SOME CONTRIBUTION TO THE THEORY OF STRONGLY AND COMPACTLY NUCLEAR TRILINEAR FORMS

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ABSTRACT

In this paper a dual nuclear locally convex space E has been studied and shown that all strongly continuous trilinear form defined (ExExE) are strongly nuclear.

Converse of the above statement has been established.

Familly i have studied all trilinear from to be compactly nuclear. It is shown that the concerning locally convex Hansdorff is dual nuclear.

BASIC CONCEPTS

a) Let E ,F and G be locally convex space. Let T(E ,F ,G) be the space of all continuous trilinear form T (X ,Y ,Z) defined on E X F X G such that relation IT (x ,y ,z) ≤PH (x) PK (Y) PM (Z) ……………………….(1) is valid for fundamental subject h, k, m of E, F ,G respectively ,them it is concluded that all trilinear forms T (X ,Y ,Z) are compactly continuous if the relation (1) holds

b) let E ,F ,G be locally convex space T(E, F,G) be the space of all continuous trilinear forms T (X,Y , Z) defined on E X F X G such that the following two relations are valid.

1 T(x,y,z)= $\sum_{n}$,an>< y , bn>< z, In > ⋯ … … … … … . (2)for x ∈ E , Y ∈ F , Z ∈ g, an ∈ E, bn ∈ $F,^{1}$ In ∈ $G^{1}$and $\sum_{n}$, $P^{1}K^{0}$(bn)$p^{1}m^{0}$ (In) < +∝ ⋯ … … … … (3) for an ∈ $H^{0}CH^{1}$, BN ∈ K^{0} CF1, LN ∈ M^{0} CG1, H = Fundamental compact subjet of E , K + Fundamental compact subject of G,

so it follows that all continous trilinear forms T(x, y, z)ϵ T (E , F G) are compactly nuclear.

c) Let E, F ,G be locally convex space and T (E ,G ,G) be the space of continuous trilinear forms T (x ,y ,z) defined on E X F X G into the field of scalars such that the relation T(x ,y, z)< PA(x)PB (y)PD (z) holds .

For X ϵ E, YVF, Z ϵ G, A∈ β(E), B ϵ β (F), D ϵ β(G) then I say that all trilinear forms T (x ,y, ,z) are strongly continuous if the relation (1) is valid .

d) Let E , F, G be locally convex space and let T (E , F , G) be the space of all continonus trilinear forms T (x, y ,z) defined on E X F X G such that the following two relation are valid.

T(x,y,z)= $\sum_{\eta}$< x, an><y , bn><z, dn> for X ϵ E , Y ϵ F , ZFG ,an ∈E^{1},dn∈F^{1}, dn ϵ G^{1} and $\sum_{\eta}$$p^{1}N^{0}$(an) $P^{1}B^{0}$(bn)$P^{1}D^{0}$(dn) < + ∝

For Aϵ β (E), B ϵ β (F), D ϵ β (G), an ϵ A^{0} CE^{1} bn ∈ $B^{0}CF^{1}$, dn ϵD^{0} C G^{1}.

So all trilinear forms T (x,y,z) ϵ T (E ,F ,G) are strongly nuclear ,and strongly nuclear trilinear form T (x , y , z) ϵ T (E,F,G) is strongly continuous.

Theorem 1 : All strongly continuous trilinear forms T (x , y ,z) ϵ T (E , F , G) are strongly

nuclear if the locally convex space E is dual nuclear.

Proof : It is obvious that there exists a fundamental system βf (e) of bounded subject Bn in E. Let there exist a bounded set B ∈ βf(E) for each bounded subject A ϵ βf (E) with A < B such that the following two relations are valid

$PA(X) \le \sum_{\eta} I < X, In > I$ … … … … … … … … … … (1)

For X ϵ A C E , In ϵ E^1 and $\sum_{\eta} P^1 B^O (In) < +\propto$ ⋯ … … … … … … … … … (2)

For In ∈ $B^O C E^1$

Since all trilinear forms T (x, x, x) ∈ T (E, E, E) are strongly continuous, there exists a relation IT

(X,X,X) I <$[PA(x)]^3$ … … … … … … … … … … … … … … … . (3)

For X ϵ E, A ϵ βf(E)

The relations (1) and (3)⟹

$\Rightarrow I\,T(X,X,X) \le I[\sum_{\eta} Y < X, In >]^3$ … … … … … … … … … … … … . . (4)

$\Rightarrow T(X,X,X) \le [\sum_{\eta} < X\, In >]^3$ … … … … … … … … ………..(5)

$\Longrightarrow T(X,X,X) \ge \sum_{\eta} (< X, In]^3$ … … … … … … … … . (6)

$\Longrightarrow T(X,X,X) = t\sum_{\eta} (< x, In]^3$ … … … … … … … … … . (7)

t ≤ I

Let $t\sum_{\eta}(< x,\ In >)^3 = \sum_{\eta}(< x, g\, In)^3$ in (7) for g = a positive number the relation (7)

$\Rightarrow T(X,X,X) = \Sigma(< X, an >)^3$ … … … … … … … …..(8)

Where g In = an ϵ E^1

The relation (2)⇒ $P^1 B^0 (In)$ = a finite number … … … … … … … (9)

The relation (9)

$\Rightarrow P^1 A^0 (In)$ = a finite number ……………………………………..(10)

$\Rightarrow [P^1 A^O (In)]^3$ = a finite number

$\Rightarrow \sum_{\eta} [P^1 A^0 (In)]^3$ = a finite number

$\Rightarrow \sum_{\eta} [\sum_{\eta} P^1 A^0 (In)]^3$ = a finite number

$\Rightarrow \sum_{\eta} [P^1 A^0 (an)]^3 <= \propto$ ⋯ … … … … … … … . (11)

For g in = an ∈ E^1

On the basis of (8) and (11) it follows that strongly continuous trilinear form T(X,X,X) ϵ T(E, E, E) are strongly nuclear .

Thus the theorem is completely proved.

THEOREM II : A locally convex space E is a dual nuclear if all trilinear forms T (X,X,X)ϵ $T(E,E,E) are$ strongly nuclear.

PROOF : All trilinear forms T (X,X,X)
$T\,(E,E,E)\ are\ strongly\ nuclear. Hence\ there\ exist\ the\ following\ two\ relations.$

T (X,X,X)= $[< X, dn >]^3$ … … … … … … ….(1)

For x ∈ E , $dn \in E^1$

and $\sum_{\eta} [P^1 A^0 (dn)]^3 < +\propto$ ⋯ … … … … … … … ….(2)

for dn∈ $A^0 C E^1, A \in \beta\, f(E)$

Also all trilinear forms T(X,X,X) $\epsilon\, T(E,E,E)$ are strongly nuclear .Hence these are strongly continuous ,there exists a relation I T(X,X,X) I [PA(x)]^3 … … … … … … … … … … … … (3)

For X ∈ $A \in \beta\, f(E)$

IT (X,X,X) ≤ $\frac{\Sigma}{\eta} I\, [< X,\ dn >]^3$ … … … … … … … … … … … … . (4)

On the basis of (3) and (4) there exists a relation [PA (X)]^3 $= \frac{\Sigma}{\eta} I\ < X, dn\ > I^3$ … … … … … … . (5)

In particular care.

The relation (5)

[PA (X)]^3 $\leq [\frac{\Sigma}{\eta} I < X, dn\ > I\ ]^3$

For X $\epsilon\, A\, \epsilon\, \beta f(E), dn\, \epsilon\, E^1$

$\Rightarrow PA(X) \leq \frac{\Sigma}{\eta} I < X, dn > I$ … … … … … … … …{6)

The relation (2)

⇒$\frac{\Sigma}{\eta}[P^1 A^0$ (dn)]^3=a finite number ……………………………………(7)

⇒$\frac{\Sigma}{\eta} P^1 A^0$(dn) = a finite number

⇒$\frac{\Sigma}{\eta} P^1 A^0$ (d n) =a finite number…………………………………………(8)

⇒$\frac{\Sigma}{\eta} P^1 B^0$ (d n) =a finite number……………………………………….(9)

⇒$\frac{\Sigma}{\eta} P^1 B^0$ (d n)^3<+ ∝⋯…………………………………………………………… (10)

For B $\epsilon\, \beta\, f(E), dn\, \epsilon\, E^1$

In this way for several fundamental bounded subsets of E the above relations are valid out of them two fundamental bounded subsets A,B$\epsilon\, \beta f(E)$ with A $< B$ such that the relation (6) and (10) are valid ……………………………………….(11)

Thus the Theorem is completely proved .

Theorem III :A locally convex Harsdorf space E is dual convex space if all Trilinear form T(x,x,x) ϵ

T (E,E,E) are.

Proof : Obviously the following two relations are valid .

T (x,x,x) =$\frac{\Sigma}{\eta}[< x, gn >]^3$ … … … … … … … … … … … … . (1)

For x $\epsilon\, E, gn\, \epsilon\, E^1$

And $\frac{\Sigma}{\eta}[P^1 H^0(gn)]^3 < +\propto \cdots$ … … … … … … … … … … (2)

Since all trilinear forms T(X,X,X) $\epsilon\, T(E,E,E)$are compactly nuclear ,hence these are compactly continuous .Consequently ,there exists a relation.

I T (X, X, X) I≤ $[PH(X)]^3$ … … … … … … … … … … … (3)

For X ∈ $H\ hf(E)$

On the basis of the relation (1) theorem follows a relation.

I T (X,X,X) I≤ $I\ < X, gn >]^3$ … … … … … … … … … . . (4)

The relation (3) and (4)

$$\Rightarrow [PH(X)]^3 = \frac{\Sigma}{\eta}[\,I < X, gn > I]^3 \ldots\ldots\ldots\ldots\ldots (5)$$

Under a particular consideration on the basis of the relation (5) it follows that

$[PH(X)]^3 \leq [\Sigma\, I < X, gn \; > I]^3$

For $x \in H \in hf(E), gn \; \in \; E^1$ Which implies

$PH(X) \leq {}^{\Sigma}_{\eta}[< X, gn > I \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)$

On the basis of (2),∃ a relation

${}^{\Sigma}_{\eta}[P^1 H^0 \; (gn)]^3 = \text{a finite number} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7)$

$\Rightarrow [{}^{\Sigma}_{\eta} P^1 H^0 (gn)]^3 =$ a finite number ....(8)

$$\Rightarrow {}^{\Sigma}_{\eta} P^1 H^0 (gn) = \text{a finite number}$$

$\Rightarrow {}^{\Sigma}_{\eta} P^1 K^0 \; (gn) \quad =$ a finite number

$$\Rightarrow \; {}^{\Sigma}_{\eta} P^1 K^0 (gn) < + \propto \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (9)$$

The relation (6) and (9) are valid nuclear countable compact of E

Two countable fundamental compact subsets are considered H,K∈ hf(E)with Hθ < K such that the relations (6)and (9)are valid.

Hence the locally convex space E is dual nuclear .thus the theorem is completely proved.

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