# SOME NEW PROPERTIES OF 5X5 MAGIC SQUARE

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*Abstract :*  In this paper, we established some new results in the area of square magic and proved some new properties, in which one is $M(A+\alpha)+ M(A-\alpha)= 2X M(A)$. Also, some new properties of Magic Constant of a Magic Square and it's derived matrix in the view of eigenvalues, eigen vectors have been established. This paper is organized as introduction, definition & preliminaries, methodology and results in relation to 5X5 square magic.

## 1.   INTRODUCTION :

The development of Magic Squares in India leads irresistibly to the conclusion that the Magic Square originated in India. The knowledge of these Squares might have gone outside India at any time between the first century and 10th century AD, but it appears to be the most probable that the west as well as China got the Magic Squares from India through the Arabs about tenth century.  Many researchers in India and world have focused on square magic in last five years in different directions. In which few are [1, 2, 3, 4, 5, and 6].

In this paper, we established some properties of 5X5 square magic having virtue in association with its magic constant, Eigen values and Eigen vectors.

## 2.   Definition & Preliminaries:

 A Magic Square of order n is a square matrix having array of $n^2$ numbers such that the sum of each row and column as well as the main diagonal and main back diagonal [1], is the same number called Magic Constant (Magic Sum or Lowest Required Sum). If X is a Magic Square and each element of another same order square matrix by addition, subtraction, multiplication or division to the corresponding element of X by the same number (not 0 for multiplication or division), then new square matrix will be a Magic Square.

An upper bound for the number of normal Magic Squares of order n can be given by $n^2 ! \big/ 8(2n+1)$ . There is only one distinct third order normal Magic Square with lowest sum.

## 3.   Methodology:

Example 3.1:   Let us consider a third-order Magic Square.

| A=<br>(Magic<br>Constant<br>=65) | 9 | 10 | 22 | 23 | 1 |
|---|---|---|---|---|---|
| | 8 | 21 | 20 | 2 | 14 |
| | 11 | 7 | 13 | 19 | 15 |
| | 12 | 24 | 6 | 5 | 18 |
| | 25 | 3 | 4 | 16 | 17 |

The Magic Constant of the Magic Square is M (A) =65.
Taking a combination of the two numbers 5 and 60 of the Magic Constant 65 such that 5+60. Now, adding 5 to each element of the Magic Square A, we get a Magic Square $A_5$ (Say).

| $A_5=$ (Magic Constant =90) | 14 | 15 | 27 | 28 | 6 |
|---|---|---|---|---|---|
| | 13 | 26 | 25 | 7 | 19 |
| | 16 | 12 | 18 | 24 | 20 |
| | 17 | 29 | 11 | 10 | 23 |
| | 30 | 8 | 9 | 21 | 22 |

Similarly,

| $A_{60}=$ (Magic Constant =365) | 69 | 70 | 82 | 83 | 61 |
|---|---|---|---|---|---|
| | 68 | 81 | 80 | 82 | 74 |
| | 71 | 67 | 73 | 79 | 75 |
| | 72 | 84 | 66 | 65 | 78 |
| | 85 | 63 | 64 | 76 | 77 |

Similarly,

| $A_{-5}=$ (Magic Constant =40) | 4 | 5 | 17 | 18 | -4 |
|---|---|---|---|---|---|
| | 3 | 16 | 15 | -3 | 9 |
| | 6 | 2 | 8 | 14 | 10 |
| | 7 | 19 | 1 | 0 | 13 |
| | 20 | -2 | -1 | 11 | 12 |

Similarly,

| $A_{-60}=$ (Magic Constant =-235) | -51 | -50 | -38 | -37 | -59 |
|---|---|---|---|---|---|
| | -52 | -39 | -40 | -58 | -46 |
| | -49 | -53 | -47 | -41 | -45 |
| | -48 | -36 | -54 | -55 | -42 |
| | -35 | -57 | -56 | -44 | -43 |

Now take,

$M(A_5) + M(A_{-5}) = 90+40= 130= 2*65= 2*M(A)$

$M(A_{60}) + M(A_{-60}) = 365-235= 130= 2*65= 2*M(A)$

$M(A_5) + M(A60) = 90+365= 455= (5+2)*65= (n+m)*M(A)$, where n= order of square magic and m= number of partitions. Here n=5 and m=2.

## 4.    Results and discussion:

(i)  If $M(A\alpha)$= Magic Constant of Magic Square $A\alpha$

$M(A-\alpha)$= Magic Constant of Magic Square $A-\alpha$

$M(A(M-\alpha))$= Magic Constant of Magic Square $A(M-\alpha)$

$M(A-(M-\alpha))$= Magic Constant of Magic Square $A-(M-\alpha)$

$M(A)$= Magic Constant of Magic Square A

n = Order of Magic Square A

m = Number of partitions of $M(A)$, Where M is any 5x5 square magic.

(ii)

It is possible to find a large number of essentially different matrices, which have different Eigen Values but nearly same Eigen vectors. But it is a difficult and time consuming task.

After Eigen decomposition process, the Eigen values and Eigen vectors of these obtained matrices are also linearly related together. Eigen vector of one matrix are the basis of an invariant subspace within the range of the corresponding linear map.

Discussion on Example 3.1

M(A)=65, multiplicity =1

$\beta_1$ =65, $\beta_2$ =-3.18+12.56i, $\beta_3$ = -3.18-12.56i , $\beta_4$ =3.18-12.56i, $\beta_5$ = 3.17+12.56i

$w_1$ =(1,1,1,1,1), $w_2$ =(0.98+1.15i, 0.28+0.002i, -0.70-0.19i, 0.39-0.96i,1)

$w_3$ = (0.98-1.15i, 0.28-0.003i, -0.70+0.19i, 0.39+0.96i,1)

$w_4$ = (-0.43-0.51i, -0.65+0.22i, 0.20+0.43i, -0.11-0.14i,1)

$w_5$ = (-0.43+0.51i, -0.65-0.22i, 0.20-0.44i, -0.11+0.14i,1)


M(A$_5$) =90, multiplicity =1

$\beta_2$ =-3.25-12.57i, $\beta_3$ = -3.25+12.57i , $\beta_4$ =3.15-12.26i, $\beta_5$ = 3.15+12.26i

$w_1$ =(1,1,1,1,1), $w_2$ =(0.99-1.15i, 0.28-0.02i, -0.71+0.21i, 0.41+0.96i,1)

$w_3$ = (0.99+1.15i, 0.28+0.02i, -0.71-0.21i, 0.41-0.96i,1)

$w_4$ = (0.44-0.51i, -0.62+0.21i, 0.16+0.41i, -0.09-0.12i,1)

$w_5$ = (0.44+0.51i, -0.62-0.21i, 0.16-0.41i, -0.09+0.12i,1)

M(A$_{-5}$)=40

$\beta_1$ =40, $\beta_2$ =-3.18+12.56i, $\beta_3$ = -3.18-12.56i , $\beta_4$ =3.18-12.56i, $\beta_5$ = 3.17+12.56i

$w_1$ =(1,1,1,1,1), $w_2$ =(0.98+1.15i, 0.28+0.002i, -0.70-0.19i, 0.39-0.96i,1)

$w_3$ = (0.98-1.15i, 0.28-0.003i, -0.70+0.19i, 0.39+0.96i,1)

$w_4$ = (-0.43-0.51i, -0.65+0.22i, 0.20+0.43i, -0.11-0.14i,1)

$w_5$ = (-0.43+0.51i, -0.65-0.22i, 0.20-0.44i, -0.11+0.14i,1)

M(A$_{60}$) =365

$\beta_1$ =365, $\beta_2$ =-3.18+12.56i, $\beta_3$ = -3.18-12.56i , $\beta_4$ =3.18-12.56i, $\beta_5$ = 3.17+12.56i

$w_1$ =(1,1,1,1,1), $w_2$ =(0.98+1.15i, 0.28+0.002i, -0.70-0.19i, 0.39-0.96i,1)

$w_3$ = (0.98-1.15i, 0.28-0.003i, -0.70+0.19i, 0.39+0.96i,1)

$w_4$ = (-0.43-0.51i, -0.65+0.22i, 0.20+0.43i, -0.11-0.14i,1)

$w_5$ = (-0.43+0.51i, -0.65-0.22i, 0.20-0.44i, -0.11+0.14i,1)

M(A$_{-60}$)= -235

$\beta_1$ =-235, $\beta_2$ =-3.18+12.56i, $\beta_3$ = -3.18-12.56i , $\beta_4$ =3.18-12.56i, $\beta_5$ = 3.17+12.56i

$w_1$ =(1,1,1,1,1), $w_2$ =(0.98+1.15i, 0.28+0.002i, -0.70-0.19i, 0.39-0.96i,1)

$w_3$ = (0.98-1.15i, 0.28-0.003i, -0.70+0.19i, 0.39+0.96i,1)

$w_4$ = (-0.43-0.51i, -0.65+0.22i, 0.20+0.43i, -0.11-0.14i,1)

$w_5$ = (-0.43+0.51i, -0.65-0.22i, 0.20-0.44i, -0.11+0.14i,1)


## 5. Conclusion:

By our methodology, one can find the another matrix of order 5x5, which have different Eigen Values but nearly same Eigen vectors if matrix is square and have property of square magic.

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