V_n$. Let $V = \{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, v_6^i, w_1^i, w_2^i \,/\, 1 \le i \le n\}$ be the vertex set of i^{th} copy of $DS(P_3 \odot K_1)$. The graph $DS(P_3 \odot K_1)$ is shown in Figure 8.

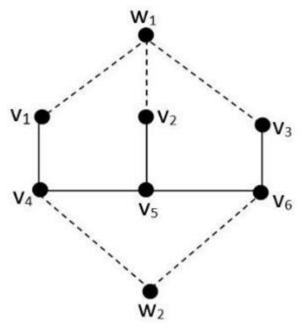

Figure:8

Define a function $f : V(G) \rightarrow \{0, 1, 2, \ldots\ldots q\}$ by

$f(v_1^1) = 7$

$f(v_1^i) = 10i - 3, 2 \le i \le n$

$f(v_2^i) = 10i - 2, 1 \le i \le n$

$f(v_3^1) = 5$

$f(v_3^i) = 10i - 1, 2 \le i \le n$

$f(v_4^1) = 0$

$f(v_4^i) = 10i - 8, 2 \le i \le n$

$f(v_5^1) = 2$

$f(v_5^i) = 10i - 5, 2 \le i \le n$

$f(v_6^i) = 10i - 6, 1 \le i \le n$

$f(w_1^i) = 10i, 1 \le i \le n$

$f(w_2^i) = 10i - 9, 1 \le i \le n$

The edges are labeled with

$f(v_1^1 v_4^1) = 7$

$f(v_1^i v_4^i) = 10i - 5, 2 \le i \le n$

$f(v_2^i v_5^i) = 10i - 4, 1 \le i \le n$

$f(v_3^1 v_6^1) = 5$

$f(v_3^i v_6^i) = 10i - 3,\ 2 \le i \le n$

$f(v_4^1 v_5^1) = 2$

$f(v_4^i v_5^i) = 10i - 7,\ 2 \le i \le n$

$f(v_5^i v_6^i) = 10i - 6,\ 1 \le i \le n$

$f(v_1^i w_1^i) = 10i - 2,\ 1 \le i \le n$

$f(v_2^1 w_1^1) = 10$

$f(v_2^i w_1^i) = 10i - 1,\ 2 \le i \le n$

$f(v_3^1 w_1^1) = 9$

$f(v_3^i w_1^i) = 10i,\ 2 \le i \le n$

$f(v_4^i w_2^i) = 10i - 9,\ 1 \le i \le n$

$f(v_6^1 w_2^1) = 3$

$f(v_6^i w_2^i) = 10i - 8,\ 2 \le i \le n$

Hence nDS ($P_3 \odot K_1$) is a Contra Harmonic Mean graph.

Example: 2.8

Contra Harmonic Mean Labeling of 4DS($P_3 \odot K_1$) is displayed in Figure 9.

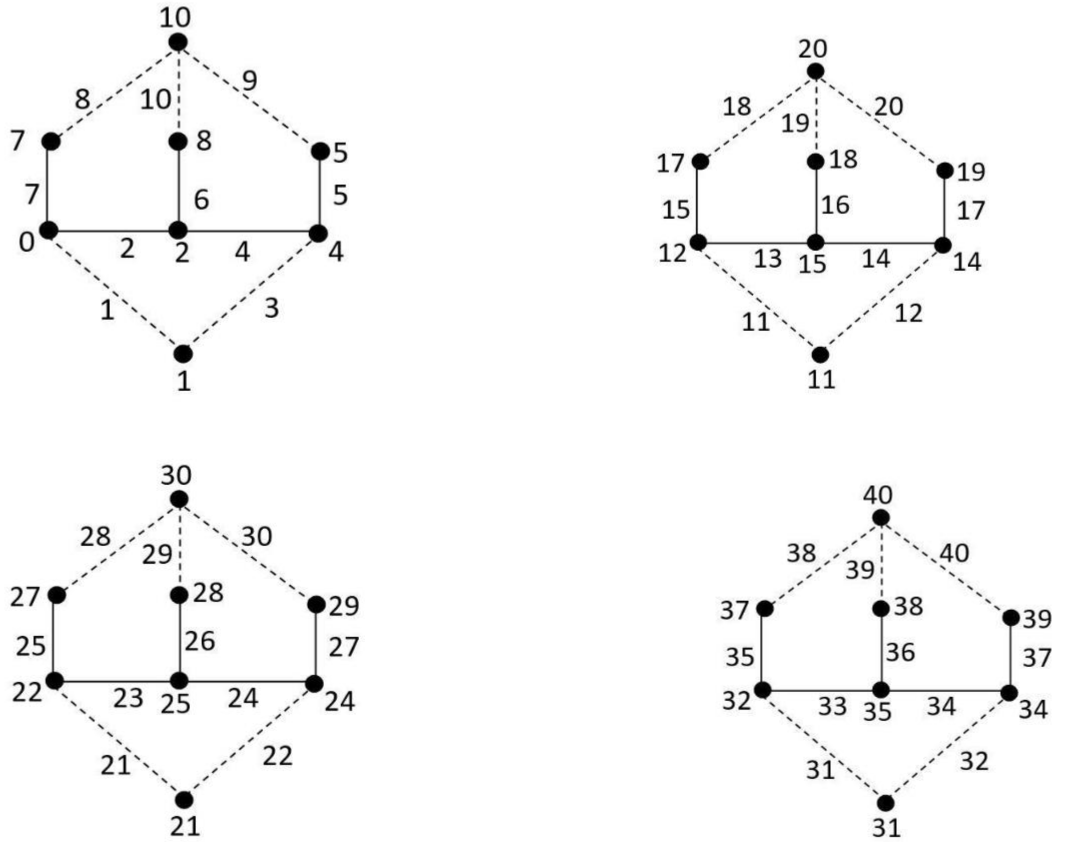

Figure: 9

Theorem : 2.9

The graph $nDS(P_2 \odot K_{1,2})$ is a Contra Harmonic Mean graph.

Proof:-

Let $G= nDS(P_2 \odot K_{1,2})$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$. Let $V=\{v_1^i,v_2^i,v_3^i,v_4^i,v_5^i,v_6^i,w_1^i,w_2^i / 1\le i\le n\}$ be the vertex set of i^{th} copy of $DS(P_2 \odot K_{1,2})$.

The graph $DS(P_2 \odot K_{1,2})$ is displayed in Figure 10.

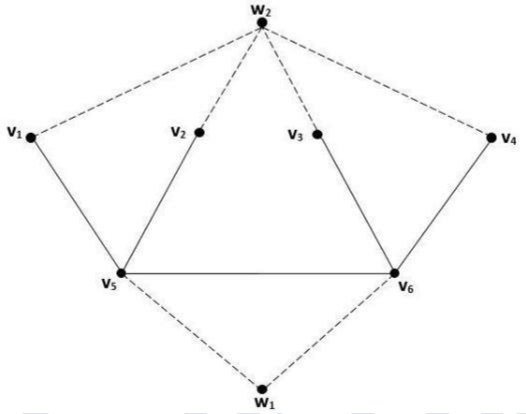

Figure:10

Define a function $f: V(G) \rightarrow \{0, 1, \ldots\ldots\ldots q\}$ by

$f(v_1^1)= 0$

$f(v_1^i)= 11i - 5, 2 \le i \le n$

$f(v_2^1)= 2$

$f(v_2^i)= 11i - 3, 2 \le i \le n$

$f(v_3^1)= 6$

$f(v_3^i)= 11i - 2, 2 \le i \le n$

$f(v_4^1)= 9$

$f(v_4^i)= 11i - 1, 2 \le i \le n$

$f(v_5^1)= 1$

$f(v_5^i)= 11i - 9, 2 \le i \le n$

$f(v_6^1)= 7$

$f(v_6^i)= 11i - 8, 2 \le i \le n$

$f(w_1^1)= 4$

$f(w_1^i)= 11i, 2 \le i \le n$

$f(w_2^1) = 11$

$f(w_2^i) = 11i - 10, 2 \leq i \leq n$

The edges are labeled with

$f(v_1^1 v_5^1) = 1$

$f(v_1^1 v_5^i) = 11i - 7, 2 \leq i \leq n$

$f(v_5^1 v_6^1) = 7$

$f(v_5^i v_6^i) = 11i - 9, 2 \leq i \leq n$

$f(v_6^1 v_4^1) = 9$

$f(v_6^i v_4^i) = 11i - 4, 2 \leq i \leq n$

$f(v_5^1 v_2^1) = 2$

$f(v_5^i v_2^i) = 11i - 6, 2 \leq i \leq n$

$f(v_6^i v_3^1) = 6$

$f(v_6^i v_3^i) = 11i - 5, 2 \leq i \leq n$

$f(v_2^1 w_1^1) = 3$

$f(v_2^i w_1^i) = 11i - 2, 2 \leq i \leq n$

$f(v_3^1 w_1^1) = 5$

$f(v_3^i w_1^i) = 11i - 1, 2 \leq i \leq n$

$f(v_1^1 w_1^1) = 4$

$f(v_1^i w_1^i) = 11i - 3, 2 \leq i \leq n$

$f(v_4^1 w_1^1) = 8$

$f(v_4^i w_1^i) = 11i, \ 2 \leq i \leq n$

Thus both vertices and edges get distinct labels. Hence $nDS(P_2 \odot K_{1,2})$ is a Contra Harmonic Mean graph.

Example: 2.10

Contra Harmonic Mean Labeling of $4DS(P_2 \odot K_{1,2})$ is shown in Figure:11

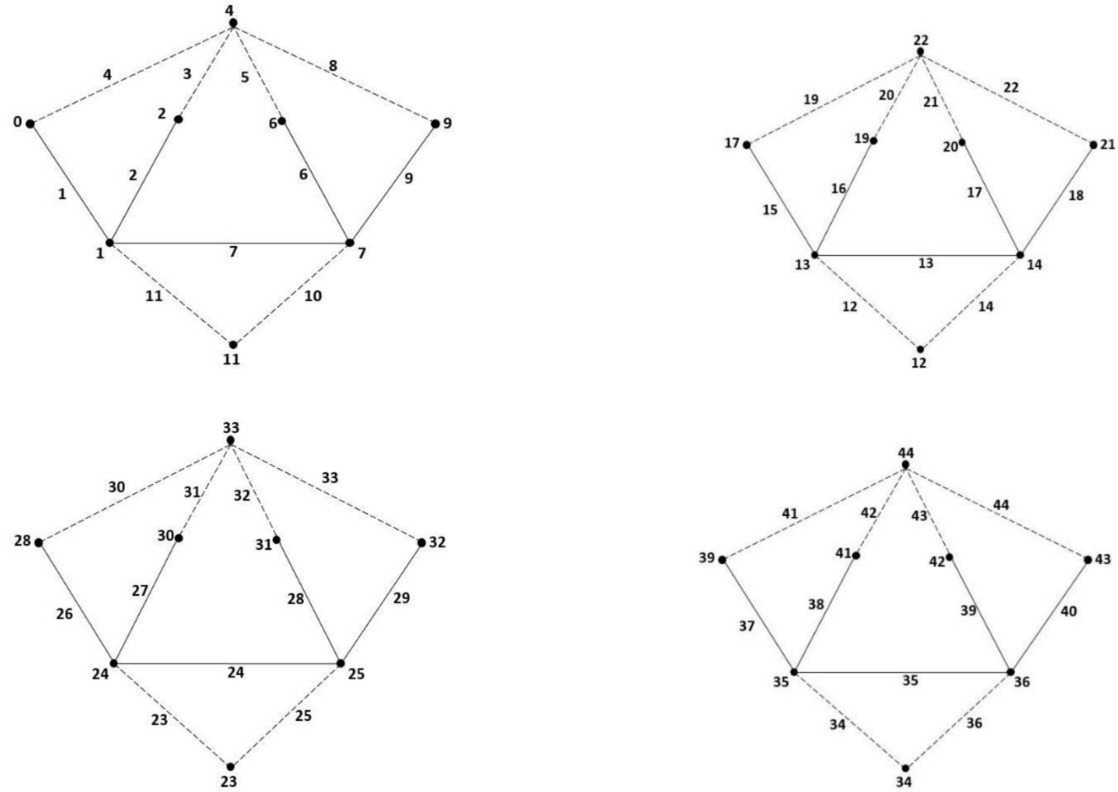

Figure:11

Theorem: 2.11

The graph $nDS(P_2 \odot K_3)$ is a Contra Harmonic Mean graph.

Proof:

Let $G = nDS(P_2 \odot K_3)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots \cup V_n$. Let $V=\{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, v_6^i, w_1^i, w_2^i \,/ 1 \le i \le n\}$ be the vertex set of i^{th} copy of $DS(P_2 \odot K_3)$

The graph $DS(P_2 \odot K_3)$ is shown in Figure 12.

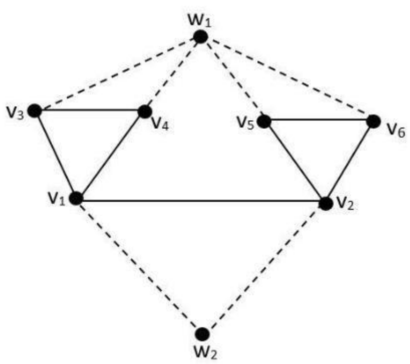

Figure:12

Define a function f : V (G) → {0, 1, 2, …..q} by

$f(v_1^1) = 5$

$f(v_1^i) = 13i - 11, 2 \leq i \leq n$

$f(v_2^1) = 8$

$f(v_2^i) = 13i - 10, 2 \leq i \leq n$

$f(v_3^1) = 0$

$f(v_3^i) = 13i - 9, 2 \leq i \leq n$

$f(v_4^1) = 3$

$f(v_4^i) = 13i - 5, 2 \leq i \leq n$

$f(v_5^1) = 12$

$f(v_5^i) = 13i - 4, 2 \leq i \leq n$

$f(v_6^1) = 13$

$f(v_6^i) = 13i - 1, 2 \leq i \leq n$

$f(w_1^1) = 1$

$f(w_1^i) = 13i, 2 \leq i \leq n$

$f(w_2^1) = 6$

$f(w_2^i) = 13i - 12, 2 \leq i \leq n$

The edges are labeled with

$f(v_3^1 v_1^1) = 5$

$f(v_3^i v_1^i) = 13i - 9, 2 \leq i \leq n$

$f(v_1^1 v_2^1) = 7$

$f(v_1^i v_2^i) = 13i - 10, 2 \leq i \leq n$

$f(v_2^1 v_6^1) = 11$

$f(v_2^i v_6^i) = 13i - 5, 2 \leq i \leq n$

$f(v_1^1 v_4^1) = 4$

$f(v_1^i v_4^i) = 13i - 8, 2 \leq i \leq n$

$f(v_2^1 v_5^1) = 9$

$f(v_2^i v_5^i) = 13i - 6, 2 \leq i \leq n$

$f(v_3^1 v_4^1) = 3$

$f(v_3^i v_4^i) = 13i - 7, 2 \leq i \leq n$

$f\,(v_5^1 v_6^1) = 12$

$f\,(v_5^i v_6^i) = 13i - 3,\ 2 \le i \le n$

$f\,(v_3^1 w_1^1) = 1$

$f\,(v_3^i w_1^i) = 13i - 4,\ 2 \le i \le n$

$f\,(v_4^1 w_1^1) = 2$

$f\,(v_4^i w_1^i) = 13i - 2,\ 2 \le i \le n$

$f\,(v_5^1 w_1^1) = 10$

$f\,(v_5^i w_1^i) = 13i - 1,\ 2 \le i \le n$

$f\,(v_6^1 w_1^1) = 13i,\ 1 \le i \le n$

$f\,(v_1^1 w_2^1) = 6$

$f\,(v_1^i w_2^i) = 13i - 12,\ 2 \le i \le n$

$f\,(v_2^1 w_2^1) = 8$

$f\,(v_2^i w_2^1) = 13i - 11,\ 2 \le i \le n$

Thus "f" admits Contra Harmonic Mean Labeling of G. Hence $nDS(P_2 \odot K_3)$ is a Contra Harmonic Mean graph.

Example: 2.12

Contra Harmonic Mean Labeling of $4DS(P_2 \odot K_3)$ shown in Figure 13.

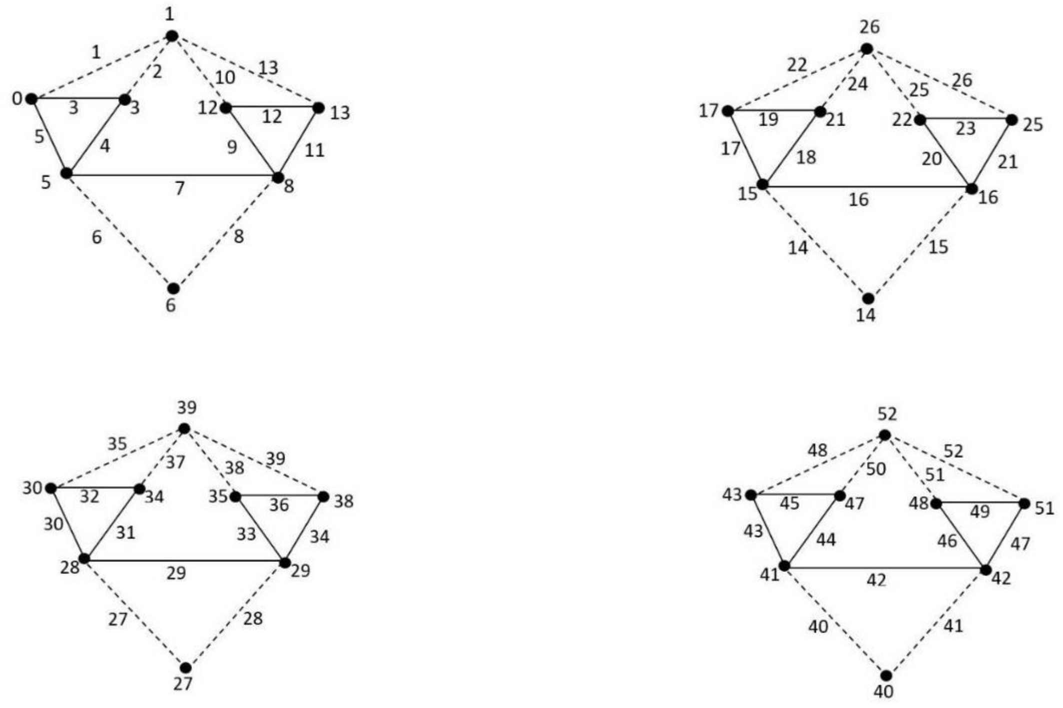

Figure:13

Theorem: 2.13

The graph nDS ($K_{1,3}$) is a Contra Harmonic Mean graph.

Proof:-

Let G = nDS ($K_{1,3}$). Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots\ldots \cup V_n$. Let $V = \{v_1^i, v_2^i, v_3^i, v_4^i, w_1^i \,/\, 1 \leq i \leq n\}$ be the vertex set of i^{th} copy of DS ($K_{1,3}$)

The graph DS($K_{1,3}$) is shown in Figure 14.

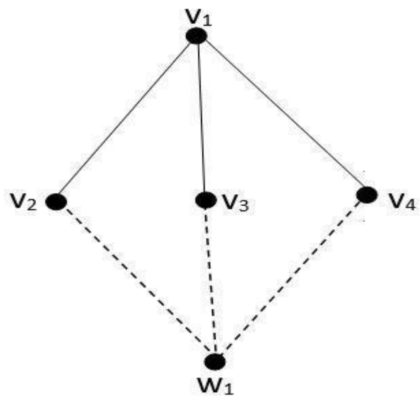

Figure: 14

Define a function f: V (G) →{0, 1, …..q} by

$f(v_1^1) = 4$

$f(v_1^i) = 6i - 5,\ 2 \leq i \leq n$

$f(v_2^1) = 6$

$f(v_2^i) = 6i - 4,\ 2 \leq i \leq n$

$f(v_3^1) = 0$

$f(v_3^i) = 6i - 2,\ 2 \leq i \leq n$

$f(v_4^1) = 1$

$f(v_4^i) = 6i - 1,\ 2 \leq i \leq n$

$f(w_1^1) = 2$

$f(w_1^i) = 6i,\ 2 \leq i \leq n$

The edges are labeled with

$f(v_1^1 v_2^1) = 6$

$f(v_1^i v_2^i) = 6i - 5,\ 2 \leq i \leq n$

$f(v_1^1 v_3^1) = 4$

$f(v_1^i v_3^i) = 6i - 4,\ 2 \le i \le n$
$f(v_1^i v_4^i) = 6i - 3,\ 1 \le i \le n$
$f(v_2^1 w_1^1) = 5$
$f(v_2^i w_1^i) = 6i - 2,\ 2 \le i \le n$
$f(v_3^1 w_1^1) = 2$
$f(v_3^i w_1^i) = 6i - 1,\ 2 \le i \le n$
$f(v_4^1 w_1^1) = 1$
$f(v_4^i w_1^i) = 6i,\ 2 \le i \le n)$

Thus nDS($K_{1,3}$) admits Contra Harmonic Mean Labeling. Hence nDS ($K_{1,3}$) is a Contra Harmonic Mean graph.

Example: 2.14

Contra Harmonic Mean Labeling of 4DS($K_{1,3}$) is displayed in Figure 15

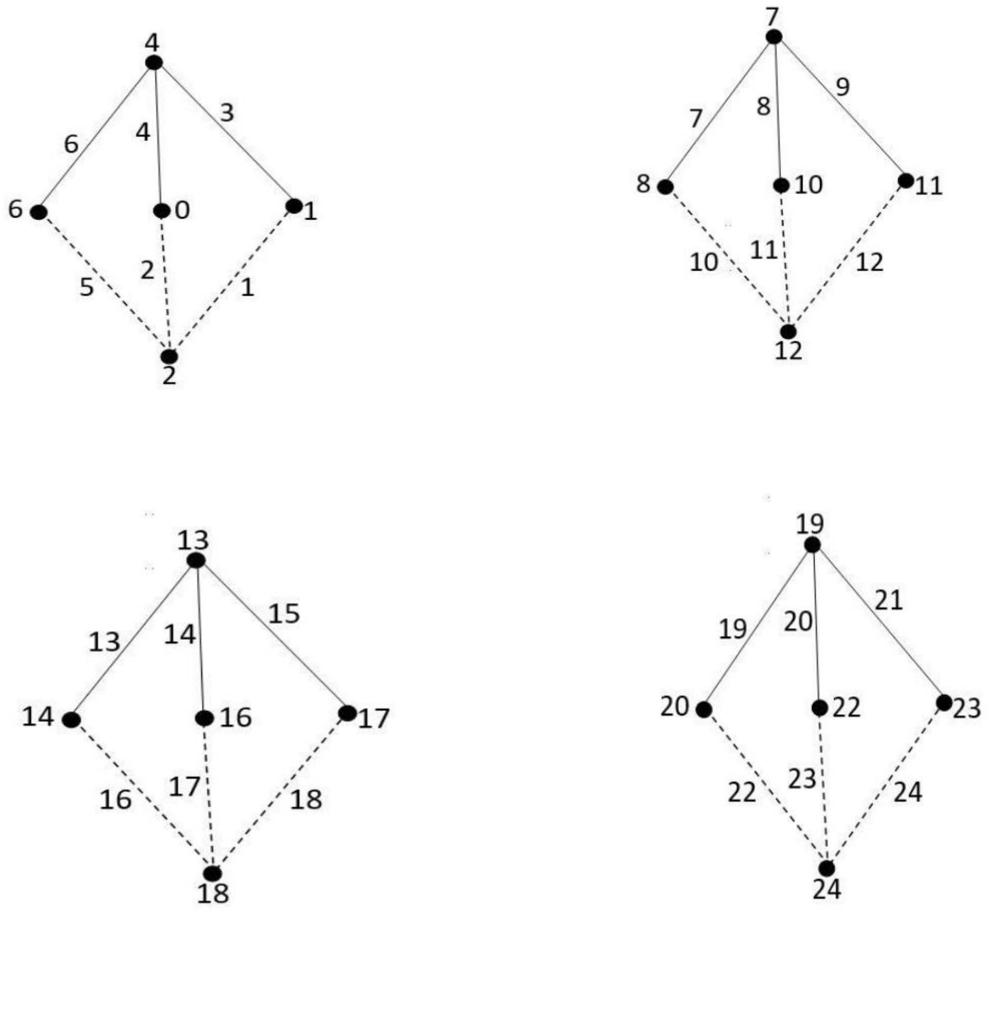

Figure: 15

Theorem: 2.15

The graph $nDS(C_3 \odot K_{1,2})$ is a Contra Harmonic Mean graph.

Proof:-

Let G = $nDS(C_3 \odot K_{1,2})$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \ldots\ldots \cup V_n$. Let $V = \{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, w_1^i, w_2^i / 1 \leq i \leq n\}$ is the vertex set of i^{th} copy of $DS(C_3 \odot K_{1,2})$.

The graph $DS(C_3 \odot K_{1,2})$ is shown in Figure16

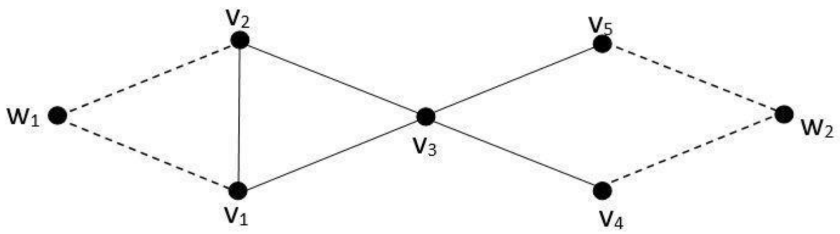

Figure:16

Define a function $f : V(G) \rightarrow \{0, 1, \ldots..q\}$ by

$f(v_1^{1)}) = 0$

$f(v_1^i) = 9i - 7, 2 \leq i \leq n$

$f(v_2^1) = 3$

$f(v_2^i) = 9i - 5, 2 \leq i \leq n$

$f(v_3^1) = 5$

$f(v_3^i) = 9i - 4, 2 \leq i \leq n$

$f(v_4^{1)}) = 8$

$f(v_4^i) = 9i - 2, 2 \leq i \leq n$

$f(v_5^1) = 6$

$f(v_5^i) = 9i - 1, 2 \leq i \leq n$

$f(w_1^i) = 9i - 8, 1 \leq i \leq n$

$f(w_2^i) = 9i, 1 \leq i \leq n$

The edges are labeled with

$f(v_1^1 v_2^1) = 3$

$f(v_1^i v_2^i) = 9i - 6, 2 \leq i \leq n$

$f(v_1^1 v_3^1) = 5$

$f(v_1^i v_3^i) = 9i - 5,\ 2 \le i \le n$

$f(v_2^1 v_3^1) = 4$

$f(v_2^i v_3^i) = 9i - 4,\ 2 \le i \le n$

$f(v_3^1 v_5^1) = 6$

$f(v_3^i v_5^i) = 9i - 2,\ 2 \le i \le n$

$f(v_3^1 v_4^1) = 7$

$f(v_3^i v_4^i) = 9i - 3,\ 2 \le i \le n$

$f(v_1^i w_1^i) = 9i - 8,\ 1 \le i \le n$

$f(v_2^1 w_1^1) = 2$

$f(v_2^i w_1^i) = 9i - 7,\ 2 \le i \le n$

$f(v_5^1 w_2^1) = 8$

$f(v_5^i w_2^i) = 9i,\ 2 \le i \le n$

$f(v_4^1 w_2^1) = 9$

$f(v_4^i w_2^i) = 9i - 1,\ 2 \le i \le n$

Thus "f" admits Contra Harmonic Mean Labeling of G. Hence nDS ($C_3 \odot K_{1,2}$) is a Contra Harmonic Mean graph.

Example:2.16

Contra Harmonic Mean Labeling of 4DS($C_3 \odot K_{1,2}$) is given in Figure 17.

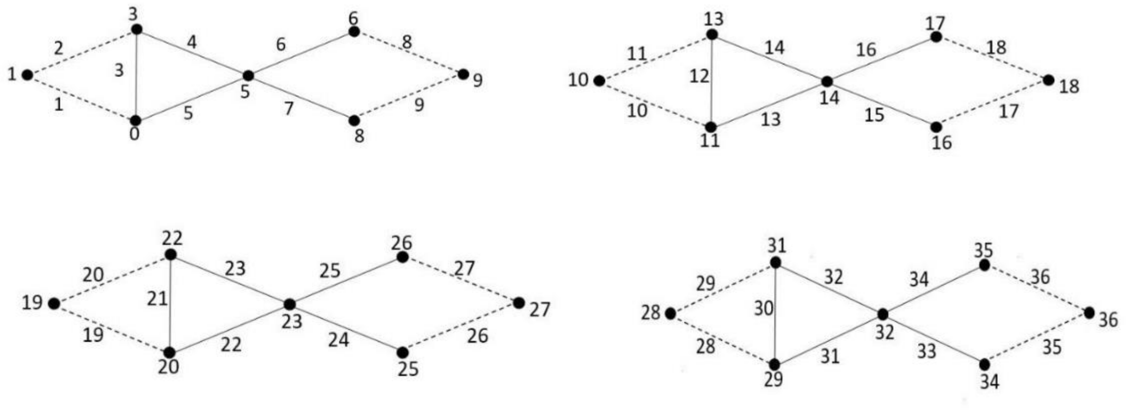

Figure: 17

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