L(3,1) - LABELING FOR SOME EXTENDED DUPLICATE GRAPHS

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Abstract: In this paper, we investigate the extended duplicate graph of triangular snake graph and quadrilateral snake graph admits L(3,1) - labeling.

AMS Subject Classification: 05C78.

Keywords: Duplicate graph, Extended duplicate graph, Triangular snake graph, Quadrilateral snake graph, L(3,1) - labeling.

1. Introduction

Graph labeling is one of the most important area in graph theory. The concept of graph labeling was introduced by Rosa in 1967 [4]. For a dynamic survey of various graph labeling we refer to J.A.Gallian[3]. Griggs and Yen[5] defined L(2,1) labeling of the graph G=(V,E) where f is a function which assigns labels to every u,v∈V from the set of positive integer such that $|f(u) – f(v)| \geq 2$ if $d(u,v) = 1$ and $|f(u) – f(v)| \geq 1$ if $d(u,v) = 2$.

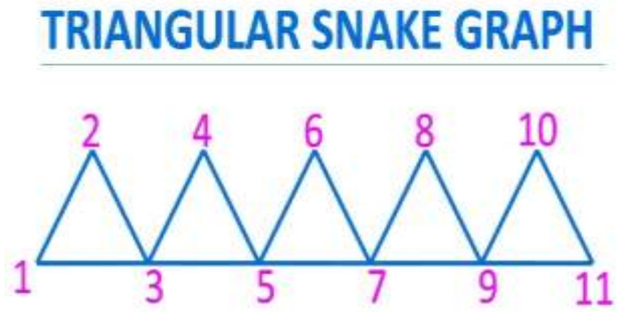
E.Sampthkumar [1] introduced the concept of duplicate graph. Thirusangu et al.[2] have introduced the concept of extended duplicate graph. G.J.Chang and D.Kuo et al., on L(d,1)-Labeling of graphs. In L(3,1)-labeling of a graph G=(V,E) where f is a function which assigns label to every u,v ∈ V from the set of positive integer such that $|f(u)-f(v)| \geq 3$ if $d(u,v) = 1$ and $|f(u) – f(v)| \geq 1$ if $d(u,v) = 2$. The L(3,1) labeling number, $\lambda(G)$ of G is the smallest number λ such that G has an L(3,1) labeling with λ as the maximum label.

2. Preliminaries

In this section, we give the basic definitions which are relevant to this paper. Let G(V,E) be a finite, simple and undirected graph with p vertices and q edges.

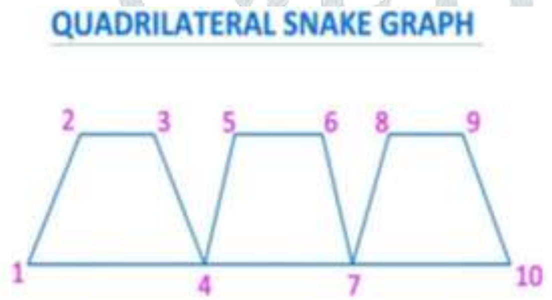
Definition :2.1 Triangular snake

A triangular snake TS_m is obtained from a path $u_1, u_2, \ldots, u_{m+1}$ by connecting u_i and u_{i+1} to a new vertex v_i, for $1 \leq i \leq m$, where 'm' is the number of edges of the path.



Definition: 2.2 Quadrilateral snake graph

A quadrilateral snake QS_m is obtained from a path $u_1,u_2,u_3,\ldots\ldots,u_n$ by joining u_i and u_{i+1} to two new vertex v_i and w_i respectively and then joining v_i and w_i , $1 \leq i \leq n-1$, where 'm' is the number of edges of the path. In general, a quadrilateral snake has 3m+1 vertices and 4m edges.



Definition : 2.3 Duplicate graph

A Simple graph G with vertex set V and edge set E. The duplicate graph of G is DG =(V_1, E_1) where the vertex set V_1= VUV′ and $V \cap V' = \phi$ and $h : V \to V'$ is bijective .The edge set E_1 of DG is defined as the edge $ab \in E$ iff both edges ab' and $a'b$ are in E_1.

Definition : 2.4 Extended duplicate graph of triangular snake

Let DG = (V_1, E_1) be a duplicate graph of the triangular snake graph G(V, E). Extended duplicate graph of triangular snake is obtained by adding the edge $v_2 v_2'$ to the duplicate graph and it is denoted by EDG (TS_m). Clearly it has 4m+2 vertices and 6m+1 edges, where 'm' is the number of edges.

Definition :2.5 Extended duplicate graph of quadrilateral snake

Let DG = (V_1,E_1) be a duplicate graph of the quadrilateral snake graph G(V,E). Extended duplicate graph of quadrilateral snake graph is obtained by adding the edge $v_2 v_2'$ to the duplicate graph and it is denoted by EDG (QS_m). Clearly it has 6m+2 vertices and 8m+1 edges, where 'm' is the number of edges.

Definition :2.6 L(2,1) – Labeling

An L(2,1) labeling or distance two labeling of a graph G is a function f from the vertex set V(G) to the set of all non-negative integers such that $|f(x)-f(y)| \geq 2$ if $d(x, y)=1$ and $|f(x)-f(y)| \geq 1$ if $d(x, y)=2$. The L(2,1) labeling number $\lambda(G)$ of G is the smallest number k such that G has an L(2,1) labeling with $\max\{f(v), v \in V(G)\} = k$.

Definition :2.7 L(3,1) – Labeling

Let G be a graph with set of vertices V and set of edges E. Let f be a function $f: V \rightarrow N$, where f is an L(3,1)- labeling of G if, for all $u, v \in V$, $|f(u)-f(v)| \geq 3$ if $d(u,v) = 1$ and $|f(u) – f(v)| \geq 1$ if $d(u,v) = 2$.

Definition : 2.8

The difference between maximum and minimum values of f for all possible f is called span of the labeling, and it is denoted by $\lambda_{3,1}(G)$ or simple $\lambda(G)$ or λ, positive integer λ to be used to label a graph G by L(3,1)-labeling.

3. MAIN RESULTS

3.1: L(3,1)-LABELING FOR TRIANGULAR SNAKE GRAPH EDG(TSm) , m≥ 1

Here, we present an algorithm and prove the existence of L(3,1)-labeling for EDG(TS_m).

Algorithm-1

Procedure - [L(3,1)-labeling for EDG(TS_m) , m ≥ 1]

$V \leftarrow \{ v_1,v_2, v_3,\ldots ,v_{2m},v_{2m+1}, v'_1,v'_2,v'_3\ldots v'_{2m} ,v'_{2m+1}\}$

$E \leftarrow \{e_1, e_2, e_3, \ldots,e_{3m} ,e_{3m+1},e'_1,e'_2,\ldots ,e'_{3m} \}$

$v_1 \leftarrow 0$, $v_2 \leftarrow 7$, $v_3 \leftarrow 6$, $v_4 \leftarrow 1$, $v_5 \leftarrow 8$

$v'_1 \leftarrow 0$, $v'_2 \leftarrow 3$, $v'_3 \leftarrow 4$, $v'_4 \leftarrow 1$, $v'_5 \leftarrow 9$

for i = 0 to (m-3)/4 do

$v_{6+8i} \leftarrow 0$; $v_{7+8i} \leftarrow 2$; $v'_{6+8i} \leftarrow 5$; $v'_{6+8i} \leftarrow 3$

end for

for i=0 to (m-4)/4 do

$v_{8+8i} \leftarrow 6$; $v_{9+8i} \leftarrow 9$; $v'_{8+8i} \leftarrow 6$; $v'_{9+8i} \leftarrow 10$

end for

for i=0 to (m-5)/4 do

$v_{10+8i} \leftarrow 0$; $v_{11+8i} \leftarrow 1$; $v'_{10+8i} \leftarrow 5$; $v'_{11+8i} \leftarrow 4$

end for

for i=0 to (m-6)/4 do

$v_{12+8i} \leftarrow 7$; $v_{13+8i} \leftarrow 10$; $v'_{12+8i} \leftarrow 6$; $v'_{13+8i} \leftarrow 11$

end for

end procedure

Theorem 3.1 : The extended duplicate graph of triangular snake graph admits L(3,1)- labeling and its number $\lambda(G)$ is 11.

Proof: Let TS_m be the triangular snake graph and $EDG(TS_m)$ be the extended duplicate graph of triangular snake graph.

Define the set of vertices and edges are

$V(G) = \{v_1, v_2, v_3, \ldots, v_{2m}, v_{2m+1}, v'_1, v'_2, v'_3, \ldots, v'_{2m}, v'_{2m+1}\}$

$E(G) = \{e_1, e_2, e_3, \ldots, e_{3m}, e_{3m+1}, e'_1, e'_2, \ldots, e'_{3m}\}$

Let $V(G) = V_1(G) \cup V_2(G)$,

Where $V_1 = \{ v_i / 1 \leq i \leq 2m+1\}$

$V_2 = \{ v'_i / 1 \leq i \leq 2m+1\}$

For V_1 and V_2 , we define a mapping $f: V(G) \rightarrow N \cup \{0\}$ such that $|f(x) - f(y)| \geq 3$ if $d(x,y) = 1$ and $|f(x) - f(y)| \geq 1$ if $d(x,y) = 2$.

Using algorithm 1, the vertices v_1 , v_2 , v_3 , v_4 , v_5 , v'_1 , v'_2 , v'_3 , v'_4 and v'_5 receive the labels 0 , 7 , 6 , 1 , 8 , 0 , 3 , 4 , 1 and 9 respectively ;

i) $f(v_{6+8i}) = 0$ and $f(v'_{6+8i}) = 5$ for $1 \leq i \leq (m-3)/4$

ii) $f(v_{7+8i}) = 2$ and $f(v'_{7+8i}) = 3$ for $1 \leq i \leq (m-3)/4$

iii) $f(v_{8+8i}) = 6$ and $f(v'_{8+8i}) = 6$ for $1 \leq i \leq (m-4)/4$

iv) $f(v_{9+8i}) = 9$ and $f(v'_{9+8i}) = 10$ for $1 \leq i \leq (m-4)/4$

v) $f(v_{10+8i}) = 0$ and $f(v'_{10+8i}) = 5$ for $1 \leq i \leq (m-5)/4$

vi) $f(v_{11+8i}) = 1$ and $f(v'_{11+8i}) = 4$ for $1 \leq i \leq (m-5)/4$

vii) $f(v_{12+8i}) = 7$ and $f(v'_{12+8i}) = 6$ for $1 \leq i \leq (m-6)/4$

viii) $f(v_{13+8i}) = 10$ and $f(v'_{13+8i}) = 11$ for $1 \leq i \leq (m-6)/4$

Thus all the vertices are labeled.

Now to prove that L(3,1)- labeling number $\lambda(G)$ is 11.

Case 1: Let x,y be any two vertices in $V_1(G)$.

Subcase (i): For m= 1

Let x and y be any two non-adjacent vertices on $V_1(G)$ such that $x = v_{m+1}$, $y = v_{m+2}$ then $f(x) = 7$ and $f(y) = 6$, $d(x,y) = 2$. Therefore $d(x,y)+ |f(x)-f(y)| = 2+|7-6| = 2+1 = 3 \geq 3$.

Subcase (ii): For m= 2

Let x and y be any two non-adjacent vertices on $V_1(G)$ such that $x = v_{m+2}$, $y = v_{m+3}$ then f(x) = 1 and f(y) = 8, d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|1-8| = 2+7 =9≥3.

Subcase (iii): For m=4n-1, nϵN.

Let x and y be any two non-adjacent vertices on $V_1(G)$ such that $x = v_{6+8i}$, $y= v_{7+8i}$ then f(x) = 0 and f(y)=2 , d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|0-2| = 2+2 = 4 ≥ 3.

Subcase (iv): For m=4n, nϵN.

Let x and y be any two non-adjacent vertices on $V_1(G)$ such that $x = v_{8+8i}$, $y= v_{9+8i}$ then f(x) = 6 and f(y)=9 , d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|6-9| = 2+3 = 5 ≥ 3.

Subcase (v): For m=4n+1, nϵN.

Let x and y be any two non-adjacent vertices on $V_1(G)$ such that $x = v_{10+8i}$, $y= v_{11+8i}$ then f(x) = 0 and f(y)=1 , d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|0-1| = 2+1 = 3 ≥ 3.

Subcase (vi): For m=4n+2, nϵN.

Let x and y be any two non-adjacent vertices on $V_1(G)$ such that $x = v_{12+8i}$, $y= v_{13+8i}$ then f(x) = 7 and f(y)=10 , d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|7-10| = 2+3 = 4 ≥ 3.

Case 2: Let x and y be any two vertices in V_2 (G).

Subcase (i): For m=1

Let x and y be any two non-adjacent vertices on $V_2(G)$ such that $x = v'_{m+1}$, $y = v'_{m+2}$ then f(x) = 3 and f(y) = 4, d(x,y) = 2. Therefore d(x,y)+|f(x) – f(y)| = 2+|3-4| = 2+1 = 3≥ 3.

Subcase (ii): For m=2

Let x and y be any two non-adjacent vertices on $V_2(G)$ such that $x = v'_{m+2}$, $y = v'_{m+3}$ then f(x) = 1 and f(y) = 9, d(x,y) = 2.Therefore d(x,y)+ |f(x) – f(y)| = 2+ |1-9| = 2+8 =10 ≥3.

Subcase (iii): For m=4n-1, nϵN.

Let x and y be any two non-adjacent vertices on $V_2(G)$ such that $x = v'_{6+8i}$, $y= v'_{7+8i}$ then f(x) = 5 and f(y)=3 , d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|5-3| = 2+2 = 4 ≥ 3.

Subcase (iv): For m=4n, nϵN.

Let x and y be any two non-adjacent vertices on $V_2(G)$ such that $x = v'_{8+8i}$, $y= v'_{9+8i}$ then f(x) = 6 and f(y)=10, d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|6-10| = 2+4 = 6 ≥ 3.

Subcase (v): For m=4n+1, nϵN.

Let x and y be any two non-adjacent vertices on $V_2(G)$ such that $x = v'_{10+8i}$, $y= v'_{11+8i}$ then f(x) = 5 and f(y)=4 , d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|5-4| = 2+1 = 3 ≥ 3.

Subcase (vi): For m=4n+2, nϵN.

Let x and y be any two non-adjacent vertices on $V_2(G)$ such that $x = v'_{12+8i}$, $y= v'_{13+8i}$ then f(x) = 6 and f(y)=11 , d(x,y) = 2. Therefore d(x,y)+ |f(x)-f(y)| = 2+|6-11| = 2+5 = 7 ≥ 3

Case 3: Let x and y be any two vertices in $V_1(G)$ and $V_2(G)$.

Subcase (i): For m=1

Let x and y be any two adjacent vertices on $V_1(G)$ & $V_2(G)$ such that $x = v_{m+1}$, $y = v'_{m+1}$ then f(x) = 7 and f(y) = 3, d(x,y) = 1. Therefore d(x,y) + |f(x) – f(y)| = 1+ |7-3| = 1+4= 5≥3.

Subcase (ii): For m=2,

Let x and y be any two adjacent vertices on $V_1(G)$ and $V_2(G)$ such that $x = v_{m+2}$, $y = v'_{m+3}$ then f(x)=1 and f(y) = 9 ,d(x,y) = 1. Therefore d(x,y) + |f(x) – f(y)| = 1+|1-9| = 1+8 = 9 ≥3.

Subcase (iii): For m=4n-1, nϵN.

Let x and y be any two adjacent vertices on $V_1(G)$ & $V_2(G)$ such that $x = v_{6+8i}$, $y= v'_{7+8i}$ then f(x) = 0 and f(y)=3 , d(x,y) = 1. Therefore d(x,y)+ |f(x)-f(y)| = 1+|0-3| = 1+3 = 4 ≥ 3.

Subcase (iv): For m=4n , nϵN.

Let x and y be any two adjacent vertices on $V_1(G)$ & $V_2(G)$ such that $x = v_{8+8i}$, $y= v'_{9+8i}$ then f(x) = 6 and f(y)=10 , d(x,y) = 1. Therefore d(x,y)+ |f(x)-f(y)| = 1+|6-10| = 1+4 = 5 ≥ 3.

Subcase (v): For m=4n+1, nϵN.

Let x and y be any two non-adjacent vertices on $V_1(G)$ &$V_2(G)$ such that $x = v_{10+8i}$, $y= v'_{11+8i}$ then f(x) = 0 and f(y)=4 ,d(x,y) = 1. Therefore d(x,y)+ |f(x)-f(y)| = 1+|0-4| = 1+4 = 5 ≥ 3.

Subcase (vi): For m=4n+2, nϵN.

Let x and y be any two adjacent vertices on $V_1(G)$ & $V_2(G)$ such that $x = v_{12+8i}$, $y= v'_{13+8i}$ then f(x) = 7 and f(y)=11 , d(x,y) = 1. Therefore d(x,y)+ |f(x)-f(y)| = 1+|7-11| = 1+4 = 5 ≥ 3.

Thus, by continuing this process of x and y , we get

$$d(x,y) + |f(x)-f(y)| = \begin{cases} \geq 3 & if \;\; d = 2 \\ \geq 4 & if \;\; d = 1 \end{cases} .$$

Hence , the extended duplicate graph of triangular snake graph admits L(3,1)- labeling and its number $\lambda(G)$ is 11.

Example 1: L(3,1)- labeling diagram in EDG(TS_5) and EDG(TS_6) is shown in figures (1) & (2)

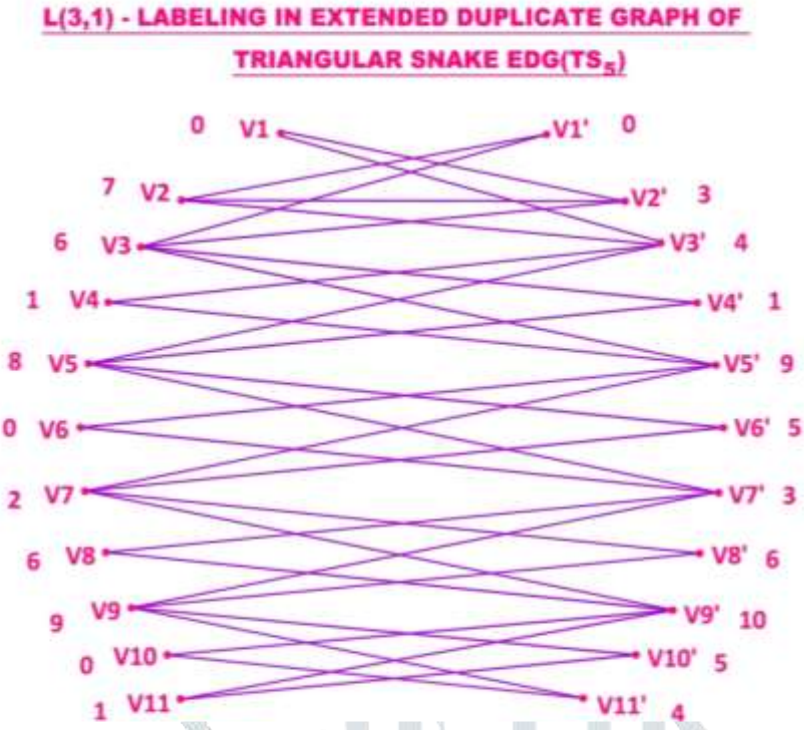


Fig 1 - EDG(TS_5)

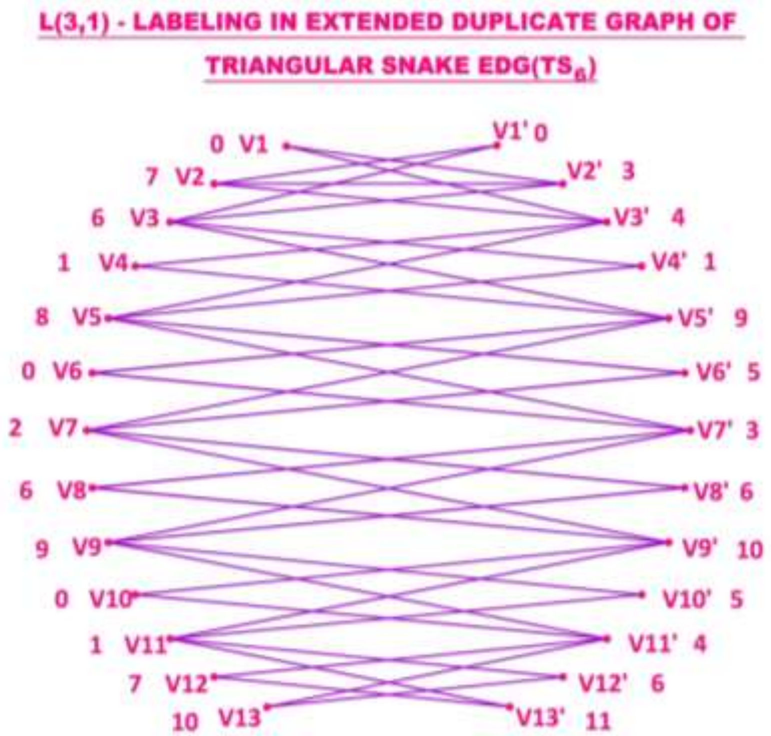


Fig 2 - EDG(TS_6)

3.2: L(3,1)-LABELING FOR QUADRILATERAL SNAKE GRAPH EDG(QSm) , m≥ 1

Here, we present an algorithm and prove the existence of L(3,1)-labeling for EDG(QS_m).

Algorithm- 2

Procedure – [L(3,1)-labeling for EDG(QS_m) , m ≥ 1]

$V \leftarrow \{ v_1,v_2, v_3\ldots v_{3m},v_{3m+1}, v'_1,v'_2,v'_3\ldots v'_{3m} ,v'_{3m+1}\}$

$E \leftarrow \{e_1, e_2, e_3 \ldots e_{4m} , e_{4m+1},e'_1,e'_2, e'_3,\ldots ,e'_{4m} \}$

$v_1 \leftarrow 0$, $v'_1 \leftarrow 0$

for i = 0 to (m-1)/3 do

$v_{2+9i} \leftarrow 6$; $v_{3+9i} \leftarrow 7$; $v_{4+9i} \leftarrow 4$; $v'_{2+9i} \leftarrow 3$; $v'_{3+9i} \leftarrow 1$; $v'_{4+9i} \leftarrow 4$

end for

for i = 0 to (m-2)/3 do

$v_{5+9i} \leftarrow 1$; $v_{6+9i} \leftarrow 0$; $v_{7+9i} \leftarrow 8$; $v'_{5+9i} \leftarrow 7$; $v'_{6+9i} \leftarrow 5$; $v_{7+9i} \leftarrow 8$

end for

for i = 0 to (m-3)/3

$v_{8+9i} \leftarrow 1$; $v_{9+9i} \leftarrow 3$; $v_{10+9i} \leftarrow 11$; $v'_{8+9i} \leftarrow 0$; $v'_{9+9i} \leftarrow 5$; $v'_{10+9i} \leftarrow 11$

end for

end procedure

Theorem 3.2 : The extended duplicate graph of Quadrilateral snake graph admits L(3,1)-labeling and its number λ(G) is 11.

Proof: Let QSm be the quadrilateral snake graph and EDG(QSm) be the extended duplicate graph of quadrilateral snake graph.

Define the set of vertices and edges are

$V(G) = \{v_1,v_2, v_3\ldots v_{3m},v_{3m+1}, v'_1,v'_2,v'_3\ldots,v'_{3m} ,v'_{3m+1}\}$

$E(G) = \{e_1, e_2, e_3 \ldots e_{4m} , e_{4m+1},e'_1,e'_2,e'_3\ldots ,e'_{4m} \}$

Assume that $V(G) = V_1 \cup V_2$,

where $V_1 = \{ v_i / 1\leq i \leq 3m+1\}$

$V_2 = \{ v'_i / 1\leq i \leq 3m+1\}$

For V_1 and V_2 , we define a mapping $f: V(G) \rightarrow N \cup \{0\}$

such that $|f(x) – f(y)| \geq 3$ if $d(x,y) =1$ and

$|f(x) – f(y) | \geq 1$ if $d(x,y) =2$.

Using the algorithm 2, the vertices v_1 and v'_1 receive the label 0 ;

i) $f(v_{2+9i}) = 0$ and $f(v'_{2+9i}) = 3$ for $1 \leq i \leq (m-1)/3$

ii) $f(v_{3+9i}) = 7$ and $f(v'_{3+9i}) = 1$ for $1 \leq i \leq (m-1)/3$

iii) $f(v_{4+9i}) = 4$ and $f(v'_{4+9i}) = 4$ for $1 \leq i \leq (m-1)/3$

iv) $f(v_{5+9i}) = 1$ and $f(v'_{5+9i}) = 7$ for $1 \leq i \leq (m-2)/3$

v) $f(v_{6+9i}) = 0$ and $f(v'_{6+9i}) = 5$ for $1 \le i \le (m-2)/3$

vi) $f(v_{7+9i}) = 8$ and $f(v'_{7+9i}) = 8$ for $1 \le i \le (m-2)/3$

vii) $f(v_{8+9i}) = 1$ and $f(v'_{8+9i}) = 0$ for $1 \le i \le (m-3)/3$

viii) $f(v_{9+9i}) = 3$ and $f(v'_{9+9i}) = 5$ for $1 \le i \le (m-3)/3$

ix) $f(v_{10+9i}) = 11$ and $f(v'_{10+9i}) = 11$ for $1 \le i \le (m-3)/3$

Thus all the vertices are labeled.

Now to prove that L(3,1)- labeling number $\lambda(G)$ is 11.

Case 1: Let x,y be any two vertices in $V_1(G)$.

Subcase (i): For $m = 3n-2$, $n \in N$.

Let x and y be any two non-adjacent vertices on $V_1(G)$, such that $x = v_{2+9i}$, $y= v_{3+9i}$ then $f(x)=6$, $f(y)=7$ and $d(x,y) = 2$. Therefore $d(x,y) + |f(x)-f(y)| = 2+|6-7| = 2+1 = 3 \ge 3$.

Subcase (ii): For $m = 3n-1$, $n \in N$.

Let x and y be any two non-adjacent vertices on $V_1(G)$, such that $x = v_{5+9i}$, $y= v_{6+9i}$ then $f(x)=1$, $f(y)=0$ and $d(x,y) = 2$. Therefore $d(x,y) + |f(x)-f(y)| = 2+|1-0| = 2+1 = 3 \ge 3$.

Subcase (iii): For $m = 3n$, $n \in N$.

Let x and y be any two non-adjacent vertices on $V_1(G)$, such that $x = v_{8+9i}$, $y= v_{9+9i}$ then $f(x)=1$, $f(y)=3$ and $d(x,y) = 2$. Therefore $d(x,y) + |f(x)-f(y)| = 2+|1-3| = 2+2 = 4 \ge 3$.

Case 2: Let x,y be any two vertices in $V_2(G)$

Subcase (i): for $m = 3n-2$, $n \in N$.

Let x and y be any two non-adjacent vertices on $V_2(G)$, such that $x = v'_{2+9i}$, $y= v'_{3+9i}$ then $f(x)=3$, $f(y)=1$ and $d(x,y) = 2$. Therefore $d(x,y) + |f(x)-f(y)| = 2+|3-1| = 2+2 = 4 \ge 3$.

Subcase (ii): for $m = 3n-1$, $n \in N$.

Let x and y be any two non-adjacent vertices on $V_2(G)$, such that $x = v'_{5+9i}$, $y= v'_{6+9i}$ then $f(x)=7$, $f(y)=5$ and $d(x,y) = 2$. Therefore $d(x,y) + |f(x)-f(y)| = 2+|7-5| = 2+2 = 4 \ge 3$.

Subcase (iii): for $m = 3n$, $n \in N$.

Let x and y be any two non-adjacent vertices on $V_2(G)$, such that $x = v'_{8+9i}$, $y= v'_{9+9i}$ then $f(x)=0$, $f(y)=5$ and $d(x,y) = 2$. Therefore $d(x,y) + |f(x)-f(y)| = 2+|0-5| = 2+5 = 7 \ge 3$.

Case 3: Let x and y be any two vertices in $V_1(G)$ and $v_2(G)$.

Subcase (i): for $m = 3n-2$, $n \in N$.

Let x and y be any two adjacent vertices on $V_1(G)$ and $V_2(G)$, such that $x = v_{2+9i}$, $y= v'_{3+9i}$ then $f(x)=6$, $f(y)=3$ and $d(x,y) = 1$. Therefore $d(x,y) + |f(x)-f(y)| = 1+|6-3| = 1+3 = 4 \ge 3$.

Subcase (ii): for $m = 3n-1, n \epsilon N$.

Let x and y be any two adjacent vertices on $V_1(G)$ and $V_2(G)$, such that $x = v_{5+9i}$, $y= v'_{6+9i}$ then $f(x)=1, f(y)=5$ and $d(x,y) = 1$. Therefore $d(x,y) + |f(x)-f(y)| = 1+|1-5| = 1+4 = 5 \geq 3$.

Subcase (iii): for $m = 3n, n \epsilon N$.

Let x and y be any two adjacent vertices on $V_1(G)$ and $V_2(G)$, such that $x = v_{8+9i}$, $y= v'_{9+9i}$ then $f(x)=1$, $f(y)=5$ and $d(x,y) = 1$. Therefore $d(x,y) + |f(x)-f(y)| = 1+|1-5| = 1+4 = 5 \geq 3$.

Thus, by continuing this process of x and y, we get

$$d(x,y) + |f(x)-f(y)| = \begin{cases} \geq 3 & if \;\; d = 2 \\ \geq 4 & if \;\; d = 1 \end{cases} .$$

Hence, the extended duplicate graph of quadrilateral snake graph admits L(3,1)- labeling and its number $\lambda(G)$ is 11.

Example 2: L(3,1)- labeling diagram in $EDG(QS_4)$ and $EDG(QS_5)$ is shown in figures (3) & (4)

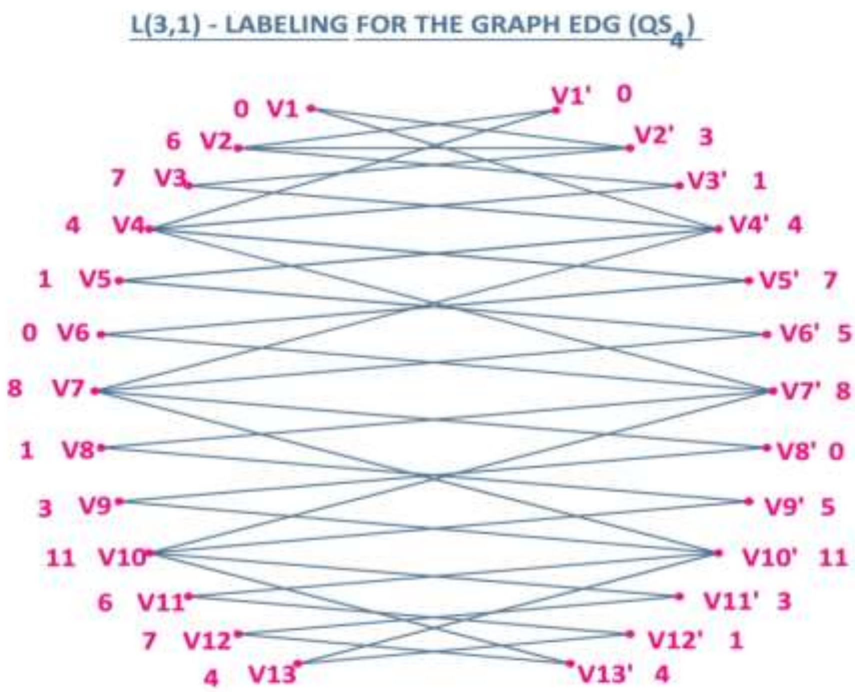


Fig 3 - $EDG(QS_4)$

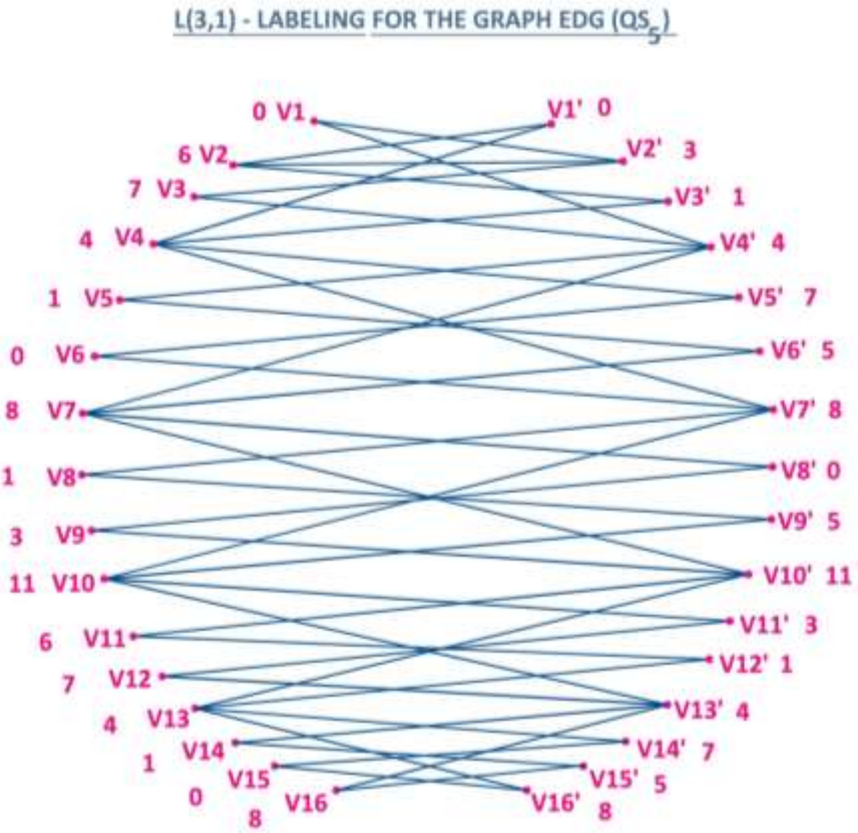


Fig 4 - EDG(QS_5)

4. Conclusion

In this paper ,we have presented algorithms and proved that the extended duplicate graph of triangular snake graph and quadrilateral snake graph admits L(3,1)-Labeling.

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