

# Dynamic Response Analysis of Preloaded Bolted Joints under Harmonic Excitation

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**Abstract:** This study has been undertaken to investigate the dynamic response analysis of preloaded bolted joints with and without gasket under harmonic excitation. Mathematical model of preloaded bolted joint of pressure vessel is developed and various parameters such as stiffness of bolts, stiffness of combined plates and damping involved are determined. With the help of mathematical model and various parameters equations of motion for bolted joint with and without gasket are formulated and response of the system in the form of motion and force transmissibilities are determined.

## 1.1 INTRODUCTION

The simple bolted joint is used to clamp two or more components which can be disassemble without destructing joint. In bolted joint preload is applied to place the bolted components in compression to resist the external tensile loads and to create friction between the components. Considering the bolted joint, nut is routed on bolt's screw thread against the joint, the bolt is extended. This extension is resisted by internal forces within the bolt and a tensile force also known as bolt preload is generated. The reaction of this force is clamping force with which joint is compressed.

The failure mode in which the separation of members in compression is very serious in the types of bolted joint. As such, it is necessary to study the dynamic response of preloaded bolted joints under the periodic excitation. Researchers in the past have carried out some theoretical and experimental studies in the area of dynamic response analysis of preloaded bolted joint under harmonic excitation. El-Zahry has carried some studies on optimum design of preloaded bolted joint under harmonic excitation. In which the dynamic response of preloaded bolted joint is investigated when subjected to harmonic excitation. The equation of motion of the joint are derived on the basis of a realistic linear mathematical model, by using simple spring-mass system analysis.

## 1.2 COMPRESSION IN BOLTED MEMBER AND TENSION IN BOLT

Pressure vessel bolted joint is considered for analysis. In bolted joint bolt is in tension and bolted cover and flange are in compression due to preload. Stiffness of bolt, cover and flange are calculated. In bolted joint if there are more than two members are included in bolt grip then these act like compression spring in series [1], and total spring rate of member is,

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_i}$$

If clamping member one of member is soft gasket, its stiffness relative to other member is usually very small so stiffness of other members can be neglected and stiffness of gasket will be considered.

And if there is no gasket is used, determination of stiffness of other two members becomes difficult to obtain, except by experimentation, because compression spread out between the bolt head and nut and area is not uniform. In most of the cases area can be determined. When area can't be determined, Professor Charles Mischke, of Iowa State University, suggested the use of a pressure cone for stiffness calculation using 45° cone angle as shown in Fig 1 [1].

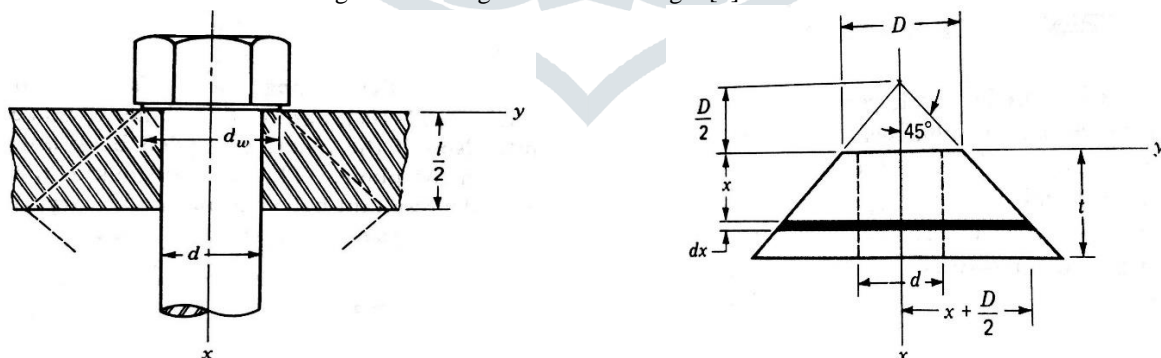


Figure 1(a) and 1(b) : Compression member assumed to be confined to the frustum of a hollow cone [1]

The top surface has a diameter  $d_w$  equal to the diameter of the washer face of the bolt. This is a simple case in which the member in the bolt grip have equal thicknesses. In general this will not be the case; in fact there may be more than two members in the grip, and they may have differing thicknesses and elastic moduli. Thus we choose the more general pressure cone shown in Fig 1(b) for analysis [1].

The elongation of an element of the cone subjected to a tensile force  $P$  is given as [1],

$$\delta = \frac{P}{\pi E d} \ln \left[ \frac{(2t + D - d)(D + d)}{(2t + D + d)(D - d)} \right] \dots \dots \dots (1)$$

The spring rate or stiffness of this frustum is [1],

$$k = \frac{P}{\delta} = \frac{\pi E d}{\ln \left[ \frac{(2t + D - d)(D + d)}{(2t + D + d)(D - d)} \right]} \dots\dots\dots (2)$$

Stiffness for cover plate and flange is determined by using Equation (2) for frustum of a cone of steel plate (SA 516 Gr 70)

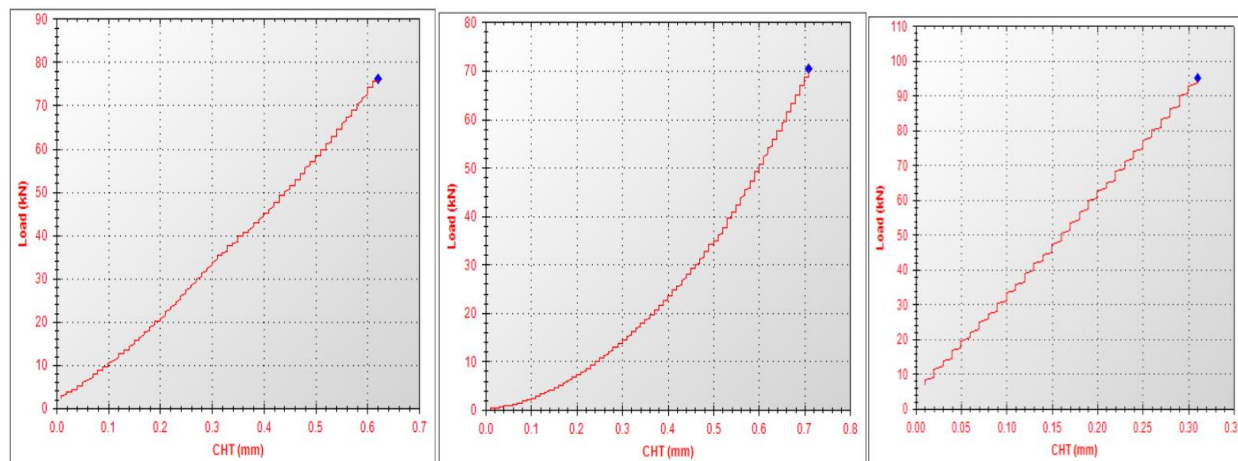
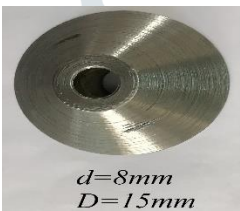
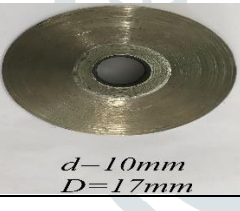
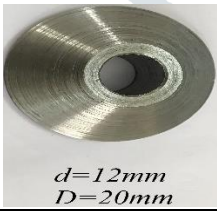


Figure 2: Compression Test of Specimen with d=12mm and D=20mm (For Average)

Table 1: Comparison between analytical and experimental results for stiffness of cover:

<i>k</i> =Stiffness	Theoretical ( <i>k</i> )	Experimental ( <i>k</i> )
 <p><i>d</i>=8mm <i>D</i>=15mm</p>	<i>k</i> =6054.55KN/mm	<i>k</i> =7.9KN/mm
 <p><i>d</i>=10mm <i>D</i>=17mm</p>	<i>k</i> =6845.97KN/mm	<i>k</i> =68.3KN/mm
 <p><i>d</i>=12mm <i>D</i>=20mm</p>	<i>k</i> =8407.987KN/mm	<i>k</i> =276.4KN/mm

The difference in theoretical and experimental values are considerable because theoretical values are obtained by using approximated formula (Eqn.2).

Stiffness of bolt is calculated with the help of tensile test on UTM as follows

$$k_b = \frac{AE_b}{L} \dots\dots\dots (3)$$

*E<sub>b</sub>*= Modulus of elasticity of bolt material= 190GPa. Bolt material [5]= ASTM SA 193 (B8M)

*A*= Area based on the nominal or major diameter. (Effect of threads is neglected)

*L*= Grip length = total thickness of the parts which have been fastened together. *k<sub>b</sub>* = Stiffness of bolt

Stiffness of M12 bolt is obtained as 716.283KN/mm.

Damping involved is calculated with the help of logarithmic decrement as

$$\text{Logarithmic decrement} = \delta = \ln \frac{X_1}{X_0} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \dots\dots\dots (4)$$

Where

$\delta$  = logarithmic decrement

$X_0$  = amplitude of damped vibration for previous cycle

$X_1$  = amplitude of damped vibration for next cycle

Damping ratio,  $\xi = 0.1617$

**1.3 DYNAMIC RESPONSE ANALYSIS OF THE BOLTED JOINT**

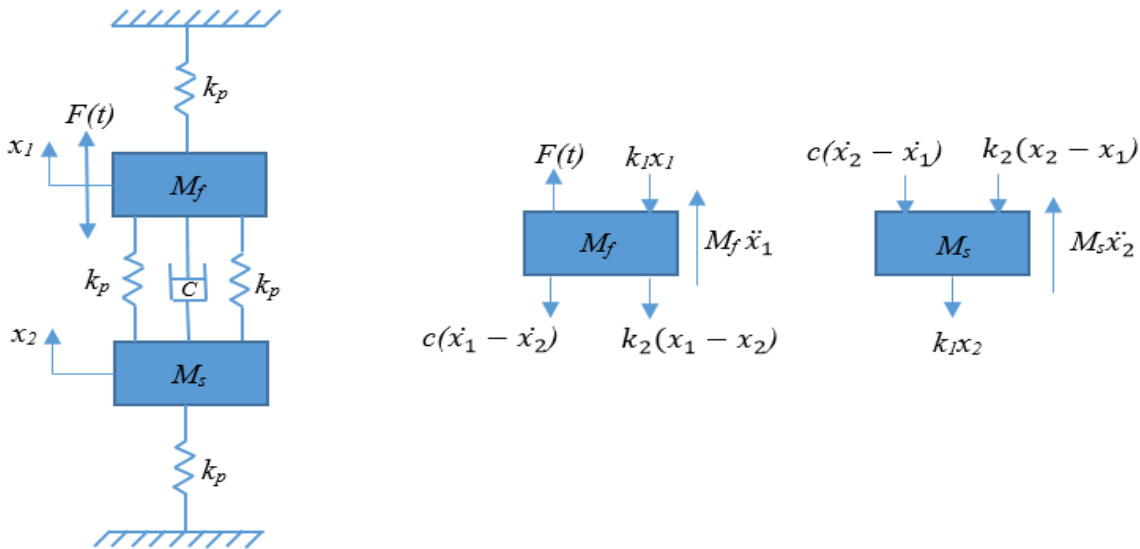


Figure 3: (a) 2dof vibration model for cover and flange (b) Free body diagram [2]

From Fig 3 following equations are obtained [2],

$$\frac{x_1}{x_0} = MT = \left[ \frac{(1+k-r_s^2)^2 + \eta^2 r_s^2}{[(1+k-r_f^2)(1+k-r_s^2)-k^2]^2 + \eta^2 r_s^2 [2-r_f^2-r_s^2]^2} \right]^{\frac{1}{2}} \dots\dots\dots (5)$$

$$\frac{F_{tr}}{F_o} = FT = \left[ \frac{(1-r_s^2)^2 (k^2 + \eta^2 r_s^2)}{[(1+k-r_f^2)(1+k-r_s^2)-k^2]^2 + \eta^2 r_s^2 [2-r_f^2-r_s^2]^2} \right]^{\frac{1}{2}} \dots\dots\dots (6)$$

Where, The equivalent static deflection of the joint plates  $x_o = \frac{F_o}{k}$

The stiffness ratio of the joint/bolt combination and stiffness of the plates  $k = \frac{k_2}{k_1} = \frac{k_b + k_p}{k_p}$

The natural frequency of the cover  $\omega_f^2 = \frac{k_1}{M_f}$ , the natural frequency of the flange  $\omega_s^2 = \frac{k_1}{M_s}$

The ratio of the excitation frequency to the natural frequency of the cover plate  $r_f = \frac{\omega}{\omega_f}$ ,

The ratio of the excitation frequency to the natural frequency of the flange  $r_s = \frac{\omega}{\omega_s}$ ,

The damping factor of bolted joint  $\eta = \frac{c}{M_s \omega_s}$ ,

The force transmissibility ratio between cover and flange  $FT = \frac{|F_{tr}|}{F_o}$  and

The motion transmissibility ratio between cover and flange  $MT = \frac{|x_1|}{x_o}$

Substituting  $k_b=716.283\text{KN/mm}$  and  $k_p=267.4\text{KN/mm}$  for  $k = \frac{k_2}{k_1} = \frac{k_b + k_p}{k_p} = 3.69$   $d=12\text{mm}$ , in Eqns. (5), and (6) while

considering the values of  $\eta = 0.1, \eta = 0.2, r_f = 0.1$  and  $r_f = 0.2$  response curve is plotted for MT vs.  $r_s$  and FT vs.  $r_s$ .

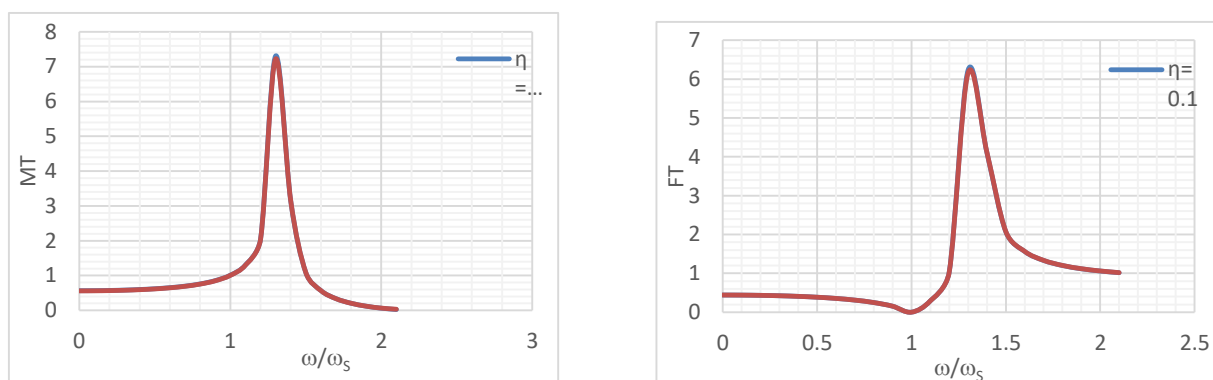


Figure 4: Motion Transmissibility MT and Force Transmissibility FT vs. Frequency ratio for and  $\omega/\omega_s$

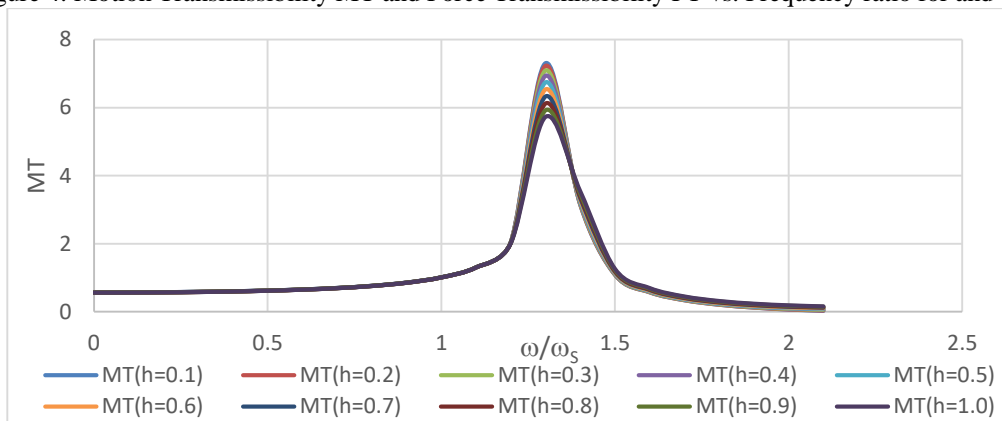


Figure 5: Motion Transmissibility MT vs. Frequency ratio  $\omega/\omega_s$  for various values of  $\eta$

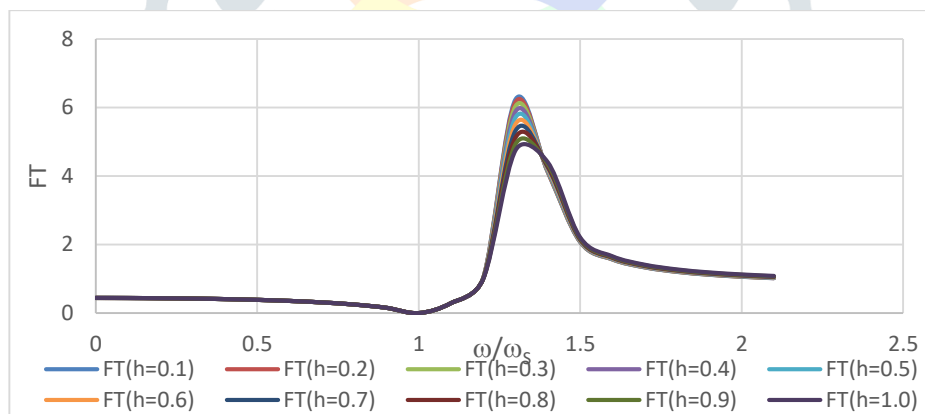


Figure 6: Force Transmissibility FT vs. Frequency ratio  $\omega/\omega_s$  for various values of  $\eta$

#### 1.4 EFFECT OF GASKET ON BOLTED JOINT

A gasket is a sealing component placed between two clamping component to create a static seal between the two flanges of a mechanical assembly. In bolted joint if there are more than two members are included in bolt grip then these act like compression spring in series, and total spring rate of member is [1],

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_i}$$

If clamping member one of member is soft gasket, its stiffness relative to other member is usually very small so stiffness of other members can be neglected and stiffness of gasket will be considered [1].

Considering the M12 bolts, stiffness of gasket  $k_g$  is calculated as follows,

$$\text{Effective area of gasket around bolt} = A_g = \frac{\pi}{4} (24^2 - 12^2) = 339.290\text{mm}^2$$

For Asbestos gasket,

$$k_g = \frac{A_g E_g}{l_g} = 54.287 \text{ KN / mm}$$

For PTFE gasket,

$$k_g = \frac{A_g E_g}{l_g} = 19.0 \text{ KN / mm}$$

For plain rubber gasket,

$$k_g = \frac{A_g E_g}{l_g} = 7.917 \text{ KN / mm}$$

For graphite gasket,

$$k_g = \frac{A_g E_g}{l_g} = 463.6963 \text{ KN / mm}$$

Gasket stiffness relative to the other member is so small therefore stiffness of other member can be neglected and only the gasket stiffness is used.

$$k = \frac{k_2}{k_1} = \frac{k_b + k_p}{k_p} \text{ in case gasket is considered } k_p = k_g, k = \frac{k_2}{k_1} = \frac{k_b + k_g}{k_g}$$

$$k_{Asbestose} = 14.25, k_{PTFE} = 38.86, k_{PlainRubber} = 91.85 \text{ and } k_{Graphite} = 2.55$$

Substituting  $k$  for each gasket response curve is plotted for MT vs.  $r_s$  and FT vs.  $r_s$

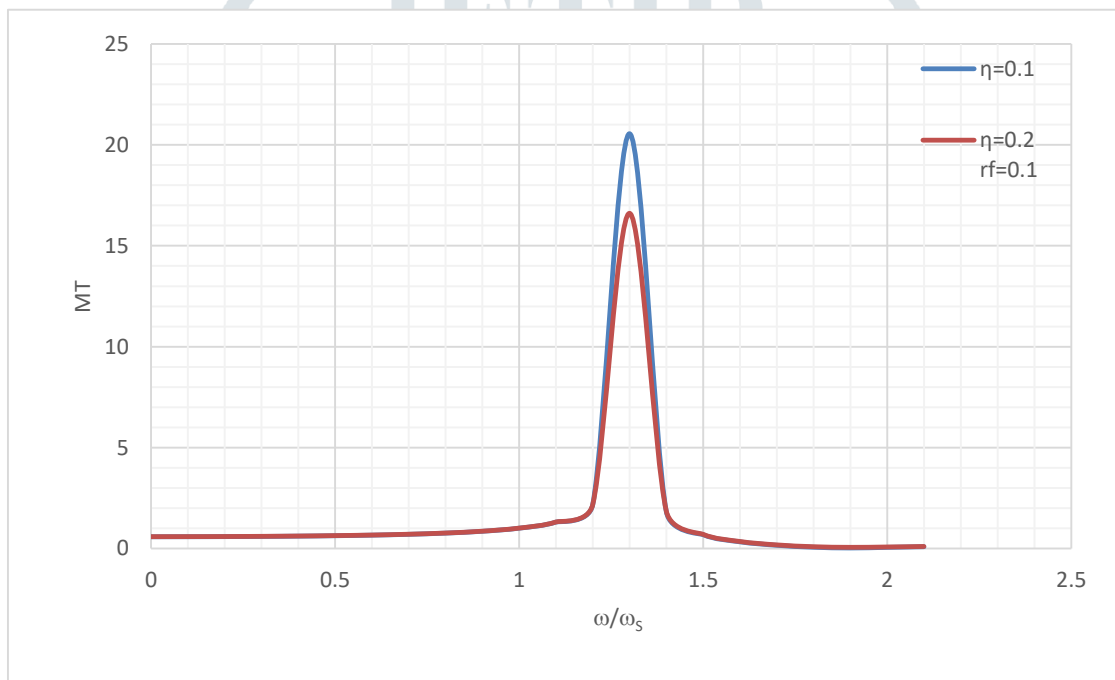


Figure 7: Motion Transmissibility MT vs. Frequency ratio  $\omega/\omega_s$  of Graphite material for  $\eta = 0.1$  and  $\eta = 0.2$  with  $r_f = 0.1$

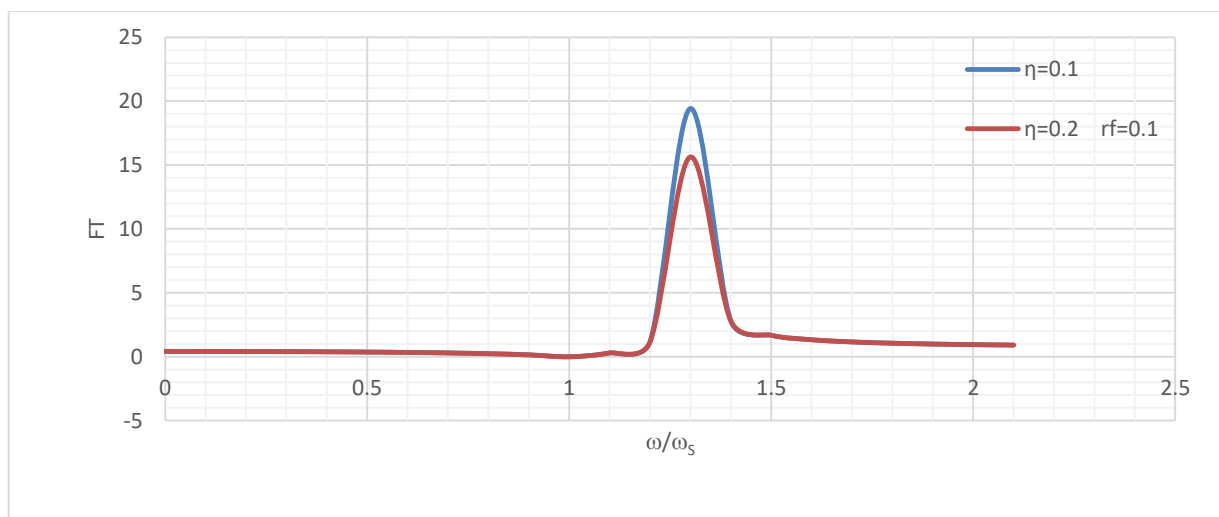


Figure 8: Force Transmissibility FT vs. Frequency ratio  $\omega/\omega_s$  of Graphite material for  $\eta = 0.1$  and  $\eta = 0.2$  with  $r_f = 0.1$

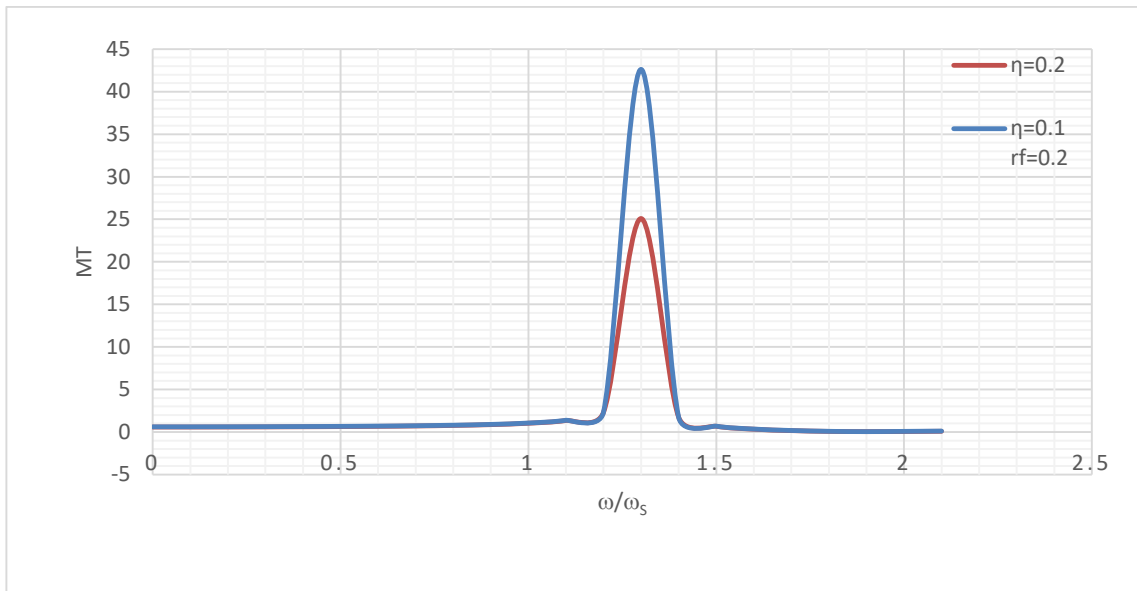


Figure 9: Motion Transmissibility MT vs. Frequency ratio  $\omega/\omega_s$  of Graphite material for  $\eta=0.1$  and  $\eta=0.2$  with  $r_f=0.2$

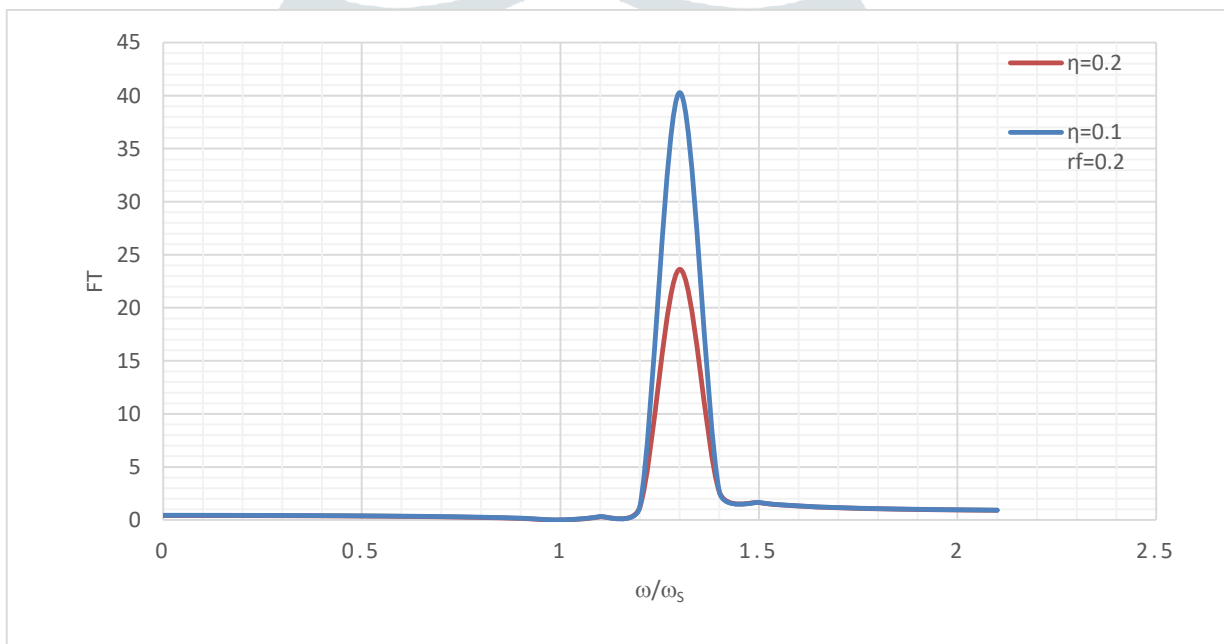


Figure 10: Force Transmissibility FT vs. Frequency ratio  $\omega/\omega_s$  of Graphite material for  $\eta=0.1$  and  $\eta=0.2$  with  $r_f=0.2$

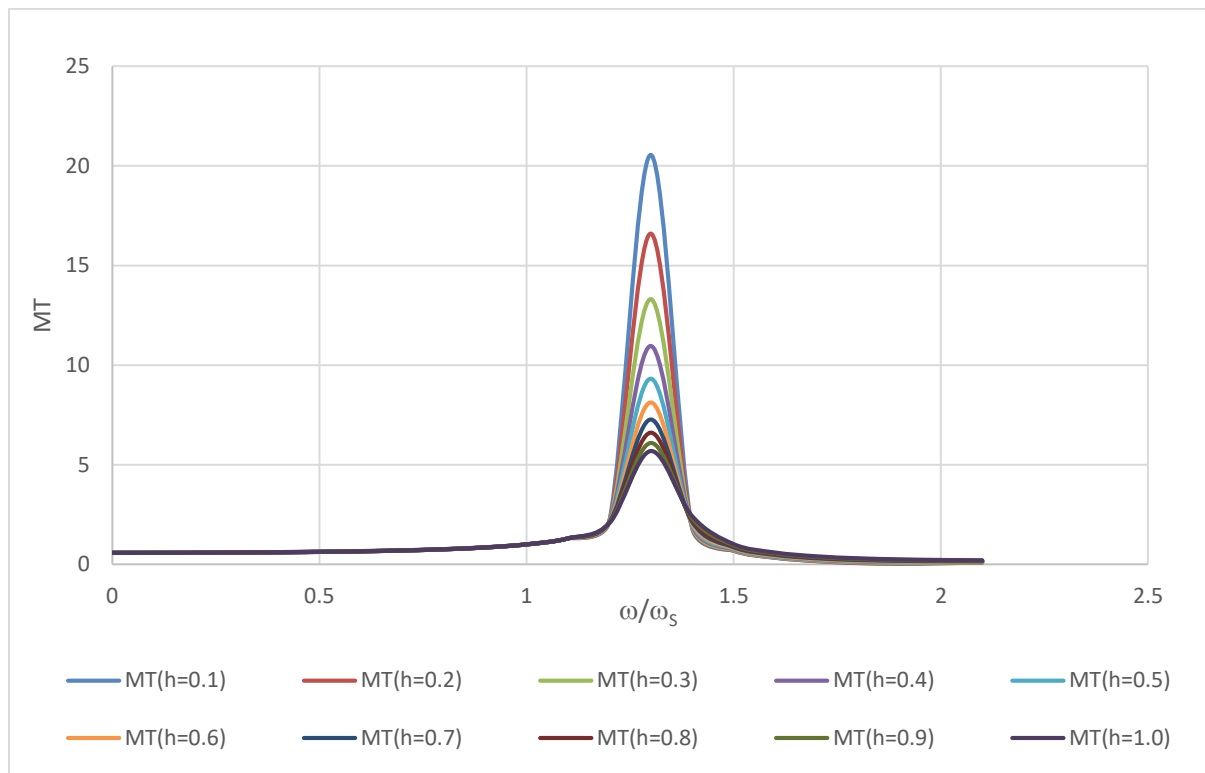


Figure 11: Motion Transmissibility MT vs. Frequency ratio  $\omega/\omega_s$  of Graphite material for various values of  $\eta$

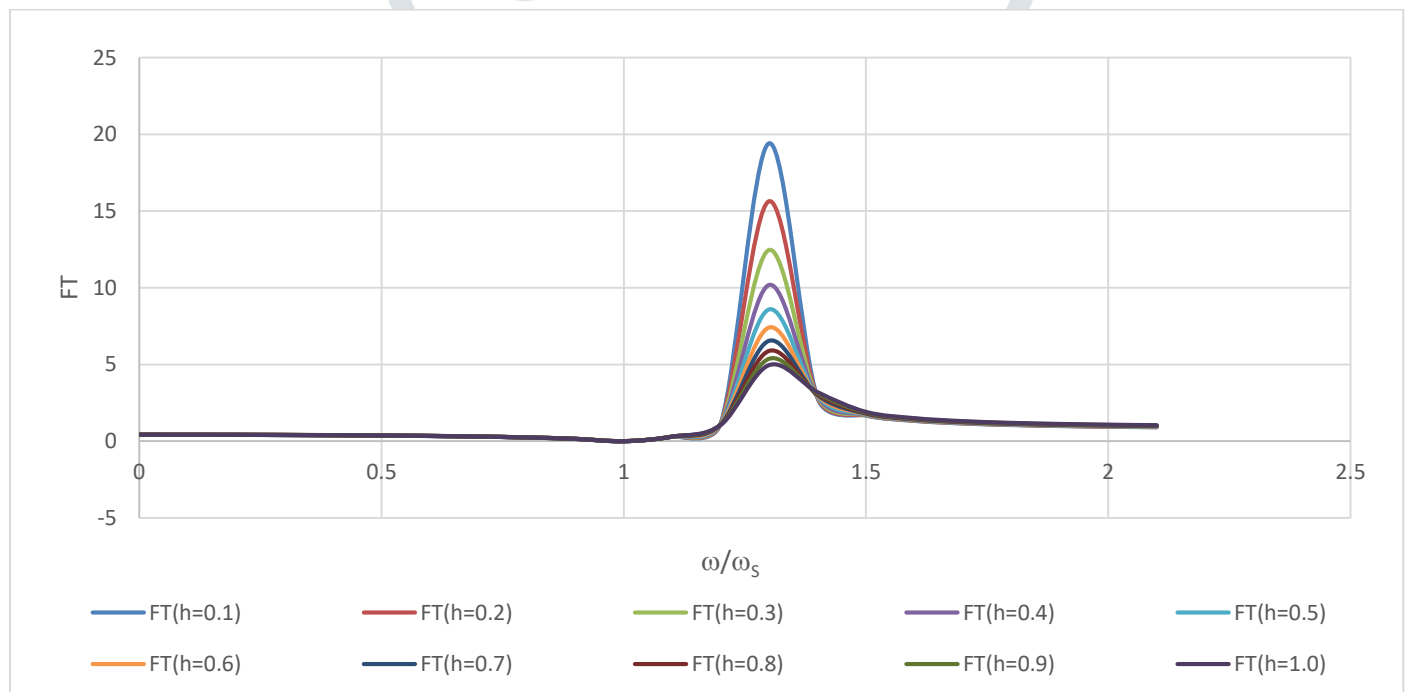


Figure 12: Force Transmissibility FT vs. Frequency ratio  $\omega/\omega_s$  of Graphite material for various values of  $\eta$

**1.5 DISCUSSION ON RESULT**

From Fig 4 and Fig 5, it is observed that for given value frequency ratio  $r_f = \omega/\omega_f = 0.1$ , when the damping factor  $\eta$  increases, the resonant amplitudes of motion transmissibility MT and force transmissibility FT comparatively does n changes. Also when  $r_f$  increases, the values of resonant amplitudes of motion and force transmissibilities decreased respectively by 27.99% and 10.44%. However, in this case also the values of amplitudes of MT and FT does not changes comparatively with the increased in value of damping factor  $\eta$ .

Table 2: Comparison of results of Motion and Force Transmissibility at resonance.

At Resonance $r_f=0.1$				At Resonance $r_f=0.2$			
MT <sub>(R)</sub>		FT <sub>(R)</sub>		MT <sub>(R)</sub>		FT <sub>(R)</sub>	
$\eta=0.1$	$\eta=0.2$	$\eta=0.1$	$\eta=0.2$	$\eta=0.1$	$\eta=0.2$	$\eta=0.1$	$\eta=0.2$
7.304	6.233	7.223	6.157	9.349	9.203	7.977	7.845

In case of bolted joint with gasket,

Table 3: MT and FT for different materials of gasket with various values of  $\eta$ 

Material of Gasket	Plain Rubber		PTFE		Asbestose		Graphite	
	MT	FT	MT	FT	MT	FT	MT	FT
$\eta=0.1$	50.352	48.848	175.919	173.160	25.417	26.162	20.555	19.422
$\eta=0.2$	50.351	48.848	175.823	173.065	25.418	26.163	16.606	15.638
$\eta=0.3$	50.351	48.848	175.662	172.907	25.421	26.165	13.307	12.463
$\eta=0.4$	50.350	48.847	175.438	172.686	25.425	26.168	10.961	10.192
$\eta=0.5$	50.349	48.846	175.151	172.403	25.431	26.172	9.316	8.585
$\eta=0.6$	50.349	48.845	174.803	172.059	25.437	26.177	8.135	7.422
$\eta=0.7$	50.348	48.844	174.394	171.656	25.445	26.182	7.265	6.555
$\eta=0.8$	50.347	48.843	173.925	171.193	25.454	26.188	6.606	5.891
$\eta=0.9$	50.345	48.842	173.399	170.676	25.463	26.196	6.097	5.371
$\eta=1.0$	50.344	48.840	172.817	170.101	25.475	26.203	5.694	4.956

When plain rubber, PTFE and asbestose materials are consider as gasket, it is observed that for the given value of frequency ratio  $r_f$  when damping factor  $\eta$  increases, the peak motion and force transmissibilities comparatively does not changes.

Also, in case of plain rubber gasket, when  $r_f$  increases the value of peak motion and force transmissibilities are increased by 95.88%,

In case of PTFE gasket, when  $r_f$  increases, the value of peak motion and force transmissibilities are decreased by 76.62%

In case of asbestose gasket, when  $r_f$  increases the value of peak motion and force transmissibilities are decreased by 42.25%,

And in case of graphite gasket, when  $r_f$  increases the value of peak motion and force transmissibilities are increased by 50.98%.

From result table given in Table 3, it is observed that, as value of damping  $\eta$  increases from 0.1 to 1 for plain rubber, asbestose, and PTFE gasket material values of peak motion and force transmissibilities comparatively does not changes while for graphite gasket peak values of motion and force transmissibilities significantly decreases as varies from 0.1 to 1.

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