Singular value decomposition and its applications

[1]Yengkhom Satyendra Singh,[2]Benaki Lairenjam

[1]Assistant Professor,[2] Assistant Professor,
[1] Department of Mathematics,
[1]REVA University, Bangalore, India.

Abstract : **Singular value decomposition (SVD) is an algebraic tool deals with singular matrices or that are much closed to singular ones. We know that SVD offers significant properties in science and engineering particularly in data science and signal processing. Here in this paper, we will discuss the importance of SVD and its applications. Also, we will present some of its properties that have not been utilized properly, say its properties in various image applications are at its early stage. This paper presents in using various properties of SVD and some new contributions that were originated from SVD properties analysis. Our aim is to highlight the importance of SVD and its applications in data science and point out its various features and important applications and forecast its future research directions.**

IndexTerms **- Singular value decomposition; fault diagnosis; signal processing and image processing.**

I. Introduction

Mathematically speaking, SVD is linear algebra technique to split a matrix into orthogonal components with which optimal sub rank approximations. Suppose we have a real valued matrix A of order $m \times n$ with rank r, where $r \leq n \leq m$. SVD is a matrix factorization to make certain subsequent matrix computations easier. SVD is define as

$$A = U\Sigma V^T$$

where A is the real $m \times n$ matrix that we wish to factorize, U is an $m \times m$ matrix, Σ is an $m \times n$ diagonal matrix, and $V^T$is an $n \times n$ matrix. The most beautiful features about SVD is that the left -singular vectors of the original matrix A is the columns of the U matrix, the right-singular vectors are the columns of V and the diagonal elements of Σ matrix is the singular values of the main matrix A.

Suppose $U^TAV = \Sigma$ be the SVD of A and if $\sigma_k \neq 0$ and $\sigma_{k+1} = 0$,were U, V are orthogonal matrices then $X = A^TA = V\Sigma U^TU\Sigma V^T = V\Sigma^2V^T$ and $Y = AA^T = U\Sigma V^TV\Sigma U^T = U\Sigma^2U^T$. The diagonal elements of the Σ matrix is the eigen values of the matrices X, Y. Here the nullity of A is the span of $v_{k+1}, v_{k+2}, \dots, v_n$ and if $A^\dagger$ is equal to $[v_1, v_2, \dots, v_k]diag\left[\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \dots, \frac{1}{\sigma_k}\right][u_1, u_2, \dots, u_k]^T$, then $x = A^\dagger b$ is a least square solution of $Ax = b$. Also, the matrix A could be reconstructed by the formula $A = [\sigma_1 u_1 {v_1}^T] + [\sigma_2 u_2 {v_2}^T] + \dots + [\sigma_k u_k {v_k}^T]$, using U, Σ and V^T i.e.,

$$A = U\Sigma V^T = [\sigma_1 u_1, \sigma_2 u_2, \dots, \sigma_k u_k, 0, \dots, 0][v_1, v_2, \dots, v_n]^T$$

$$= \sum_{j=1}^{k} \sigma_j u_j v_j^T = [u_1, u_2, \dots, u_k]\, diag[\sigma_1, \sigma_2, \dots, \sigma_k][v_1, v_2, \dots, v_n]^T$$

In data science perspective we consider matrix A as data matrix, say

$$A = \begin{bmatrix} | & | & \dots & | \\ x_1 & x_2 & \dots & x_n \\ | & | & \dots & | \end{bmatrix} = U\Sigma V^T$$

$$= \begin{bmatrix} | & | & \dots & | \\ u_1 & u_2 & \dots & u_m \\ | & | & \dots & | \end{bmatrix} \begin{bmatrix} \sqrt{\sigma_1} & 0 & \dots & 0 \\ 0 & \sqrt{\sigma_2} & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & \sqrt{\sigma_n} \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} | & | & \dots & | \\ v_1 & v_2 & \dots & v_n \\ | & | & \dots & | \end{bmatrix}$$

Here U consists the data information of the columns space of the data matrix A and V consists the data information of the row space of the data matrix A.

And so, data matrix $A = \sum_{i=1}^{n} u_i \sigma_i v_i^T$ where u_i are the linearly independent columns and v_i^T are linearly independent rows of the data matrix A i.e., eigen vectors of a correlation matrices of AA^T and A^TA . We can truncate at rank r if we are having lots of negligible small singular values. SVD is a very useful tool (unsupervised learning tool) to determine clusters, latent variables for dimensionality reduction by processing source data. So SVD is used to convert experimental data into a well-organized non redundant, noise free to extract hidden information..

II. Applications

Diagnose the running state of rolling bearings is very essential in order to avoid accidents or malfunction in rotating machine by frequent observation. Fault diagnosis of rolling bearings requires a deep feature extraction. In [1] authors, use a novel technique call empirical mode decomposition (EMP) to decompose the vibrating signals under different states of rolling bearings. Secondly, the use SVD to extract more representative high-level features from the intrinsic mode functions in order to determine the singular value parameters. Then the singular value parameters are used as the input parameters of the stated kernel sparse auto encoder network. The method opted by them does not require prior knowledge of fault diagnosis and even does not need the signal denoising processing that makes easier the traditional method of feature extraction of rolling bearing fault diagnosis. In [2], the authors combined SVD and Kalman filter to identify broken rotor bars for induction motor. They used SVD to detect the frequency allocation of the fault by decomposing the covariance matrix of the stator current. Secondly, they estimate the phases and amplitudes of the selective harmonics. We all know that Induction motor is the most economical low-cost device commonly used in industries that can withstand explosions, fire or the aftermath of human errors [3]. But different fault might occur on account of internal resistance due to external forces [4]. Broken rotor bar (BRB) is the most common faults in the Induction motor due to load fluctuation; increase in temperature or due to motor vibration. The most used technique in the industries to detect BRB faults is Periodogram method, but it is non reliable when the speed and load fluctuates [5,6]. We have seen that SVD and Kalman Filter independently have shown good results in solving many problems related to signal processing and image processing. In order to overcome and identify the issue related to BRB with better result the authors adopted hybrid approach using SVD and Kalman Filter on a limited frequency band and the have shown better result as compared with traditional method called Periodogram. In [7] the authors have shown a methodology using SVD for high resolution range profile target recognition. High resolution radar is widely implemented in the fields of military, remote sensing, air traffic control, measuring ocean surface waves, detection of speeding traffic, satellite, etc. They have shown SVD helps improving the computational complexity by removing the redundant and noisy information. They also proved that determining the angle between signal subspaces and singular vectors is not enough for obtaining good recognition, but weighted element helps to increase in identification rates. In [8] the authors used Laplacian pyramid and SVD technique to enhance the visual quality of subtle organs as well as segmenting capabilities from MR images. Comparing with other enhancement techniques that gives the Gibbs phenomenon at edges their method shows better result. They used Laplacian pyramid to capture edge efficiently and it has been enhanced by SVD technique. They have shown that I the segmentation of internal auditory canal (IAC) and its nerves, SVD could be an effective tool for sharpening and enhancing the visual quality of the subtle organ. In [9] the authors introduced a method based on SVD to compute H-basis which is quite numerically stable. They used SVD to enhance computing H-basis tremendously by computing a minimal generating set for the Syzygy of a given degree of the underlying ideal. They also revealed that minimal generating set for Syzygies could be computed for each degree, say for first Syzygy, second and so forth for the given ideal and generate a finite chain of Syzygies. And those free resolutions i.e., the finite chain Syzygies, helps to generate important invariants such as Krull-dimension, Hilbert regularity and Betti numbers, etc. [10]. In [11] the authors presented a new compression algorithm for matrix to enhance the efficiency in computing the nested complex source beam (NCSB) method. They used truncated singular value decomposition to the disaggregation, multilevel aggregation and translation operations in NCSB. Using truncated far-field matching based on the directional far-field radiation property of the complex source beams (CSBs), they determined the aggregation /disaggregation matrices. Translation matrices are also determined from the beam width of CSBs. They have shown that the square of the singular values is equivalent to the radiation power of the new sources. This helps improve the efficiency of the original NCSB method by neglecting the insignificant singular values. An algorithm based on fuzzy neural network (FNN) using SVD was proposed by the authors in [12] for diagnosing and fault features extraction of diesel engine crankshaft bearings effectively. By decomposing the vibrational signals of crankshaft bearings in known state under the same working condition by using EMD the authors extract fault feature information. Finally, the fault features have been trained using Fuzzy neural network. And they observed that numerical identification of the crankshaft bearing in different states of the fuzzy neural network using SVD has very efficient output in terms of accuracy and stability. In [13] using SVD to the raw image, a novel static visual feature, namely, right and left singular feature vectors and singular value feature vector, were proposed by the authors. They have shown that the features extracted from this new method and by using SVD to the low-level features of images (like in PCA or LAS techniques) have a great difference. The proposed SVD features integrated edge, color and texture information simultaneously and was sorted in accordance with their functionality and importance for the concept detection. The authors extracted features in multi-granularity manner and classification was done for each partition of each granularity separately whereas classification is carried out for the whole image not for each partition separately in existing systems. The multigranularity partitioning and classification had no effect on each other on the results of classifications on partitions with and without the target concept, which improves the concept detection. Due to high dimensionality after reducing the dimension, the feature vectors undergo classification using KNN algorithm with a novel distance function which is very stable in high dimensional space and has good result in low dimensional space. The authors have shown that their proposed method had advantages over the spatial pyramid matching method and classification on the whole image and the superiority of the proposed SVD features over the widely used local and global features for the concept detection even though it consumes more training and testing time. In the proposed learning algorithms [14], they used prototype selection algorithm to compose the hidden neurons for the RBFN or the ELM using a pseudo-inverse matrix (by SVD). As compared with other known machine learning algorithms like SVM, RBFN, ELM or KNN their novel algorithm shows significantly better result. They have shown that this algorithm does not require tuning manually, the algorithm automatically extracts the kernels and their number and so it does not require inner cross validation. They have shown that that estimating the number of kernels with time consuming training and testing procedures are no longer needed and hence best results of learning could be obtained using pseudo inverse learning with SVD algorithm. In [15] the author used a novel technique called incremental singular value decomposition (SVD) to complete the missing data, selecting one having minimal rank. From various datasets like nonstationary datasets, noisy, extremely large, partially incomplete, etc., the author investigated the problems of determining good low rank subspace models. By manipulating numerical noise, uncertainty transformation for map inference and rank minimizing accusations the author developed an algorithm with better accuracy, reliable and time and space complexity as compared with other known methods and he levels it as parsimonious online SVD methods. He has examined this algorithm in semi-dense optical flow on nonrigid surfaces with occlusions, automatic handling of occlusions in structure-from-motion and eigen spaces and shown that recovery rate is extremely fast as compared with known methods. In [16] the authors introduced a secure, robust, and blind adaptive audio watermarking algorithm using Singular Value Decomposition in the digital watermarking technique using synchronization code. They have shown that their result has better payload and performance when compared with MP3 compression to earlier audio watermarking methods. Their algorithm is easy in user friendly, easy in computation and hence good for practical use. In [17] the authors introduced a novel approach for vector-sensor signal modelling and processing using quaternion algebra that provides a simple notation for vector signal manipulation and processing. Their new approach helps to determine the best rank-k approximation of the quaternion matrix and allow wave separation over vector-sensor array in subspace method. They have shown that generalized SVD that is SVDQ when comparing with the traditional SVD method, the new method has more efficiency in polarized wave separation problem.

III. CONCLUSION

One of the most challenging tools in machine learning is to develop effective tools for analyzing the datasets. SVD is a very useful and very efficient tools among the analysis techniques. SVD is very efficient method and could be able to determine best sub-rank approximations by decomposing given matrix into orthogonal dominant and subdominant subspaces effectively [18]. This property helps in watermarking and noise reduction. So, using the largest singular value one can easily determine the low rank approximation. We can represent a sum of rank one matrices from a given matrix by using singular value decomposition. We can truncate the matrix A to a specific rank k matrix by using SVD and store the approximated A_k of the matrix in place of the entire matrix A. This technique is often used in watermarking, noise reduction, image compression, etc. SVD can be used to extract mechanical meaningful signals from rotating machine from the data. For example, in [2], SVD is used to detect the frequency allocation of the fault by decomposing the covariance matrix of the stator current. SVD is also used for diagnosing and fault features extraction of diesel engine crankshaft bearings effectively. High resolution radar is widely implemented in the fields of military, remote sensing, air traffic control, measuring ocean surface waves, detection of speeding traffic, satellite. SVD helps improving the computational complexity by removing the redundant and noisy information for high resolution range profile target recognition. SVD technique is also used to enhance the visual quality of subtle organs as well as segmenting capabilities from MR images. Singular value decomposition (SVD) could be used to complete the missing data by selecting one having minimal rank. SVD is used to enhance computing H-basis tremendously by computing a minimal generating set for the Syzygy of a given degree of the underlying ideal and therefore it will enhance the computation of Groebner basis [18]. Even though lots of work have been done on SVD, SVD in image processing, fault detection, medical imaging is still in early stage. As there are many unexplored properties of SVD, our aim is to highlight those characteristics of SVD and gives a contribution for future research challenges. Briefly speaking SVD is a very efficient technique which is fast and easy to implement. It provides a practical solution to fault detection, image compression and recognition problems.

.

REFERENCES

[1] Fengtao Wang , Gang Deng, Chenxi Liu, Wensheng Su, Qingkai Han and Hongkun Li, A deep feature extraction method for bearing fault diagnosis based on empirical mode decomposition and kernel function, Advances in Mechanical Engineering 2018, Vol. 10(9) 1–12, 2018 DOI: 10.1177/1687814018798251 journals.sagepub.com/home/ade.

[2] Azouzi K., Boudinar A. H., Aimer F. A., Bendiabdellah A., Use of a combined SVD-Kalman filter approach for induction motor broken rotor bars identification, Journal of Microwaves, Optoelectronics and Electromagnetic Applications, Vol. 17, No. 1, March 2018 DOI: http://dx.doi.org/10.1590/2179-10742018v17i11136.

[3] Karvelis P., Georgoulas G., Tsoumas I.P. , Antonino-Daviu J.A., Climente-Alarcón V., and Stylios C.D., "ASymbolic Representation Approach for the Diagnosis of Broken Rotor Bars in Induction Motors", IEEE on Industrial Informatics, Vol 11 No.5, pp. 1028 – 1037, 2015.

[4] Bendiabdellah A., Boudinar A.H. , Benouzza N. , and Khodja M. , "The enhancements of broken bar fault detection in induction motors ", In IEEE of the International Conference on Electrical Machines and Power Electronics , International Conference on Optimization of Electrical & Electronic Equipment and International Symposium on Advanced Electromechanical Motion Systems , Side., Turkey, pp. 81–86, 2015.

[5] Kia S.H., Henao H., and Capolino G.A, "Zoom-MUSIC frequency estimation method for three phase induction machine fault detection", in IEEE Annual Conference of Industrial Electronics Society, pp.2603-2608, 2005.

[6] Hernandez A. C., Daviu J. A., Llinares J. P., Alarcon V. C. and Galiana V. F., "Transient-Based Rotor Cage Assessment in Induction Motors Operating With Soft Starters," IEEE Transactions on Industry Applications, vol. 51, No. 5, pp. 3734-3742, Sept.-Oct. 2015.

[7] Patricia López-Rodríguez, David Escot-Bocanegra, Raúl Fernández-Recio and Ignacio Bravo, "Non-Cooperative Target Recognition by Means of Singular Value Decomposition Applied to Radar High Resolution Range Profiles" Sensors 2015, 15, 422-439; doi:10.3390/s150100422.

[8] Jeevankala S., Brintha Therese A., Sharpening enhancement technique for MR images to enhance the segmentation, Biomedical Signal Processing and Control 41(2018)21-30.

[9] Masoumeh Javanbakht · Tomas Sauer, Numerical computation of H–bases, https://arxiv.org/abs/1803.06203v1.

[10] Peeva I.: Graded Syzygies. Springer (2011).

[11] Lianning Song and Zaiping Nie, A Truncated Singular Value Decomposition Enhanced Nested Complex Source Beam Method, Hindawi International Journal of Antennas and Propagation Volume 2017, Article ID 9072421, 7 pages https://doi.org/10.1155/2017/9072421.

[12] Jingbo Gai and Yifan Hu, Research on Fault Diagnosis Based on Singular Value Decomposition and Fuzzy Neural Network, Hindawi Shock and Vibration Volume 2018, Article ID 8218657, 7 pages https://doi.org/10.1155/2018/8218657.

[13] Jafar Mansouri, Kamran Farajzadeh, Esmail Zarezadeh, Concept Detection in Images Using SVD Features and MultiGranularity Partitioning and Classification, Journal of Information Systems and Telecommunication, Vol. 5, No. 3, pp. 72-82, July-September 2017.

[14] NORBERT JANKOWSKI, comparison of prototype selection algorithms used in construction of neural networks learned by svd, Int. J. Appl. Math. Comput. Sci., 2018, Vol. 28, No. 4, 719–733 DOI: 10.2478/amcs-2018-0055.

[15] Matthew Brand, Incremental Singular Value Decomposition of Uncertain Data with Missing Values, A. Heyden et al. (Eds.): ECCV 2002, LNCS 2350, pp. 707–720, 2002. c Springer-Verlag Berlin Heidelberg 2002.

[16] Vivekananda Bhat K, Indranil Sengupta, Abhijit Das, An adaptive audio watermarking based on the singular value decomposition in the wavelet domain, Digital Signal Processing 20 (2010) 1547–1558, doi:10.1016/j.dsp.2010.02.006.

[17] Nicolas Le Bihan, Jér ôme Mars, Singular value decomposition of quaternion matrices: a new tool for vector-sensor signal processing, Signal Processing 84 (2004) 1177 – 1199.

[18] Yengkhom Satyendra Singh, Benaki Lairenjam, *Groebner Basis and its Applications*, International Journal of Engineering and Advanced Technology (IJEAT) ISSN: 2249 – 8958, Volume-9 Issue-1, October 2019.