EFFICIENT RESOURCE ALLOCATION OF A MIMO-OFDM SYSTEM USING CONVEX OPTIMIZATION

MOHAMMAD ASIF[1], *DR.DALVEER KAUR*[2], *DR.AVTAR SINGH BUTTAR*[3]

[1]M.Tech Student, Department of Electronics and Communication, I.K.G.Punjab Technical University Kapurthala,India,

[2]Assistant professor, Department of Electronics and Communication, I.K.G.Punjab Technical University Kapurthala,India,

[3]H.O.D. Department of Electronics and Communication, I.K.G.Punjab Technical University Kapurthala,India.

Abstract

For higher order Multiple Input Multiple Output (MIMO) networks with and without Zero Forcing (ZF) technique, we consider the problem of sum rate maximization. The aim is to maximize the achievable communication sum rate by formulating the user dependent MIMO channel matrix. To obtain a quality to the existing non-convex problem, we devise a method based on convex optimization by applying the negative sum rate of the non-convex optimization. Furthermore, to obtain the optimal power allocation channel coefficients, the KKT condition is applied. Simulations are performed for 4×4, 8×8, 16×16, 32×32, 64×64, 128×128 and 256×256 MIMO systems. Simulation results prove that ZF technique performs better at higher power levels.

Introduction

On the efficient radio resource allocation schemes, several works have already been carried out for 5G wireless communication systems. To attain much higher data throughput and improved spectral efficiency without the requirement for increased bandwidth and redundant base stations, the non-orthogonal multiple access (NOMA), multiple input and multiple output and relaying technologies have been discussed in [1]. Considering the typical indoor environment, cooperative and coordinated multi-cell resource allocation methods for 5G ultra reliable low latency connection, has been presented by the authors in [2]. Moreover for 5G networks with application to device to device (D2D) and machine to machine (M2M) communications various other resource allocation schemes have been presented in [3–5]. Efficient resource allocation for MIMO and OFDM in 5G is a non-convex optimization problem [6]. Using the convex optimization routine in MATLAB, by considering the negative sum rate of the optimization objective function a less complex closely approaching optimal solution for such problems can be obtained.

Problem Description

For the 5G wireless communication systems, the promising technologies are Massive MIMO, cooperative communication, etc. To efficiently allocate the channel resources and to support the spectrum reuse, at both the base station and the mobile station, a large array of high directive/gain antennas are employed, multiplexed spatially for highly directed beam-forming in massive MIMO. The resource allocation scheme does not guarantee the optimal solution of the problem for MIMO and OFDM based wireless system architectures as the resource allocation scheme here is a non-convex optimization problem. In this section, for obtaining the solution of the non-convex optimization problems we introduce a duality counterpart by minimizing the negative sum rate of the objective function. By minimizing the objective function or cost function subject to certain real-time constraints (inequality or equality constraints) the optimal solutions for all the convex optimization problems related to judicious allocation of network resources can be obtained. The convex optimization techniques form the basis for the efficient resource allocation like spectrum reuse, energy efficiency, bit error rates, etc.

Problem Solution

We formulate analytically and mathematically the various optimization parameters like optimal MIMO-OFDM power allocation, OFDM rate optimization, MIMO rate optimization [7, 8] and optimization problems related to effect of multiple antennas in cooperative communication for optimum resource allocation in MIMO and multicarrier (OFDM, FBMC) based 5G wireless communication systems. We first introduce the concept of typical convex optimization problem in this section and then using the convex optimization approach and LaGrange's function we will analyse various optimization parameters. Note that the number of Lagrangian coefficients is always equal to number of the constraints.

Convex Function and Convex Optimization

If the domain of the optimization function $g(x)$ is a convex set, i.e., $x, y \in \text{dom } g$, the function $g(x)$ is a convex function satisfying the following inequality:

$$g(\alpha x + (1-\alpha)y) \leq \alpha g(x) + (1-\alpha)g(y);$$

$$0 \leq \alpha \leq 1 \qquad (1)$$

Any convex optimization problem will have the form

$$\text{Minimize } g_0(x)$$

$$\text{Subject to } g_i(x) \leq 0, i = 1, 2, \dots, m$$

$$h_i(x) = 0, i = 1, 2, \dots, n \qquad (2)$$

This describes a convex optimization problem for finding the values of the variable x that minimizes $g_0(x)\forall x$ satisfying the constraints $g_i(x) \leq 0,\ i = 1, 2, \dots, m$ and $h_i(x) = 0,\ i = 1, 2, \dots, n$. The variable $x \in R^n$ is termed as the optimization variable and the function $g_0 : R^n \rightarrow R$ the cost function or optimization function. The inequalities $g_i(x) \leq 0$ are inequality constraints corresponding to inequality constraints functions $g_i : R^n \rightarrow R$. The equality constraint $h_i(x) = 0$ corresponds to the equality constraint functions $h_i : R^n \rightarrow R$. The optimization problem is called unconstrained if there are no constraints,.

Graphically a convex function represents a chord passing through two points $(x, g(x))$ and $(y, g(y))$ from x to y. An optimization function g is strictly convex, if strict inequality holds i.e. whenever $x \neq y$ and $0 \leq \alpha \leq 1$. If g is concave then $-g$ is convex and g is strictly concave if $-g$ is strictly convex. This is a generalized fact that an optimization functions like a MIMO rate optimization function, OFDM rate optimization function, optimal MIMO-OFDM power allocation functions and optimization problems related to effect of multiple antennas in cooperative communication are concave optimization problems but the negative sum rate of these functions are convex functions. The convex optimization techniques can now be employed to obtain the optimal values of the said functions.

System Model for MIMO Rate Optimization

We consider a standard MIMO system consisting of 't' transmit antennas at the base station side and 'r' decentralized receive antennas. The MIMO channel can be equivalently modelled as:

$$\bar{Y} = H\bar{X} + \bar{N}. \qquad (3)$$

where $\bar{Y} = [y_1, y_2, y_3, \dots, y_r]$ is the 'r' dimensional receive vector at the MIMO receiver, $\bar{X} = [x_1, x_2, x_3, \dots, x_t]$ is a 't' dimensional transmit vector with each symbol transmitted through each transmit antenna [9]. $\bar{H} = [h_1, h_2, h_3, \dots, h_{rt}]$ is the $r \times t$ channel coefficient vector and $\bar{N} = [n_1, n_2, n_3, \dots, n_r]$ is the 'r' dimensional noise vector. The subscripts to the parameters y, x, h, n corresponds to the antenna numbers at transmit and receive sides of the MIMO channel.

The MIMO system introduced represents the parallelization of the MIMO channel with 't' symbols transmitted in parallel and spatially multiplexed. The signal power received at the receiver corresponding to each MIMO channel is given as

$$\sigma_i^2\{E|\bar{X}|^2\} \qquad (4)$$

where σ_i represents the singular values of the channel coefficient matrix $\bar{H}$ [10] of the MIMO channel. The SVD of $\bar{H}$ is given below

$$H = U \sum V^H \qquad (5)$$

where the matrices U, $\sum V$are $r \times t, t \times t$ and $t \times t$ dimensional respectively [11]. The noise power received at the receiver corresponding to each MIMO channel is given by σ_n^2 computed as the value of the covariance of the noise matrix. Therefore the signal to noise ratio at the input of the receiver is given as

$$SNR = \frac{\sigma_i^2\{E|\bar{X}|^2\}}{\sigma_n^2} \quad (6)$$

From the above SNR expression for the ith channel, the Shannon capacity C_i of the channel can be derived as given below

$$C_i = \log_2\left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right) \quad (7)$$

The optimal MIMO power allocation problem can now be formulated as

$$\text{Maximize} \sum_{i=1}^{t} \log_2\left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right). \quad (8)$$

$$\text{Subject to} \sum_{i=1}^{t} P_i \leq P \quad (9)$$

where P is the total transmit power.

The above optimization problem with the given inequality constraint is a non-convex optimization problem and hence the convex optimization techniques cannot be applied directly to obtain the optimal solution for the MIMO power allocation problem. A non-convex optimization problem is transformed into a convex optimization problem by taking the negative sum rate of the non-convex optimization expression. The optimal MIMO power allocation problem can further be modified and formulated as a convex optimization problem by taking the negative sum rate of $\sum_{i=1}^{t} \log_2\left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right)$ as under

$$\text{Minimize.} -\sum_{i=1}^{t} \log_2\left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right) \quad (10)$$

$$\text{Subject to} \sum_{i=1}^{t} P_i \leq P \quad (11)$$

To solve the above convex optimization problem a series of steps are followed as under:

Step1: Finding the Lagrangian cost function $f(\bar{P}, \mu)$

It is important to note that the number of Lagrangian multiples is equal to the number of constraints- inequality of equality constraints. The Lagrangian cost function $f(\bar{P}, \mu)$ for the given optimization problem can be formulated as under

$$f(\bar{P}, \mu) = \sum_{i=1}^{t} \log_2\left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right) + \mu\left(P - \sum_{i=1}^{t} P_i\right)$$

(12)

Step2: Finding the maxima of the Lagrangian cost function.

Differentiating the above obtained Lagrangian cost function $f(\bar{P}, \mu)$ with respect to power associated with the ith MIMO channel P_i and setting the result equal to 0, we get

$$\frac{\partial}{\partial x} f(\bar{P}, \mu) = 0 \quad (13)$$

$$\Rightarrow \frac{\frac{\sigma_i^2}{\sigma_n^2}}{1+\frac{P_i\sigma_i^2}{\sigma_n^2}} - \mu = 0 \quad (14)$$

Step3: Finding the optimal Pi using KKT conditions

The Kurush Kuhn Tucker (KKT) conditions states that if $\frac{\partial}{\partial x} f(\bar{P}, \mu) = 0$ then there exist local minima P^* for a unique value of the Lagrangian multiple μ as μ^* subject to $\mu^* \geq 0$. Solving the above differential equation $\frac{\partial}{\partial x} f(\bar{P}, \mu) = 0$ yields

$$\frac{\frac{\sigma_i^2}{\sigma_n^2}}{1+\frac{P_i\sigma_i^2}{\sigma_n^2}} - \mu = 0 \quad (15)$$

$$\Rightarrow \frac{\sigma_i^2}{\sigma_n^2}\frac{1}{\mu} = 1 + \frac{P_i\sigma_i^2}{\sigma_n^2} \quad (16)$$

$$\Rightarrow P_i = \left(\frac{1}{\mu} - \frac{\sigma_n^2}{\sigma_i^2}\right)^+ \quad (17)$$

P_i represents the power associated with the ith MIMO channel, the function $\left(\frac{1}{\mu} - \frac{\sigma_n^2}{\sigma_i^2}\right)^+$ always accounts for the positive value of the channel powers, because channel powers cannot be negative.

$P_i = \left(\frac{1}{\mu} - \frac{\sigma_n^2}{\sigma_i^2}\right)^+$ is positive if $\frac{1}{\mu} \geq \frac{\sigma_n^2}{\sigma_i^2}$ and 0 otherwise. P_i Can now be formulated as a piecewise optimization function as under

$$P_i = \begin{cases} \frac{1}{\mu} - \frac{\sigma_n^2}{\sigma_i^2}, & x \geq 0 \\ 0, & x < 0 \end{cases} \qquad (18)$$

Step4: Finding the Lagrangian multiplier 'μ' for optimal P_i.

The power allocated to the ith MIMO channel is directly dependent on user defined function σ_i^2. Increasing the σ_i will increase the power allocated to the ith MIMO channel coefficient. Thus the resulting power allocation will result in the water filling phenomenon subject to the constraint $\sum_{i=1}^{t} P_i \leq P$. Employing the same constraint will yield the value of 'μ' for optimal power allocation to the ith MIMO channel.

$$P_i = \left(\frac{1}{\mu} - \frac{\sigma_n^2}{\sigma_i^2}\right), \quad \forall \frac{1}{\mu} \geq \frac{\sigma_n^2}{\sigma_i^2} \qquad (19)$$

Solving the above expression for 'μ' we get

$$\mu = \frac{\sigma_i^2}{P_i \sigma_i^2 + \sigma_n^2} \qquad (20)$$

For optimal P_i, the Lagrangian multiplier 'μ' should be minimum subject to the condition $\sum_{i=1}^{t} P_i \sigma_i \leq P$.

Results and Discussions

The figures from 1–7 represents the capacity (Mbps) associated with the ith MIMO channel as a function of the corresponding power of the same channel with Zero-Padding (ZP) and with Cyclic-Prefix (CP).

Figure 1 shows capacity (Mbps) versus Power (dB) graph with ZP technique and with CP for 4 × 4MIMO system. The figure clearly reveals that with ZP and CP technique capacity increases with the increase in power, resulting in water filling phenomenon. It is observed that as the power increases from 5 to 60 dB capacity increases to 45 Mbps.

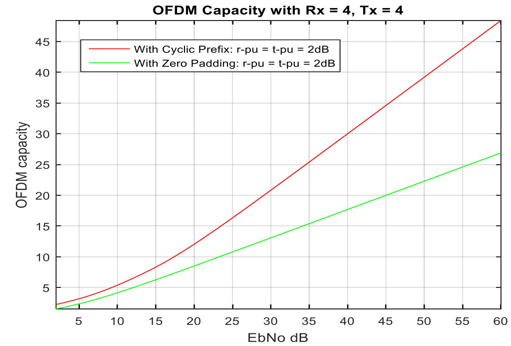

Fig. 1 Capacity (Mbps) versus power (dB) for 4 × 4 MIMO systems

Figure 2 shows capacity (Mbps) versus Power (dB) graph with ZP technique and with CP for 8×8 MIMO system. The figure reveals that at low power (<5 db) there is no improvement in capacity using ZP technique. However after 5 dB power level ZP technique shows an improvement in capacity enhancement, again resulting in water filling phenomenon. At 10 dB there is an overall increase in capacity in both ZP and CP.

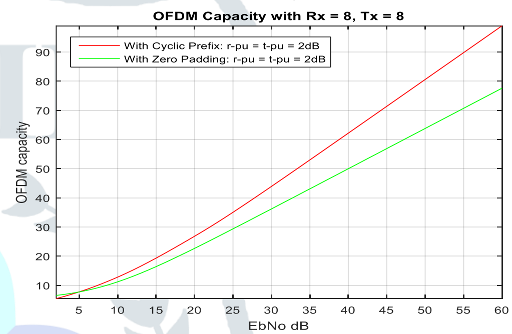

Fig. 2 Capacity (Mbps) versus power (dB) for 8 × 8 MIMO systems

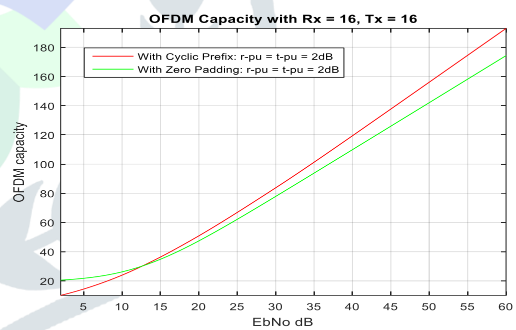

Fig. 3 Capacity (Mbps) versus power (dB) for 16 × 16 MIMO systems

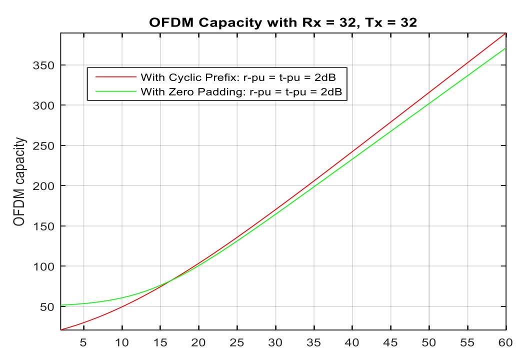

Fig. 4 Capacity (Mbps) versus power (dB) for 32 × 32 MIMO systems

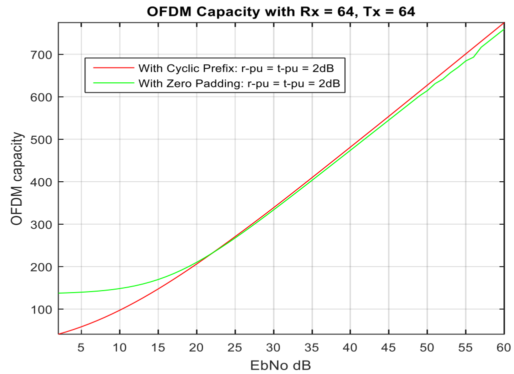

Fig. 5 Capacity (Mbps) versus power (dB) for 64 × 64 MIMO systems

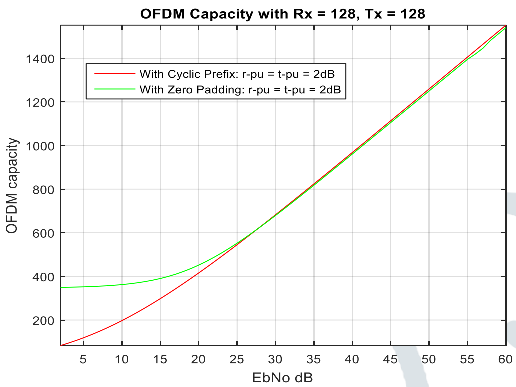

Fig. 6 Capacity (Mbps) versus power (dB) for 128 × 128 MIMO systems

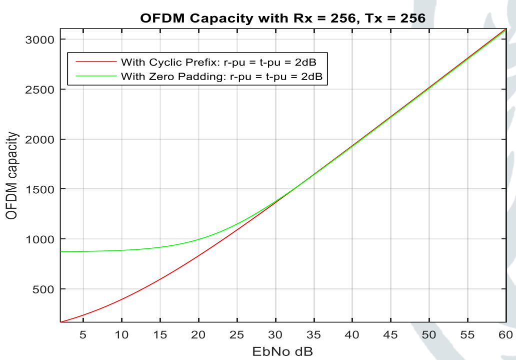

Fig. 7 Capacity (Mbps) versus power (dB) for 256 × 256 MIMO systems

Figure 3 shows capacity (Mbps) versus Power (dB) graph with ZP technique and with CP for 16 × 16 MIMO system. The Figure reveals that at low values of power the performances of CP technique deteriorates as compared to ZP technique. After 15 db power level CP technique again performs better than ZP technique. Thus massive MIMO systems will perform better at higher power levels where SINR values will be optimal. This is again shown in Fig. 4 where we again plot capacity vs. power for 32 × 32 MIMO system. ZP technique performs almost closer to that of CP technique after 20 dB power levels. Same is the case in 64 × 64 MIMO System and when we further increase the MIMO order, the graph for capacity vs. power coincides for both CP and ZP technique after 25 dB. Thus ZP technique will definitely perform better when MIMO order is further increased.

Conclusion

To evaluate the performance of proposed ZP technique using convex optimization method a number of iterations has been done in this paper. The iteration method was based on applying negative sum rate of non-convex optimization. Via CVX toolbox the sum rate maximization of proposed ZF technique is dealt. As depicted from our plots in the proposed technique the optimal values for power allocation coefficient and Lagrangian coefficient obtained result in water filling phenomenon. Compared to CP technique, at higher power levels for higher MIMO order configurations the proposed ZP technique performs better due to lesser interference and noise limited environment.

References

1. L. Dai, B. Wang, Y. Yuan, S. Han, C. I, Z. Wang, Non-orthogonal multiple access for 5G: solutions, challenges, opportunities, and future research trends. IEEE Commun. Mag. 53(9), 74–81 (2015)
2. Z. Ding, Z. Yang, P. Fan, H.V. Poor, On the systems with randomly deployed users. IEEE Sig. Process. Lett. 21(12), 1501–1505 (2014)
3. X. Liu, X.Wang, Highly efficient 3D resource allocation techniques in 5G for NOMA enabled massive MIMO and relaying systems. IEEE J. Sel. Areas Commun. 35(12), 2785–2797 (2017)
4. G.D. Swetha, G.R. Murthy, Fair resource allocation for D2D communication in mmWave 5G networks. IEEE (2017).
5. P. Zhao, L. Feng, P. Yu, W. Li, X. Qiu, A social-aware resource allocation for 5G device-todevice multicast communication. IEEE 2169-3536 (c) (2017).
6. L. Sharnagat, H. Dakhore, Method of resource allocation in OFDMA using convex optimization. IEEE. (2015).
7. S. Shi, M. Schubert, H. Boche, Rate optimization for multiuser MIMO systems with linear processing. IEEE Trans. Sig. Process. 56(8), 4020–4030 (2008).
8. J.A. Sheikh et al., Towards green capacity in massive MIMO based 4G-LTE a cell using beam forming vector based sectored relay planning. Wirel. Pers. Commun.
9. H.V. Nguyen, V.-D. Nguyen, H.M. Kim, O.-S. Shin, A convex optimization for sum rate maximization in a MIMO cognitive radio network, in Eighth International Conference on Ubiquitous and Future Networks (ICUFN) (2016).
10. J. Zhu, R. Schober, V.K. Bhargava, Linear precoding of data and artificial noise in secure massive MIMO systems. IEEE Trans. Wirel. Commun. 15(3), 2245–2261 (2016).
11. H. Yang, B.-C. Seet, S.F. Hasan, P.H.J. Chong, M.Y. Chung, Radio resource allocation for D2D-enabled massive machine communication in the 5G era, 2016