K-ECCENTRIC INDICES OF POLYHEX NANOTUBES $TUAC_6(p, q)$ AND $TUZC_6(p, q)$

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Abstract: In this paper, we evaluate some eccentricity based topological indices known as K-eccentric indices for armchair polyhex nanotubes $TUAC_6(p, q)$ and zigzag polyhex nanotubes $TUZC_6(p, q)$.

Keyword: Eccentricity index, Line graph, K-eccentric indices, polyhex nanotubes.

1.Introduction

Let G be a simple, finite graph with n vertices and m edges with vertex set V(G) and edge set E(G). The edge connecting the vertices u and v $\in$ V(G) is denoted by e = uv. The vertices and edges of a graph are called elements of G.

We construct a graph L(G) in the following way: The vertex set of L(G) is in 1-1 correspondence with the edge set of G and two vertices of L(G) are joined by an edge if and only if the corresponding edges of G are adjacent in G. The graph L(G) (which is always a simple graph) is called the line graph or edge graph of G.

Let G be a connected graph and v be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v); u \in V\}$. The radius r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) is the maximum eccentricity.

The topological indices are one of the mathematical models that can be defined by assigning a real number to the chemical molecule. The physical-chemical characteristics of the molecules can be analyzed by taking benefit from the topological indices and such properties as boiling point, entropy, enthalpy of vaporization, standard enthalpy of vaporization, enthalpy of formation, acetic factor, etc can be predicted.

In [1], Bhanumathi and Easu Julia Rani, introduced the K- Eccentricity indices. They defined the first and second K-eccentric indices as

$B_1E(G) = \sum_{ue}[(e_G(u)+e_{L(G)}(e)]$

$B_2E(G) = \sum_{ue}[(e_G(u).e_{L(G)}(e)]$ and the first and second K-Hyper Eccentricity indices as

$HB_1E(G) = \sum_{ue}[(e_G(u)+e_{L(G)}(e)]^2$

$HB_2E(G) = \sum_{ue}[(e_G(u).e_{L(G)}(e)]^2$

In [2], Bhanumathi and Easu Julia Rani, introduced Multiplicative K - Eccentric Indices. They were defined as

$B\Pi_1E(G) = \prod_{ue}[e_G(u)+e_{L(G)}(e)]$

$B\Pi_2E(G) = \prod_{ue}[e_G(u).e_{L(G)}(e)]$ and the first and second Multiplicative K-Hyper Eccentric Indices as

$HB\Pi_1E(G) = \prod_{ue}[e_G(u).e_{L(G)}(e)]^2$ and $HB\Pi_2E(G) = \prod_{ue}[e_G(u).e_{L(G)}(e)]^2$,where e $\in$ E(G) is incident with u in G, $e_G(u)$ denotes the eccentricity of u $\in$ V(G) in G and $e_{L(G)}(e)$ denotes the eccentricity of e in L(G).

2. K-Eccentricity indices of Armchair polyhex nanotubes $TUAC_6(p, q)$

In this section, we compute First K-Eccentric Index, Second K-Eccentric Index, First K- Hyper Eccentric Index, Second K-Hyper Eccentric Index, Multiplicative First K-Eccentric Index, Multiplicative Second K-Eccentric Index, Multiplicative First K-Hyper Eccentric Index, and Multiplicative Second K-Hyper Eccentric Index of Armchair polyhex nanotubes.

Let $G = TUAC_6(p, q)$ be the armchair polyhex nanotube, where p is the number of hexagons in each row and q is the number of rows in the molecular graph G. The graph $TUAC_6(p, q)$ has 4pq vertices and 6pq – 2p edges.

The eccentricity of e = uv in L(G) and the eccentricity of vertices u and v in G are given in the following table.

Sl.No	Number of edges e = uv in G	Eccentricity of e in L(G)	Eccentricity of the end vertices (e(u), e(v)) in G
1	p 2p	p + 2q – 2 p + 2q – 2	(p + 2q – 1, p + 2q – 1) (p + 2q – 1, p + 2q – 2)
2	p 2p	p + 2q – 3 p + 2q – 3	(p + 2q – 2, p + 2q – 2) (p + 2q – 2, p + 2q – 3)
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q	p 2p	p + q – 1 p + q –1	(p + q, p + q) (p + q, p + q)
q + 1	p 2p	p + q – 1 p + q	(p + q, p + q) (p + q, p + q + 1)
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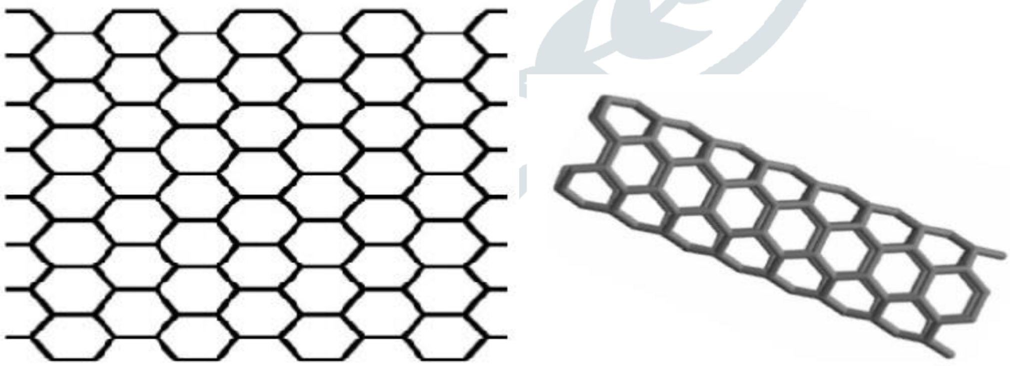
q + 1 + q – 2 = q – 1	p 2p	p + 2q – 3 p + 2q – 2	(p + 2q – 2, p + 2q – 2) (p + 2q – 2, p + 2q – 1)
q + 1 + q – 1 = 2q	p	p + 2q – 2	(p + 2q – 1, p + 2q – 1)

Table 2.1

Example: Let $G = TUAC_6(4, 8)$. The graph $TUAC_6(4, 8)$ has 128 vertices and 184 edges.
The eccentricity of e = uv in L(G) and the eccentricity of vertices u and v in G are given in Table 2.2



(a) **(b)**

Fig (a) The 2-dimensional lattice **(b) $TUAC_6(4, 8)$ nanotube**

Sl.No	Number of edges e = uv in G	Eccentricity of e in L(G)	Eccentricity of the end vertices (e(u), e(v)) in G
1	4 8	18 18	(19, 19) (19, 18)
2	4 8	17 17	(18, 18) (18, 17)
3	4 8	16 16	(17,17) (17,16)
4	4 8	15 15	(16, 16) (16, 15)
5	4 8	14 14	(15, 15) (15,14)
6	4 8	13 13	(14, 14) (14, 13)
7	4 8	12 12	(13, 13) (13,12)
8	4 8	11 11	(12,12) (12, 12)
9	4 8	11 12	(12, 12) (12, 13)
10	4 8	12 13	(13, 13) (13, 14)
11	4 8	13 14	(14, 14) (14, 15)
12	4 8	14 15	(15, 15) (15, 16)
13	4 8	15 16	(16, 16) (16, 17)
14	4 8	16 17	(17, 17) (17, 18)
15	4 8	17 18	(18, 18) (18, 19)
16	4	18	(19, 19)

Table 2.2

(i) $B_1E(G) = \sum_{e=uv\in E(G)} [e_G(u) + 2e_{L(G)}(e) + e_G(v)]$

= 2{4(19 + 2(18) + 19) + 8(19 + 2(18) + 18) + 4(18 + 2(17) +18) + 8(18 + 2(17) + 17) + 4(17 + 2(16) +17) + 8(17 + 2(16) + 16) + 4(16 + 2(15) +16) + 8(16 + 2(15) + 15) + 4(15 + 2(14) +15) + 8(15 + 2(14) + 14) + 4(14 + 2(13) +14) + 8(14 + 2(13) + 13) + 4(13 + 2(12) +13) + 8(13 + 2(12) + 12) + 4(12 + 2(11) +12)} + 8(12 + 2(11) + 12.

= 2{4 × 74 + 8 ×73 + 4 × 70 + 8 × 69 + 4 × 66 + 8 × 65 + 4 × 62 + 8 × 61 + 4× 58 + 8 × 57 + 4 × 54 + 8 × 53 + 4× 50 + 8 × 49 + 4 × 46 } + 8(46)

= 2{296 + 584 + 280 + 552 + 264 + 520 + 248 + 488 + 232 + 456 + 216 + 424 + 200 + 392 + 184} + 368 = 11040

Similarly, we can find out that,

(ii) $B_2E(G)$ = 84528 (iii) $HB_1E(G)$ = 338368 (iv) $HB_2E(G)$ = 210931

(v) $B\Pi_1E(G) = (1.589599375 \times 10^{48})^2\,(4232)$

(vi) $B\Pi_2E(G) = (4.163486727 \times 10^{81})^2\,(139392)$

(vii) $HB\Pi_1E(G) = (6.17993583 \times 10^{99})^2\,(2238728)$

(viii) $HB\Pi_2E(G) = (5.477059928 \times 10^{79})^2(2.302804326 \times 10^{72})^2\,(2428766208)$

First and Second K-Eccentric Indices and The First and Second K-Hyper Eccentric Indices

Theorem 2.1: First K - Eccentric Index of G = $TUAC_6(p, q)$ is

$$B_1E(G) = 2\sum_{k=0}^{q-1} 2p((p+q+k)+(p+q+(k-1))) + 2\sum_{k=1}^{q-1} 2p[(p+q+k)+3(p+q+(k-1))]$$

+ 4p((p + q) + (p + q – 1))

Proof: $B_1E(G) = \sum_{ue} [(e_G(u) + e_{L(G)}(e)]$

$= \sum_{e=uv\in E(G)} [e_G(u) + 2e_{L(G)}(e) + e_G(v)]$

= {p[(p + 2q – 1) + 2(p + 2q – 2) +(p + 2q – 1)]} + {2p[(p + 2q – 1) + 2(p + 2q – 2) +(p + 2q – 2)]} + {p[(p + 2q – 2) + 2(p + 2q – 3) + (p + 2q – 2)]} +…+ {2p[(p + q + 1) + 2(p + q) + (p + q)]} + {p[(p + q) + 2(p + q – 1) + (p + q)]} + {2p[(p + q) + 2(p + q –1) + (p + q)]} +…+ {2p[(p + 2q – 2) + 2 (p + 2q – 2) + (p + 2q – 1)]} + {p[(p + 2q – 1) + 2 (p + 2q – 2) + (p + 2q – 1)]}

= {p[2(p + 2q – 1) + 2(p + 2q – 2)]}+ {2p[(p + 2q – 1) + 3(p + 2q – 2)]} + {p[2(p + 2q – 2) + 2(p + 2q – 3)]} +...+ {2p[(p + q + 1) + 3(p + q)]} + {p[2(p + q) + 2(p + q – 1)]} + {2p[2(p + q) + 2(p + q – 1)]} + ...+ {2p[3(p + 2q – 2) + (p + 2q – 1)]} + {p[2(p + 2q – 1) + 2(p + 2q – 2)]}

$= 2\sum_{k=0}^{q-1} 2p((p+q+k)+(p+q+(k-1))) + 2\sum_{k=1}^{q-1} 2p[(p+q+k)+3(p+q+(k-1))]$

+ 4p((p + q) + (p + q – 1))

Theorem 2.2: Second K-Eccentric Index of G = $TUAC_6(p, q)$ is $B_2E(G) = 2\sum_{k=0}^{q-1} p[2(p+q+k)(p+q+(k-1))] + 2\sum_{k=1}^{q-1} 2p[(p+q+(k-1))^2 + (p+q+k)(p+q+(k-1))] +$ 4p((p + q)(p + q – 1))

Proof: $B_2E(G) = \sum_{ue} [(e_G(u).e_{L(G)}(e)]$

$= \sum_{e=uv\in E(G)} [((e_G(u).e_{L(G)}(e) + (e_G(v).e_{L(G)}(e))]$

= {p[(p + 2q – 1) (p + 2q – 2) +(p + 2q – 1) (p + 2q – 2)]} + {2p[(p + 2q – 1)(p + 2q – 2) + (p + 2q – 2) (p + 2q – 2)]} + {p[(p + 2q – 2)(p + 2q – 3) + (p + 2q – 2) (p + 2q – 3)]} +…+ {2p[(p + q + 1)(p + q) + (p + q) (p + q)]} + {p[(p + q)(p + q – 1) + (p + q) (p + q – 1)]} + {2p[(p + q)(p + q – 1) + (p + q) (p + q – 1)]} +…..+ {2p[(p + 2q – 2)(p + 2q – 2) + (p + 2q – 1) (p + 2q – 2)]} + {p[(p + 2q – 1)(p + 2q – 2) + (p + 2q – 1) (p + 2q – 2)]}

= {p[2(p + 2q – 1) (p + 2q – 2)]} + {2p[(p + 2q – 2)^2 +(p + 2q – 2) (p + 2q – 1)]} + {p[2(p + 2q – 2) (p + 2q – 3)]} +….+ {2p[(p + q)^2 +(p + q + 1) (p + q)]} + {p[2(p + q)(p + q – 1]} + {2p[2(p + q)(p + q – 1]} +…+ {2p[(p + 2q – 2)^2 +(p + 2q – 1) (p + 2q – 2)]} + {p[2(p + 2q – 1) (p + 2q – 1)]}

$= 2\sum_{k=0}^{q-1} p[2(p+q+k)(p+q+(k-1))] + 2\sum_{k=1}^{q-1} 2p[(p+q+(k-1))^2 + (p+q+k)(p+q+(k-1))]$

+ 4p((p + q)(p + q – 1))

Theorem 2.3: First K-Hyper Eccentric Index of G = $TUAC_6(p, q)$ is

$HB_1E(G) = 2\sum_{k=0}^{q-1} 2p[(p+q+k)+(p+q+(k-1))]^2 +$

$2\sum_{k=1}^{q-1} 2p[[(p+q+k)+(p+q+(k-1))]^2 + 4(p+q+(k-1))^2]$ + 4p[(p + q) +(p + q – 1)]^2

Proof: $HB_1E(G) = \sum_{ue} [(e_G(u) + e_{L(G)}(e)]^2$

$= \sum_{ue} [(e_G(u) + e_{L(G)}(e)]^2 + \sum_{ve} [(e_G(v) + e_{L(G)}(e)]^2$

= {p[((p + 2q – 1) + (p + 2q – 2))^2 +((p + 2q – 2) + (p + 2q – 1)]^2]} + {2p[(p + 2q – 1) + (p + 2q – 2))^2 +[(p + 2q – 2) +(p + 2q – 2)]^2]} + {p[((p + 2q – 2) + (p + 2q – 3))^2 + ((p + 2q – 3) + (p + 2q – 2)]^2]} +…+ {2p[((p + q + 1) + (p + q))^2 + ((p + q) +(p + q)]^2]} + {p[((p + q) + (p + q –1))^2 + [(p + q)+ (p + q –1)]^2]} + {2p[((p + q) + (p + q –1))^2 +((p + q) + (p + q – 1)]^2]}+…+ {2p[((p + 2q – 2) + (p + 2q – 2))^2 +[(p + 2q – 2) + (p + 2q – 1)]^2]}+{p[((p + 2q – 1) +(p + 2q – 2))^2 + ((p + 2q – 2) + (p + 2q – 1)]^2]}

= {2p[(p + 2q – 1) + (p + 2q – 2)]^2}+ {2p[((p + 2q – 1) (p + 2q – 2))^2+ 4(p + 2q – 2)]^2}+ {2p[(p + 2q – 2) + (p + 2q – 3)]^2}+…+ {2p[((p + q + 1) (p + q))^2 + 4(p + q)]^2} + {2p[(p + q) + (p + q – 1]^2} + ...+ {2p[4(p + 2q – 2)^2 +[(p + q – 2) (p + 2q – 1)]^2}+ {2p[(p + 2q – 1) + (p + 2q – 2)]^2}

$$= 2\sum_{k=0}^{q-1} 2p[(p+q+k)+(p+q+(k-1))]^2 +$$

$$2\sum_{k=1}^{q-1} 2p[[(p+q+k)+(p+q+(k-1))]^2 + 4(p+q+(k-1))^2] + 4p[(p + q) +(p + q - 1)]^2$$

Theorem 2.4: Second K-Hyper Eccentric Index of G = $TUAC_6(p, q)$ is

$$HB_2E(G) = 2\sum_{k=0}^{q-1} 2p[(p+q+k)(p+q+(k-1))]^2$$

$$+2\sum_{k=1}^{q-1} 2p[(p+q+(k-1))^4 + ((p+q+k)(p+q+(k-1)))^2] + 4p((p + q)(p + q - 1))^2$$

Proof: $HB_2E(G) = \sum_{ue}[(e_G(u).e_{L(G)}(e)]^2$

$$= \sum_{e=uv\in E(G)}[(e_G(u).e_{L(G)}(e)) + (e_G(v).e_{L(G)}(e))]^2$$

= {p[[(p + 2q – 1) (p + 2q – 2)]^2+[(p + 2q – 1) (p + 2q – 2)]^2]}+ {2p[[(p + 2q – 1)(p + 2q – 2)]^2 +[(p + 2q – 2) (p + 2q – 2)]^2]}+ {p[[(p + 2q – 2)(p + 2q – 3)]^2 + [(p + 2q – 2) (p + 2q – 3)]^2]} +...+ {2p[[(p + q + 1)(p + q)]^2 + [(p + q) (p + q)]^2]}+{p[[(p + q)(p + q – 1)]^2 + [(p + q)(p + q – 1)]^2] }+ {2p[[(p + q)(p + q – 1)]^2 + [(p + q) (p + q – 1)]^2]}+…+ {2p[[(p + 2q – 2)(p + 2q – 2)]^2 + [(p + 2q – 1) (p + 2q – 2)]^2]}+ {p[[(p + 2q – 1) (p + 2q – 2)]^2 + [(p + 2q – 1) (p + 2q – 2)]^2]}

= {2p[(p + 2q – 1) (p + 2q – 2)]^2}+ {2p[(p + 2q – 2)^4 +[(p + 2q – 2) (p + 2q – 1)]^2]}+ {2p[(p + 2q – 2) (p + 2q – 3)]^2}+….+ {2p[[(p + q)^4 +[(p + q + 1) (p + q)]^2} + 2p[(p + q)(p + q – 1)]^2} + {4p[(p + q)(p + q – 1)]^2}+…+{2p[(p + 2q – 2)^4 +[(p + 2q – 1)(p + 2q – 2)]^2]}+ {2p[(p + 2q – 1) (p + 2q – 2)]^2}

$$= 2\sum_{k=0}^{q-1} 2p[(p+q+k)(p+q+(k-1))]^2$$

$$+2\sum_{k=1}^{q-1} 2p[(p+q+(k-1))^4 + ((p+q+k)(p+q+(k-1)))^2] + 4p((p + q)(p + q - 1))^2$$

Multiplicative First and Second K-Eccentric Indices and Multiplicative First and Second K-Hyper Eccentric Indices

Theorem 2.5: Multiplicative First K-Eccentric Index of G = $TUAC_6(p, q)$ is

$$B\Pi_1E(G) = \prod_{k=0}^{q-1}\left(p[(p+q+k)+(p+q+(k-1))^2\right)^2 \prod_{k=1}^{q-1}\left(2p[(p+q+(k-1)+(p+q+k))2(p+q+(k-1))]\right)^2 \times$$

$2p((p + q) + (p + q – 1))^2$

Proof: $B\Pi_1E(G) = \prod_{ue}[e_G(u) + e_{L(G)}(e)]$

$$= \prod_{e=uv\in E(G)}[(e_G(u) + e_{L(G)}(e)).(e_G(v) + e_{L(G)}(e))]$$

={ p[[(p + 2q – 1) + (p + 2q – 2)] [(p + 2q – 1) + (p + 2q – 2)]]} {2p[[(p + 2q – 1) + (p + 2q – 2)] [(p + 2q – 2) + (p + 2q – 2)]]} {p[[(p + 2q – 2) + (p + 2q – 3)] [(p + 2q – 2) + (p + 2q – 3)]]} ... {2p[[(p + q + 1) + (p + q)] [(p + q) +(p + q)]]} {p[[(p + q)+(p + q – 1)] [(p + q)+ (p + q – 1)]} {2p[(p + q) + (p + q – 1)] [(p + q) + (p + q –1)]]} ... {2p[[(p + 2q – 2) +(p + 2q – 2)] [(p + 2q – 1) + (p + 2q – 2)]]} {p[[(p + 2q – 1) + (p + 2q – 2)][(p +2q –1) + (p + 2q – 2)]]}

= {p[(p + 2q – 1) + (p + 2q – 2)]^2} {2p[((p + 2q – 1) + (p + 2q – 2)) 2(p + 2q – 2)]} {p[(p + 2q – 2) + 2(p + 2q – 3)] }... {2p[((p + q + 1) + (p + q)) 2(p + q)]} {p[(p + q)+(p + q – 1)]^2} {2p[((p + q)+(p + q – 1))^2]} ... {2p[((p + 2q – 2) + (p + 2q – 1)) 2(p + 2q – 2)]} + {p[(p + 2q – 1) + (p + 2q – 2)]^2}

$$= \prod_{k=0}^{q-1} \left(p[(p+q+k)+(p+q+(k-1))^2 \right)^2$$

$$\prod_{k=1}^{q-1} \left(2p[(p+q+(k-1)+(p+q+k))2(p+q+(k-1))] \right)^2 \times 2p((p+q)+(p+q-1))^2$$

Theorem 2.6: Multiplicative second K-Eccentric Index of G = TUAC$_6$(p, q) is $B\Pi_2E(G)$

$$= \prod_{k=0}^{q-1} p^2[(p+q+k)(p+q+(k-1))]^4 \prod_{k=1}^{q-1} \left(2p[(p+q+k)(p+q+(k-1))^3] \right)^2$$

$\times$ 2p((p + q)(p + q – 1))^2

Proof: $B\Pi_2E(G) = \prod_{ue} [e_G(u).e_{L(G)}(e)]$

$$= \prod_{uv} [(e_G(u).e_{L(G)}(e)).(e_G(v).e_{L(G)}(e))]$$

= {p[[(p + 2q – 1) (p + 2q – 2)] [(p + 2q – 1) (p + 2q – 2)]]} {2p[[(p + 2q – 1) (p + 2q – 2)] [(p + 2q – 2) (p + 2q – 2)]]} {p[[(p + 2q – 2) (p + 2q – 3)] [(p + 2q – 2) (p + 2q – 3)]]} … {2p[[(p + q + 1) (p + q)] [(p + q) (p + q)]]} {p[[(p + q)(p + q – 1)] [(p + q) (p + q –1)]]} {2p[(p + q) (p + q – 1)] [(p + q) (p + q –1)]]} … {2p[[(p + 2q – 2) (p + 2q – 2)] [(p + 2q – 1) (p + 2q – 2)]]} {p[[(p + 2q – 1) (p + 2q – 2)] [(p + 2q – 1)(p + 2q – 2)]]}

= {p[(p + 2q – 1)(p + 2q – 2)]^2} {2p[(p + 2q – 1) (p + 2q – 2) (p + 2q – 2)^2]} {p[(p + 2q – 2) (p + 2q – 3)]^2}… {2p[(p + q + 1)(p + q) (p + q)^2]} {p[(p + q)(p + q – 1)]^2} {2p[(p + q)(p + q – 1)]^2]} ... {2p[(p + 2q –2)^2[(p + 2q – 2) + (p + 2q – 1)]]} {p[(p + 2q – 1) (p + 2q – 1)]^2}

$$= \prod_{k=0}^{q-1} p^2[(p+q+k)(p+q+(k-1))]^4 \prod_{k=1}^{q-1} \left(2p[(p+q+k)(p+q+(k-1))^3] \right)^2$$

$\times$ 2p((p + q)(p + q – 1))^2

Theorem 2.7: Multiplicative First K-Hyper Eccentric Index of G = TUAC$_6$(p, q) is $HB\Pi_1E(G)$

$$= \prod_{k=0}^{q-1} \left(p[(p+q+k)+(p+q+(k-1))]^4 \right)^2 \prod_{k=1}^{q-1} \left(2p[4(p+q+(k-1))^2((p+q+k)+(p+q+(k-1))]^2] \right)^2 \times$$

2p((p + q) + (p + q – 1))^4

Proof: $HB\Pi_1E(G) = \prod_{ue} [e_G(u)+e_{L(G)}(e)]^2$

$$= \prod_{e=uv\in E(G)} [(e_G(u)+e_{L(G)}(e)).(e_G(v)+e_{L(G)}(e))]^2$$

= {p[[(p + 2q – 1) +(p + 2q – 2)]^2[(p + 2q – 1) +(p + 2q – 2)]^2] } {2p[[(p + 2q – 1) +(p + 2q– 2)]^2 [(p + 2q – 2) +(p + 2q – 2)]^2]} {p[[(p + 2q – 2) + (p + 2q – 3)]^2[(p + 2q – 2) + (p + 2q – 3)]^2]}… {2p[[(p + q + 1) + (p + q)]^2 [(p + q) +(p + q)]^2]} {p[[(p + q)+(p + q –1)]^2 [(p + q)+ (p + q – 1)]^2]} {2p[(p + q) + (p + q – 1)]^2 [(p + q) + (p + q – 1)]^2]} … {2p[[(p + 2q – 2) +(p + 2q – 2)]^2 [(p + 2q – 1) + (p + 2q – 2)]^2]} {p[[(p + 2q – 1) + (p + 2q – 2)]^2 [(p + 2q – 1) + (p + 2q – 2)]^2]}

$$= \prod_{k=0}^{q-1} \left(p[(p+q+k)+(p+q+(k-1))]^4 \right)^2$$

$$\prod_{k=1}^{q-1} \left(2p[4(p+q+(k-1))^2((p+q+k)+(p+q+(k-1))]^2] \right)^2 \times 2p((p+q)+(p+q-1))^4$$

Theorem 2.8: Multiplicative First K-Hyper Eccentric Index of G = TUAC$_6$(p, q) is $HB\Pi_2E(G)$

$$= \prod_{k=1}^{q-1} \left(p[(p+q+k)(p+q+(k-1))]^4 \right)^2 \prod_{k=1}^{q-1} \left(2p[(p+q+k)^2(p+q+(k-1))^6 \right)^2 2p((p+q)(p+q-1))^4$$

Proof: $HB\Pi_2E(G) = \prod_{ue} [e_G(u).e_{L(G)}(e)]^2$

$$= \prod_{ue} [(e_G(u).e_{L(G)}(e)).(e_G(v).e_{L(G)}(e))]^2$$

= {p[[(p + 2q – 1) (p + 2q – 2)]^2 [(p + 2q – 1) (p + 2q – 2)]]^2]} {2p[[(p + 2q – 1) (p + 2q – 2)]^2 [(p + 2q – 2) (p + 2q – 2)]]^2]} {p[[(p + 2q – 2) (p + 2q – 3)]^2 [(p + 2q – 2) (p + 2q –

$3)]]^2]\}$ … $\{2p[[(p + q + 1)(p + q)]^2 \; [(p + q)(p + q)]]^2]\} \; \{p[[(p + q)(p + q - 1)]^2 [(p + q)(p + q - 1)]^2]\} \; \{2p[(p + q)(p + q - 1)]^2 \; [(p + q)(p + q - 1)]]^2]\} \ldots \{2p[[(p + 2q - 2)(p + 2q - 2)]^2 \; [(p + 2q - 1)(p + 2q - 2)]]^2]\} \; \{p[[(p + 2q - 1)(p + 2q - 2)]^2 \; [(p + 2q - 1)(p + 2q - 2)]]^2]\}$

$= \{p[(p + 2q - 1)(p + 2q - 2)]^4\} \; \{2p[((p + 2q - 1)(p + 2q - 2))^2 ((p + 2q - 2)^4)]\} \; \{p[(p + 2q - 2)(p + 2q - 3)]^4\} \ldots \{2p[((p + q + 1)(p + q))^2 \; ((p + q)^4)]\} \; \{p[((p + q)(p + q - 1))^4\} \; \{2p[(p + q)(p + q - 1)]^4\} \ldots \{2p[(p + 2q - 2)^4[(p + 2q - 2) + (p + 2q - 1)]^2\} \; \{p[(p + 2q - 1)(p + 2q - 2)]^4\}$

$$= \prod_{k=1}^{q-1} \left(p[(p+q+k)(p+q+(k-1))]^4\right)^2 \prod_{k=1}^{q-1} \left(2p[(p+q+k)^2(p+q+(k-1))^6\right)^2$$

$2p((p + q)(p + q - 1))^4$

3. K-Eccentricity indices of Zigzag polyhex nanotubes $TUZC_6(p, q)$

In this section, we compute First K-Eccentric Index, Second K-Eccentric Index, First K- Hyper Eccentric Index, Second K-Hyper Eccentric Index, Multiplicative First K-Eccentric Index, Multiplicative Second K-Eccentric Index, Multiplicative First K-Hyper Eccentric Index, Multiplicative Second K-Hyper Eccentric Index of Zigzag polyhex nanotubes.

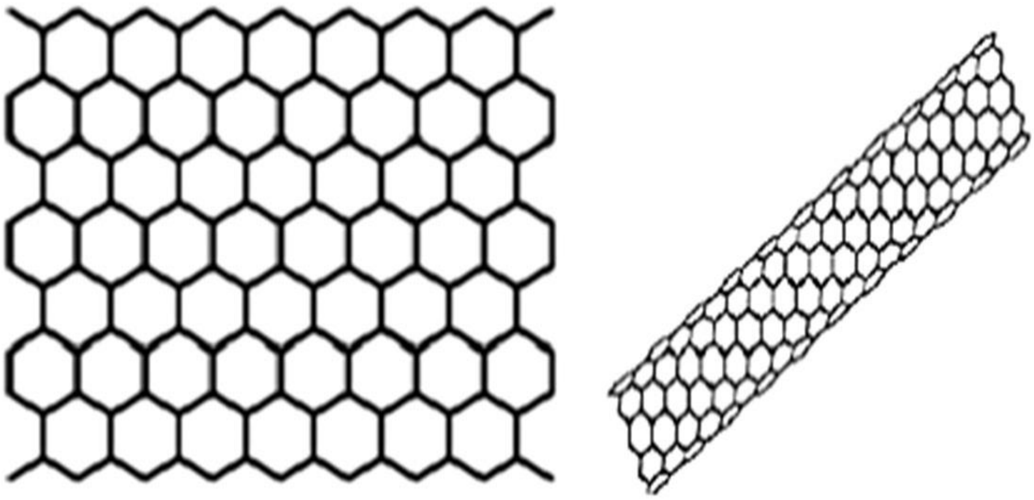
Let G = $TUZC_6(p, q)$ be a zigzag polyhex nanotube, where p is the number of hexagons in each row and q is the number of zigzag lines in the column of molecular graph of G. The graph $TUZC_6(p, q)$ has 2pq vertices and 3pq – p edges.

The eccentricity of e = uv in L(G) and the eccentricity of vertices u and v in G are given in the following table:

Sl.No	Number of edges e = uv in G	Eccentricity of e in L(G)	Eccentricity of end vertices (e(u), e(v)) in G
1	2p p	(p + q/2) + (q/2 – 1) (p + q/2) + (q/2 – 2)	(p + q/2 + (q/2 – 1), p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1), p + q/2 + (q/2 – 2))
2	2p p	(p + q/2) + (q/2 – 2) (p + q/2) + (q/2 – 3)	(p + q/2 + (q/2 – 2), p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2), p + q/2 + (q/2 – 3))
3	2p p	(p + q/2) + (q/2 – 3) (p + q/2) + (q/2 – 4)	(p + q/2 + (q/2 – 3), p + q/2 + (q/2 – 3)) (p + q/2 + (q/2 – 3), p + q/2 + (q/2 – 4))
-----	-------------	------------------	-------------------------
q/2 – 1	2p p	p + q/2 +1 p + q/2	(p + q/2 + 1, p + q/2 + 1) (p + q/2 + 1, p + q/2)
q/2	2p p	p + q/2 p + q/2 – 1	(p + q/2, p + q/2) (p + q/2, p + q/2)
-----	-------------	---------------	-------------------
q – 1	2p p	(p + q/2) + (q/2 – 2) (p + q/2) + (q/2 – 2)	(p + q/2 + (q/2 – 2), p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2), p + q/2 + (q/2 – 1))
q	2p	(p + q/2) + (q/2 – 1)	(p + q/2 + (q/2 – 1), p + q/2 + (q/2 – 1))

Table 3.1

Example: Let G = $TUZC_6(8, 8)$. The graph $TUZC_6(8, 8)$ has 128 vertices and 184 edges.
The eccentricity of e = uv in L(G) and the eccentricity of vertices u and v in G are given in Table 3.2



(a) (b)

Fig (a) The 2-dimensional lattice (b) TUZC$_6$nanotube of TUZC$_6(8,8)$ nanotube

Sl. No	Number of edges e = uv in G	Eccentricity of e in L(G)	Eccentricity of end vertices (e(u), e(v)) in G
1	16 8	15 14	(15, 15) (15, 14)
2	16 8	14 13	(14, 14) (14, 13)
3	16 8	13 12	(13, 13) (13, 12)
4	16 8	12 11	(12, 12) (12, 12)
5	16 8	12 12	(12, 12) (12, 13)
6	16 8	13 13	(13, 13) (13, 14)
7	16 8	14 14	(14, 14) (14, 15)
8	16	15	(15, 15)

Table 3.2

(i) $B_1E(G) = \sum_{e=uv\in E(G)}[e_G(u)+2e_{L(G)}(e)+e_G(v)]$

= 2{16(15+2(15) +15) + 8(15 + 2(14) +14) + 16(14 + 2(14) +14) + 8(14 +2(13) + 13) + 16(13 +2(13) + 13) + 8(13 + 2(12) + 12) + 16(12 + 2(12) +12)} + 8(12 +2(11) +12)

= 2{960 + 456 +896 + 424 + 832 + 392 + 768} + 368 = 2 × 4728 + 368 = 9824

Simillarly, we can find out that

(ii) $B_2E(G)$ = 66000 (iii) $HB_1E(G)$ = 264064 (iv) $HB_2E(G)$ = 12160776

(v) $B\Pi_1E(G) = (3.152985945 \times 10^{27})^2$ (4232)

(vi) $B\Pi_2E(G) = (1.088475516 \times 10^{40})^2$(139392)

(vii) $HB\Pi_1E(G) = (2.962744347 \times 10^{46})^2$(2238728)

(viii) $HB\Pi_2E(G) = (4.141055578 \times 10^{29})^2$(2428766208)

First and Second K-Eccentric Indices and the First and Second K-Hyper Eccentric Indices

Theorem 3.1: First K-Eccentric Index of G = TUZC$_6$(p, q) is

$$B_1E(G) = 2\sum_{k=0}^{q/2-1} 2p(4(p+q/2+k)) + 2\sum_{k=1}^{q/2-1} p(3(p+q/2+(k-1)) + (p+q/2+k)) + 2p((p + q/2)+(p + q/2-1))$$

Proof: $B_1E(G) = \sum_{ue}[(e_G(u)+e_{L(G)}(e)]$

$$= \sum_{e=uv\in E(G)}[e_G(u)+2e_{L(G)}(e)+e_G(v)]$$

= {2p((p + q/2 + (q/2 – 1)) +2(p + q/2 + (q/2 – 1)) + (p + q/2 + (q/2 – 1)))} + {p((p + q/2 + (q/2 – 1)) +2(p + q/2 + (q/2 – 2)) + (p + q/2 + (q/2 – 2)))} + {2p((p + q/2 + (q/2 – 2)) + 2(p + q/2 + (q/2 – 2)) + (p + q/2 + (q/2 – 2)))} + ...+ {2p((p + q/2) + 2(p + q/2) + (p + q/2))} + {p((p + q/2) + 2(p + q/2 – 1) + (p + q/2))} + …+ {p((p + q/2 + (q/2 – 2)) + 2(p + q/2 + (q/2 – 2)) + (p + q/2 + (q/2 – 1)))} + {2p((p + q/2 + (q/2 – 1)) + 2(p + q/2 + (q/2 – 1)) + (p + q/2 + (q/2 – 1)))}

= {2p(4(p + q –1))} + {p((p + q – 1) + 3(p + q – 2))} + {2p(4(p + q – 2))} + ...+ {2p(4(p + q/2))} + {p(3(p + q/2) + (p + q/2 + 1))} + …+ {p(3(p + q – 2) + (p + q – 1))} + {2p(4(p + q – 1))}

$$= 2\sum_{k=0}^{q/2-1}2p(4(p+q/2+k)) + 2\sum_{k=1}^{q/2-1}p(3(p+q/2+(k-1))+(p+q/2+k))$$

+ 2p((p + q / 2) + (p + q / 2 - 1))

Theorem 3.2: Second K-Eccentric Index of G = $TUZC_6(p, q)$ is

$$B_2E(G) = 2\sum_{k=0}^{q/2-1}2p(2(p+q/2+k)^2) + 2\sum_{k=1}^{q/2-1}p((p+q/2+k)(p+q/2+(k-1))+(p+q/2+(k-1))^2)$$

+ 2p((p + q / 2)(p + q / 2 – 1))

Proof: $B_2E(G) = \sum_{ue}[(e_G(u).e_{L(G)}(e)]$

$$= \sum_{e=uv\in E(G)}[((e_G(u).e_{L(G)}(e)+(e_G(v).e_{L(G)}(e))]$$

= {2p((p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1)) + (p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1))) + {p((p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 2)) + (p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2))} + {2p((p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2)) + (p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2)))} + …+ {2p((p + q/2) (p + q/2) + (p + q/2) (p + q/2))} + {p((p + q/2) (p + q/2) + (p + q/2 – 1) (p + q/2)} + …+ {p((p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2)) + (p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 2))} + {2p((p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 1)) + (p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1)))}

= $\{2p(2(p + q –1)^2)\} + \{p((p + q – 1)(p + q – 2) + (p + q – 2)^2)\} + \{2p(2(p + q – 2)^2)\} + …+ \{2p(2(p + q/2)^2)\} + \{p((p + q/2)^2 +(p + q/2 – 1) (p + q/2))\} + …+ \{p((p +q – 2)^2 + (p + q– 1)(p + q– 2))\} + \{2p(2(p + q –1)^2)\}$

$$=2\sum_{k=0}^{q/2-1}2p(2(p+q/2+k)^2) + 2\sum_{k=1}^{q/2-1}p((p+q/2+k)(p+q/2+(k-1))+(p+q/2+(k-1))^2) +$$

p(2(p + q / 2)(p + q / 2 – 1))

Theorem 3.3: First K - Hyper Eccentric Indices of G = $TUZC_6(p, q)$ is

$$HB_1E(G)=2\sum_{k=0}^{q/2-1}2p(8(p+q/2+k)^2)+2\sum_{k=1}^{q/2-1}p(4(p+q/2+(k-1))^2+((p+q/2+k)+(p+q/2+(k-1)))^2$$

+ $2p((p + q / 2) + (p + q / 2 – 1))^2$

Proof: $HB_1E(G) = \sum_{uv}[(e_G(u)+e_{L(G)}(e)]^2$

$$=\sum_{ue}[(e_G(u)+e_{L(G)}(e)]^2+\sum_{ve}[(e_G(v)+e_{L(G)}(e)]^2$$

= $\{2p((p + q/2 + (q/2 – 1) +(p + q/2 + (q/2 – 1))^2 + ((p + q/2 + (q/2 – 1) + (p + q/2 + (q/2\ 1)))^2)\} + \{p((p + q/2 +(q/2 – 1) +(p + q/2 + (q/2 – 2))^2 + ((p + q/2 + (q/2 – 2))+ (p + q/2 + (q/2 – 2)))^2\}+ \{2p((p + q/2 + (q/2 – 2) +(p + q/2 + (q/2 – 2))^2 +((p + q/2 + (q/2 – 2)) +(p + q/2 + (q/2 – 2)))^2)\}+ ...+ \{2p((p + q/2) + (p + q/2))^2 +((p + q/2) +(p + q/2))^2)\}+ \{p((p + q/2) + (p + q/2 – 1))^2 + ((p + q/2) + (p + q/2 – 1))^2\}+ …+ \{p((p + q/2 + (q/2 – 2)) + (p + q/2 + (q/2 – 2)))^2 + ((p + q/2 + (q/2 – 2) + (p + q/2 + (q/2 – 1)))^2)\}+ \{2p((p + q/2 + (q/2 – 1)) + (p + q/2 + (q/2 – 1)))^2 +((p + q/2 + (q/2 – 1)) + (p + q/2 + (q/2 – 1)))^2)\}$

= $\{2p(8(p + q –1)^2)\}+ \{p(((p + q – 1) + (p + q – 2))^2 + 4(p + q – 2)^2)\}+ \{2p(8(p + q – 2)^2)\}+ ...+ \{2p(8(p + q/2)^2)\}+ \{p((p + q/2) + (p + q – 1))^2 + ((p + q / 2) + (p +q/2 + 1))^2)\}+ …+\{p(4(p + q – 2)^2 + (p + q – 1) + (p + q – 2))^2\}+ \{2p(8(p+q –1)^2)\}$

$$= 2\sum_{k=0}^{q/2-1} 2p(8(p+q/2+k)^2)\ 2\sum_{k=1}^{q/2-1} p(4(p+q/2+(k-1))^2 + ((p+q/2+k)+(p+q/2+(k-1)))^2$$

+ 2p((p + q / 2) + (p + q / 2 – 1))^2

Theorem 3.4: Second K-Hyper Eccentric Index of G = $TUZC_6$(p, q) is

$$HB_2E(G) = 2\sum_{k=0}^{q/2-1}\left(4p((p+q/2+k)^4)\right) + 2\sum_{k=1}^{q/2-1}\left(p((p+q/2+k)(p+q/2+(k-1)))^2 + (p+q/2+k)^4)\right)$$

+ 2p((p + q /2)(p + q / 2 – 1))^2

Proof: $HB_2E(G) = \sum_{ue}[(e_G(u).e_{L(G)}(e)]^2$

$$= \sum_{e=uv\in E(G)}[(e_G(u).e_{L(G)}(e)) + (e_G(v).e_{L(G)}(e))]^2$$

= {2p[[(p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 1))^2 + ((p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 1))]]^2)}+ {p[[(p + q/2 + (q/2 – 1)(p + q/2 + (q/2 – 1))^2 + ((p + q/2 + (q/2 – 2) (p + q/2 + (q/2 – 1))]]^2)} + {2p[[(p + q/2 + (q/2 – 2) (p + q/2 + (q/2 – 2))^2 + ((p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2))]]^2)}+ … + {2p[((p + q/2) (p + q/2))^2 + ((p + q/2)(p + q/2))^2]}+ {p[((p + q/2)(p + q/2 – 1))^2 + ((p + q/2 – 1)(p + q/2))^2]}+ ...+ {p[((p + q/2 + (q/2 – 2) (p + q/2 + (q/2 – 1))^2 + ((p + q/2 +(q/2 – 1) (p + q/2 + (q/2 – 1))]^2]}+ {2p[[(p + q/2 + (q/2 – 1) (p + q/2 + (q/2 – 1))^2 + ((p + q/2 + (q/2 – 1)(p + q/2 + (q/2 – 1))]^2]}

= {4p(p + q – 1)^4} + {p((p + q – 1)^4+ ((p + q – 2)(p + q – 1))^2}+ {4p((p + q – 2)^4}+... +{4p(p +q/2)^4 } {p(((p + q/2)(p + q/2 +1))^2 + (p + q/2 +1)^4)} +…+ {p(((p + q – 2) (p + q – 1))^2 + (p + q – 1)^4)} {4p(p + q – 1)^4}

$$= 2\sum_{k=0}^{q/2-1}\left(4p((p+q/2+k)^4)\right) + 2\sum_{k=1}^{q/2-1}\left(p((p+q/2+k)(p+q/2+(k-1)))^2 + (p+q/2+k)^4)\right)$$

+ 2p((p + q /2)(p + q / 2 – 1))^2

Multiplicative First and Second K-Eccentric Indices and Multiplicative First and Second K - Hyper Eccentric Indices

Theorem 3.5: Multiplicative First K-Eccentric Indices of G = $TUZC_6$(p, q) is

$$B\Pi_1E(G) = \prod_{k=0}^{q/2-1}\left(2p(4(p+q/2+k)^2)\right)^2\ \prod_{k=1}^{q/2-1}\left(p[2(p+q/2+(k-1)[(p+q/2+k)+(p+q/2+(k-1))]]\right)^2 \times$$

p((p + q/ 2) + (p + q / 2 – 1))^2

Proof: $B\Pi_1E(G) = \prod_{ue}[e_G(u) + e_{L(G)}(e)]$

$$= \prod_{e=uv\in E(G)}[(e_G(u) + e_{L(G)}(e)).(e_G(v) + e_{L(G)}(e))]$$

= {2p((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 1)))} {p((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2) + p + q/2 + (q/2 – 2)))} {2p((p + q/2 + (q/2 – 2) + p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2) + p + q/2 + (q/2 – 2)))}…{2p((p + q/2 + p + q/2) (p + q/2 + p + q/2))} {p((p + q/2 + p + q/2 – 1) (p + q/2 – 1 + p + q/2))} … {p((p + q/2 +(q/2 – 2) + p + q/2 + (q/2 – 2) (p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 2)))} {2p((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 1)))}

$$= \prod_{k=0}^{q/2-1}\left(2p(4(p+q/2+k)^2)\right)^2\ \prod_{k=1}^{q/2-1}\left(p[2(p+q/2+(k-1)[(p+q/2+k)+(p+q/2+(k-1))]]\right)^2$$

× p((p + q/ 2) + (p + q / 2 – 1))^2

Theorem 3.6: Multiplicative second K-Eccentric Indices of G = $TUZC_6$(p, q) is

$$B\Pi_2E(G) = \prod_{k=0}^{q/2-1}\left(2p((p+q/2+k)^4\right)^2\ \prod_{k=1}^{q/2-1}\left(p((p+q/2+k)(p+q/2+(k-1))(p+q/2+(k-1))^2)\right)^2$$

× p((p + q / 2)^2 (p + q / 2 – 1)^2)

Proof: $B\Pi_2E(G) = \prod_{ue}[e_G(u).e_{L(G)}(e)]$

$= \prod_{uv}[(e_G(u).e_{L(G)}(e)).(e_G(v).e_{L(G)}(e))]$

= {2p[(p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 1))]} {p[(p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 2))(p + q/2 + (q/2 – 2))(p + q/2 + (q/2 – 2))]} {2p[(p + q/2 + (q/2 – 2))(p + q/2 + (q/2 – 2))(p + q/2 + (q/2 – 2))(p + q/2 + (q/2 – 2))]} ... {2p[((p + q/2) (p + q/2)) ((p + q/2) (p + q/2))]} {p[((p + q/2) (p + q/2 – 1)) ((p + q/2 – 1) (p + q/2))]} … {p[((p + q/2 + (q/2 – 2)) ((p + q/2 + (q/2 – 2))] ((p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 2)))]} {2p[[(p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1))] [(p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 1))]]}

= $\{2p(p + q – 1)^4\}$ $\{p((p + q – 1)(p + q – 2)(p + q – 2)^2)\}$ $\{2p((p + q – 1)^4\}$ ... $\{2p(((p + q/2)^4\}$ $\{p(((p + q/2)(p + q/2 –1))(p + q/2 – 1)^2)\}$ …$\{p(((p + q – 1) (p + q – 2)) (p + q – 2)^2)\}$ $\{2p(p + q – 1)^4\}$

$$= \prod_{k=0}^{q/2-1}\left(2p((p+q/2+k)^4\right)^2 \prod_{k=1}^{q/2-1}\left(p((p+q/2+k)(p+q/2+(k-1))(p+q/2+(k-1))^2)\right)^2$$

$\times\ p((p + q / 2)^2 (p + q / 2 – 1)^2)$

Theorem 3.7: Multiplicative First K-Hyper Eccentric Indices of G = $TUZC_6(p, q)$ is $HB\Pi_1E(G)=$
$$\prod_{k=0}^{q/2-1}\left(2p(16(p+q/2+k)^4\right)^2 \prod_{k=1}^{q/2-1}\left(p((p+q/2+k)+(p+q/2+(k-1)))^2(4(p+q/2+(k-1))^2)\right)^2$$
$\times\ p((p + q/ 2) + (p + q / 2 – 1))^4$

Proof: $HB\Pi_1E(G) = \prod_{ue}[e_G(u)+e_{L(G)}(e)]^2$

$$= \prod_{e=uv\in E(G)}[(e_G(u)+e_{L(G)}(e)).(e_G(v)+e_{L(G)}(e))]^2$$

= $\{2p((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 1))^2$ $((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 1))^2)\}$ $\{p((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 2))^2$ $((p + q/2 + (q/2 – 2) + p + q/2 + (q/2 – 2))^2)\}$ $\{2p((p + q/2 + (q/2 – 2) + p + q/2 + (q/2 – 2))^2$ $((p + q/2 + (q/2 – 2) + p + q/2 + (q/2 – 2))^2)\}$ … $\{2p((p + q/2 + p + q/2)^2$ $(p + q/2 + p + q/2)^2)\}$ $\{p((p + q/2 + p + q/2 –1)^2$ $(p + q/2 – 1 + p + q/2))^2\}$… $\{p((p + q/2 +(q/2 –2) + p + q/2 + (q/2 – 2))^2$ $((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 2))^2)\}$ $\{2p((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 1))^2$ $((p + q/2 + (q/2 – 1) + p + q/2 + (q/2 – 1))^2)\}$

$$= \prod_{k=0}^{q/2-1}\left(2p(16(p+q/2+k)^4\right)^2$$

$$\prod_{k=1}^{q-1}\left(p((p+q/2+k)+(p+q/2+(k-1)))^2(4(p+q/2+(k-1))^2)\right)^2$$

$\times\ p((p + q/ 2) + (p + q / 2 – 1))^4$

Theorem 3.8: Multiplicative First K-Hyper Eccentric Indices of G = $TUAC_6(p, q)$ is $HB\Pi_2E(G)$ =
$$\prod_{k=0}^{q/2-1}\left(2p(p+q/2+k)^8\right)^2 \prod_{k=1}^{q/2-1}\left(p(((p+q/2+k)(p+q/2+(k-1)))^2(p+q/2+(k-1)^4)\right)^2$$
$\times\ p((p + q/ 2)(p + q / 2 – 1))^4$

Proof: $HB\Pi_2E(G) = \prod_{ue}[e_G(u).e_{L(G)}(e)]^2$

$$= \prod_{ue}[(e_G(u).e_{L(G)}(e)).(e_G(v).e_{L(G)}(e))]^2$$

= $\{2p[[(p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 1))]^2[(p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 1))]^2]\}$ $\{p[[(p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 2))]^2$ $[(p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2))]^2]\}$ $\{2p[[(p + q/2 + (q/2 – 2))$ $(p + q/2 + (q/2 – 2))]^2$ $[(p + q/2 + (q/2 – 2))$ $(p + q/2 + (q/2 – 2))]^2]\}$ …$\{2p[((p + q/2) (p + q/2))^2$ $((p + q/2) (p + q/2))^2]\}$ $\{p[((p + q/2) (p + q/2))^2$ $((p + q/2 + 1)(p + q/2))^2]\}$ … $\{p[((p + q/2 + (q/2 – 2)) ((p + q/2 + (q/2 – 2))]^2$ $((p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 2)))^2]\}$ $\{2p[[(p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1))]^2$ $[(p + q/2 + (q/2 – 1))(p + q/2 + (q/2 – 1))]^2]\}$

$= \{2p(p + q - 1)^8\} \{p([(p + q - 1)(p + q - 2)]^2(p + q - 2)^4)\} \{2p((p + q - 1)^8)\}... \{2p(p + q/2)^8\} \{p(((p + q/2)(p + q/2 +1))^2(p + q/2 +1)^4)\}...\{p(((p + q - 1) (p + q - 2))^2 (p + q - 2)^4)\{2p(p + q - 1)^8\}$

$$= \prod_{k=0}^{q/2-1}\left(2p(p+q/2+k)^8\right)^2 \prod_{k=1}^{q/2-1}\left(p(((p+q/2+k)(p+q/2+(k-1)))^2(p+q/2+(k-1))^4)\right)^2$$

$$\times\ p((p + q/ 2)(p + q / 2 - 1))^4$$

4.Conclusion

In this paper we have computed First K-Eccentric Index, Second K-Eccentric Index, First K-Hyper Eccentric Index, Second K-Hyper Eccentric Index, Multiplicative First K-Eccentric Index, Multiplicative Second K-Eccentric Index, Multiplicative First K-Hyper Eccentric Index, Multiplicative Second K-Hyper Eccentric Index of armchair polyhex nanotubes and zigzag polyhex nanotubes.

5.References

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