Analysis of Motion Transmissibility and Current Transmissibility of an Electromechanical Vibration Absorber

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Abstract: **A Tuned Vibration Absorber (TVA) is used to minimize the vibrations of mechanical machines and structures subjected to harmonic force excitation. However, it is achieved at the expense of large vibration amplitude of absorber mass. Nagem et al. have addressed this problem by replacing the TVA by an Electromechanical Vibration Absorber (EMVA). In this paper, in the approach of Nagem et al., the equations of motion transmissibility and current transmissibility of an EMVA are obtained using the parameters of mechanical subsystem and the electrical subsystem. Non-dimensional frequency response curves for the motion transmissibility and current transmissibility are plotted for different values of mechanical damping ratio ξ_1 and electrical damping ratio ξ_e of EMVA. It is seen that with the increase in the values of ξ_1 and ξ_e, significant reduction in the resonant response of primary system is achieved when the primary system is subjected to harmonic force excitation. The excitation frequency band over which the EMVA is effective is also increased. The resonant values of current in the shunted electrical RLC circuit are considerably reduced resulting in less energy loss in the electrical circuit.**

***Keywords* – Electromechanical vibration absorber, Motion transmissibility, Current transmissibility**

1. INTRODUCTION

The engineering systems processing mass and springiness vibrate when subjected to dynamic excitation. When the natural frequency of such a system coincides with the frequency of excitation, i.e. at resonance, the system vibrates with very large amplitudes and may lead, in many cases, to failure. In the literature, many approaches have been reported to control these resonant vibrations. The extensively studied approach is the use of Tuned Vibration Absorber (TVA). An excellent account of various types of passive, adaptive and active TVAs has been given in review paper by Sun et al. [1]. However, in these TVAs the problem is the large amplitudes of absorber mass. One of the solution of this problem is the use of Electromechanical Vibration Absorber (EMVA). The EMVA comprises of an electromechanical transducer and resonant electrical circuit. By tuning the electrical circuit, as per the requirement, the vibration amplitude of the primary system can be considerably reduced. In this case, the large electrical oscillations in the resonant circuit take place instead of large amplitude of absorber mass vibration in the TVA.[2] [3] [4]. Also in an EMVA, the variation of absorber parameter values is possible. It is in this context, in this paper, the performance curves of motion transmissibility and current transmissibility of an EMVA have been obtained using the equations developed for the same in the approach of Nagem et al.

The EMVA system mainly consists of a fixed magnet and a movable electric coil. The EMVA operates as a generator, converting the vibration of primary mass system into electrical energy where an electromotive force e.m.f is induced in the coil due to relative motion between the magnet and the coil. This generated e.m.f. creates an opposing force that is proportional to the velocity of the coil, causing the viscous damping effect.

2. MOTION AND CURRENT TRANSMISSIBILITY OF EMVA

The schematic mathematical model of an EMVA is shown in fig. (1). It comprises of a mechanical subsystem having primary mass m, spring stiffness k, and mechanical damping c_m. Its electrical subsystem consists of a permanent magnet and electric coil. The coil connected to primary mass 'm' slides with respect to fixed magnet as shown in fig. (1). The primary mass displacement is x(t) under the harmonic force excitation f(t) = Fsin(ωt), where ω is the excitation frequency. The relative velocity between coil and magnet is $\dot{x}$. An electrical resonant circuit comprising of inductance L, capacitance C, and resistance R, is connected across the coil terminals. The induced e.m.f. e across the coil due to relative motion of magnet and coil is supplied to the shunted electrical circuit. The electrical resonant circuit across the coil terminals controls the electrical damping of the EMVA. The magnet may be neodymium or samarium cobalt having large residual magnetic field strength. The cylindrical coil is formed using copper material.

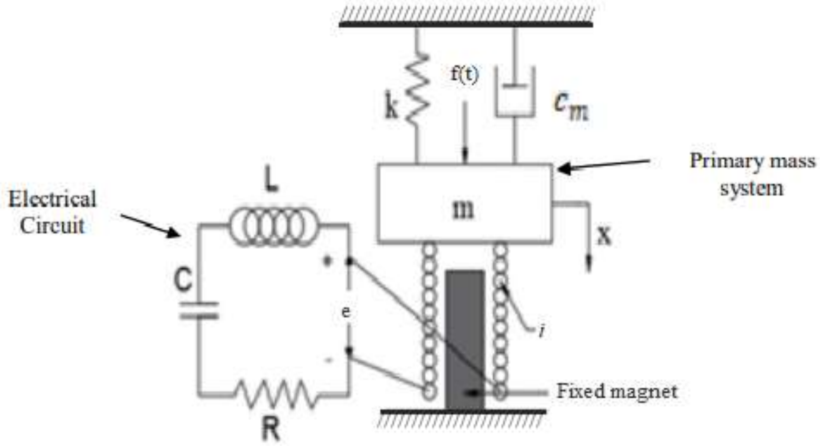

Fig.1 Schematic of an Electro-Mechanical Vibration Absorber (EMVA)

2.1 Mechanical subsystem

In the mechanical subsystem, the motion of mass 'm' under excitation force f(t) = Fsin(ωt), is opposed by damping force $c_m\dot{x}$, spring force kx, and magnetic force f_m, hence one can write, accelerating force on the mass as,

$$m\ddot{x} = -kx - c_m\,\dot{x} - f_m + Fsin(\omega t) \quad \text{Or} \quad m\ddot{x} + kx + c_m\,\dot{x} + f_m = Fsin(\omega t) \tag{1}$$

The magnetic force f_m produced due to relative movement of magnet and coil, is proportional to the proportional to current i in the coil, therefore,

$$f_m \propto i \quad \text{Or} \quad f_m = \alpha i \tag{2}$$

Where, α is the transduction parameter.

The relative movement of coil and magnet converts kinetic or vibrational energy of mass of the main system into electrical energy producing the voltage across the coil terminals. Because which also relates the voltage 'e' generated across the coil terminals to relative velocity of magnet and coil, thus it acts as a transducer, therefore,

$$e \propto \alpha \quad \text{Or} \quad e = \alpha\dot{x} \tag{3}$$

The value of α is determined by electrical coil design and flux density of the magnet selected. The value of α is calculated as,

$$\alpha = 2\pi nrB \tag{4}$$

Where, n is the effective number of turns in the coil, r is radius of coil, B is the uniform radial magnetic field strength in the annular gap between coil and magnet (A brief derivation of 'α' is given in appendix I). Using equation (2) in equation (1), we get

$$m\ddot{x} + kx + c_m\,\dot{x} + \alpha i = fsin(\omega t)$$

$$m\frac{d^2x}{dt^2} + c_m\frac{dx}{dt} + kx + \alpha i = fsin(\omega t) \tag{5}$$

2.2 Electrical subsystem

According to Kirchhoff's law, for electrical subsystem, one can write,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i.dt - e = 0$$

Using equation (3) in above equation, we get

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i.dt - \alpha\dot{x} = 0 \tag{6}$$

Differentiating eq. (6) with respect to time 't', eq. (6) becomes,

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i - \alpha\frac{d^2x}{dt^2} = 0 \tag{7}$$

2.3 Transmissibility Equations

Taking the displacement x(t) of mass m as $x = Xe^{j\omega t}$, current in the coil i as $i = Ie^{j\omega t}$ and the excitation force as $f = Fe^{j\omega t}$, one can obtain equations (5) and (7) in the form as,

$$-mX\omega^2 + jc_mX\omega + kX + \alpha I = F \tag{8}$$

$$-LI\omega^2 + jRI\omega + \frac{1}{C}I + \alpha X\omega^2 = 0 \tag{9}$$

From equations (8) and (9) the amplitude X of the mass displacement and amplitude I of current are obtained as,

$$X = \frac{F - \alpha I}{(-m\omega^2 + jc_m\omega + k)} \tag{10}$$

$$I = \frac{-\alpha X\omega^2}{(-L\omega^2 + jR\omega + \frac{1}{C})} \tag{11}$$

Putting the value of I from equation (11) in equation (10) we get,

$$X(-m\omega^2 + jc_m\omega + k) + \alpha\left[\frac{-\alpha X\omega^2}{\left(-L\omega^2 + jR\omega + \frac{1}{C}\right)}\right] = F$$

$$X(-m\omega^2 + jc_m\omega + k)\left(-L\omega^2 + jR\omega + \frac{1}{C}\right) - \alpha^2X\omega^2 = F\left(-L\omega^2 + jR\omega + \frac{1}{C}\right)$$

$$X\left(mL\omega^4 - mR\omega^3 - \frac{m\omega^2}{c} - jc_mL\omega^3 + j^2bR\omega^2 + j\frac{b\omega}{c} - kL\omega^2 + jkR\omega + \frac{k}{C} - \alpha^2\omega^2\right) = F\left(-L\omega^2 + jR\omega + \frac{1}{C}\right)$$

Multiplying numerator and denominator by C/k, amplitude X of mass displacement is given as

$$X = \frac{\frac{F}{k}(-LC\omega^2 + jRC\omega + 1)}{\frac{m}{k}LC\omega^4 - j\frac{m}{k}RC\omega^3 - \frac{m\omega^2}{k} - j\frac{c_m}{k}LC\omega^3 + j^2\frac{c_m}{k}RC\omega^2 + j\frac{c_m\omega}{k} - LC\omega^2 + jRC\omega + 1 - \alpha^2\frac{C}{k}\omega^2} \tag{12}$$

Where, $\omega_1 = \sqrt{\frac{k}{m}}$= natural frequency of mechanical system, $\omega_e = \frac{1}{\sqrt{LC}}$ = natural frequency of electrical system, and defining $\omega_\alpha = \sqrt{\frac{k}{\alpha^2 C}}$,dimensionless parameter, $\xi_1 = \frac{c_m}{2\sqrt{mk}} = \frac{c_m}{2\frac{k}{\omega_1}} = \frac{\omega_1 c_m}{2k}$ = damping ratio of mechanical system, $\xi_e = \frac{R}{2\frac{L}{\sqrt{LC}}} = \frac{\omega_e RC}{2}$ = damping ratio of electrical system, $r_e = \frac{\omega_1}{\omega_e}$ = frequency ratio for eletro-mechanical system, $r_\alpha = \frac{\omega_1}{\omega_\alpha}$ = frequency ratio for coil transducer, and static deflection $\delta_{st} = \frac{F}{k}$

Rearranging the terms in equation (12) we get,

$$X = \frac{\frac{F}{k}\left(-\frac{\omega^2}{\omega_e^2}.\frac{\omega_1^2}{\omega_1^2} + j2\frac{\omega_eRC}{2}.\frac{\omega}{\omega_e}.\frac{\omega_1}{\omega_1} + 1\right)}{\frac{\omega^4}{\omega_e^2\omega_1^2}.\frac{\omega_1^2}{\omega_1^2} - j2\frac{\omega_eRC}{2}.\frac{\omega^3}{\omega_e\omega_1^2}.\frac{\omega_1}{\omega_1} - \frac{\omega^2}{\omega_1^2} - j2\frac{\omega_1c_m}{2k}.\frac{\omega^3}{\omega_1\omega_e^2}.\frac{\omega_1^2}{\omega_1^2} + j^24\frac{\omega_1c_m}{2k}.\frac{\omega_eRC}{2}.\frac{\omega^2}{\omega_1\omega_e}.\frac{\omega_1}{\omega_1} + j2\frac{\omega_1c_m}{2k}.\frac{\omega}{\omega_1} - \frac{\omega^2}{\omega_e^2}.\frac{\omega_1^2}{\omega_1^2} + j2\frac{\omega_eRC}{2}.\frac{\omega}{\omega_e}.\frac{\omega_1}{\omega_1} + 1 - \frac{\omega^2}{\omega_\alpha^2}\frac{\omega_1^2}{\omega_1^2}}$$

$$X = \frac{\frac{F}{k}\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_er_e\frac{\omega}{\omega_1}\right)}{r_e^2\frac{\omega^4}{\omega_1^4} - 2j\xi_er_e\frac{\omega^3}{\omega_1^3} - \frac{\omega^2}{\omega_1^2} - 2j\xi_1r_e^2\frac{\omega^3}{\omega_1^3}. + 4j^2\xi_1\xi_er_e\frac{\omega^2}{\omega_1^2} + 2j\xi_1\frac{\omega}{\omega_1} - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_er_e\frac{\omega}{\omega_1} + 1 - r_\alpha^2\frac{\omega^2}{\omega_1^2}} \tag{13}$$

Rearranging the terms in equation (13) we get amplitude X of the displacement of mass m as,

$$X = \frac{\frac{F}{k}\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_e r_e\frac{\omega}{\omega_1}\right)}{\left(1 - \frac{\omega^2}{\omega_1^2} + 2j\xi_1\frac{\omega}{\omega_1}\right)\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_e r_e\frac{\omega}{\omega_1}\right) - r_\alpha^2\frac{\omega^2}{\omega_1^2}} \qquad (14)$$

By putting the value of X in equation (11) we get,

$$I = \frac{\frac{F}{k}\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_e r_e\frac{\omega}{\omega_1}\right).\alpha\omega^2}{\left(-L\omega^2 + R\omega + \frac{1}{C}\right)\left(1 - \frac{\omega^2}{\omega_1^2} + 2j\xi_1\frac{\omega}{\omega_1}\right)\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_e r_e\frac{\omega}{\omega_1}\right) - r_\alpha^2\frac{\omega^2}{\omega_1^2}}$$

Multiplying numerator and denominator by C/k and after rearranging the terms we get amplitude I of the current,

$$I = \frac{\frac{F}{k}\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_e r_e\frac{\omega}{\omega_1}\right).\alpha^2\frac{C}{k}\omega^2.\frac{\omega_1^2}{\omega_1^2}}{\frac{1}{k}\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_e r_e\frac{\omega}{\omega_1}\right)\left(1 - \frac{\omega^2}{\omega_1^2} + 2j\xi_1\frac{\omega}{\omega_1}\right)\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_e r_e\frac{\omega}{\omega_1}\right) - r_\alpha^2\frac{\omega^2}{\omega_1^2}}$$

$$I = \frac{\frac{F}{\alpha}r_\alpha^2\frac{\omega^2}{\omega_1^2}}{\left(1 - \frac{\omega^2}{\omega_1^2} + 2j\xi_1\frac{\omega}{\omega_1}\right)\left(1 - r_e^2\frac{\omega^2}{\omega_1^2} + 2j\xi_e r_e\frac{\omega}{\omega_1}\right) - r_\alpha^2\frac{\omega^2}{\omega_1^2}} \qquad (15)$$

The equations (14) and (15), after rearranging terms, can be rewritten as,

Motion Transmissibility $\frac{X}{\left(\frac{F}{k}\right)}$ as,

$$\frac{X}{\left(\frac{F}{k}\right)} = \frac{\left(1 - r_e^2\frac{\omega^2}{\omega_1^2}\right) + j\left(2\xi_e r_e\frac{\omega}{\omega_1}\right)}{\left(1 + r_e^2\frac{\omega^4}{\omega_1^4} - r_e^2\frac{\omega^2}{\omega_1^2} - \frac{\omega^2}{\omega_1^2} - 4\xi_1\xi_e r_e\frac{\omega^2}{\omega_1^2} - r_\alpha^2\frac{\omega^2}{\omega_1^2}\right) + j\left(2\xi_1\frac{\omega}{\omega_1} + 2\xi_e r_e\frac{\omega}{\omega_1} - 2\xi_e r_e\frac{\omega^3}{\omega_1^3} - 2\xi_1 r_e^2\frac{\omega^3}{\omega_1^3}\right)} \qquad (16)$$

Current Transmissibility $\frac{I}{\left(\frac{F}{\alpha}\right)}$ as,

$$\frac{I}{\left(\frac{F}{\alpha}\right)} = \frac{r_\alpha^2\frac{\omega^2}{\omega_1^2}}{\left(1 + r_e^2\frac{\omega^4}{\omega_1^4} - r_e^2\frac{\omega^2}{\omega_1^2} - \frac{\omega^2}{\omega_1^2} - 4\xi_1\xi_e r_e\frac{\omega^2}{\omega_1^2} - r_\alpha^2\frac{\omega^2}{\omega_1^2}\right) + j\left(2\xi_1\frac{\omega}{\omega_1} + 2\xi_e r_e\frac{\omega}{\omega_1} - 2\xi_e r_e\frac{\omega^3}{\omega_1^3} - 2\xi_1 r_e^2\frac{\omega^3}{\omega_1^3}\right)} \qquad (17)$$

The equation (16) can be written in the form as,

$$\frac{X}{\left(\frac{F}{k}\right)} = \frac{a_1 + ja_2}{a_3 + ja_4} \quad or \quad \frac{X}{\left(\frac{F}{k}\right)} = \sqrt{\frac{({a_1}^2 + {a_2}^2)}{({a_3}^2 + {a_4}^2)}} \qquad (18)$$

Where, $a_1 = \left(1 - r_e^2\frac{\omega^2}{\omega_1^2}\right)$, $a_2 = \left(2\xi_e r_e\frac{\omega}{\omega_1}\right)$, $a_3 = \left(1 + r_e^2\frac{\omega^4}{\omega_1^4} - r_e^2\frac{\omega^2}{\omega_1^2} - \frac{\omega^2}{\omega_1^2} - 4\xi_1\xi_e r_e\frac{\omega^2}{\omega_1^2} - r_\alpha^2\frac{\omega^2}{\omega_1^2}\right)$, $a_4 = \left(2\xi_1\frac{\omega}{\omega_1} + 2\xi_e r_e\frac{\omega}{\omega_1} - 2\xi_e r_e\frac{\omega^3}{\omega_1^3} - 2\xi_1 r_e^2\frac{\omega^3}{\omega_1^3}\right)$

The equation (17) can be written in the form as,

$$\frac{I}{\left(\frac{F}{\alpha}\right)} = \frac{a_5}{a_3 + ja_4} \quad or \quad \frac{I}{\left(\frac{F}{\alpha}\right)} = \sqrt{\frac{({a_5}^2)}{({a_3}^2 + {a_4}^2)}} \qquad (19)$$

Where, $a_5 = r_\alpha^2\frac{\omega^2}{\omega_1^2}$

2.4 Frequency response curves

From the equation (18) and equation (19), the transmissibility curves in the non-dimensional form have been plotted in figs.(2 to 7), in the range 2.5Hz-25Hz of the excitation and for the values of mechanical damping ratio $\xi_1 = 0.05$, $\xi_1 = 0.1$ and $\xi_1 = 0.15$ when the electrical damping ratio $\xi_e = 0.1$, $\xi_e = 0.15$ and $\xi_e = 0.2$ and $r_e = r_\alpha = 1$.

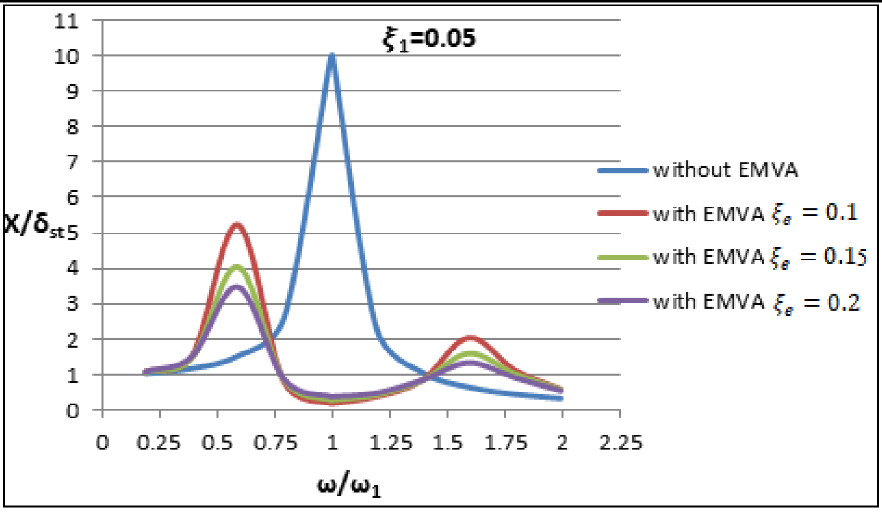

Fig.2 Motion transmissibility (X/δ_{st}) vs. frequency ratio (ω/ω_1)

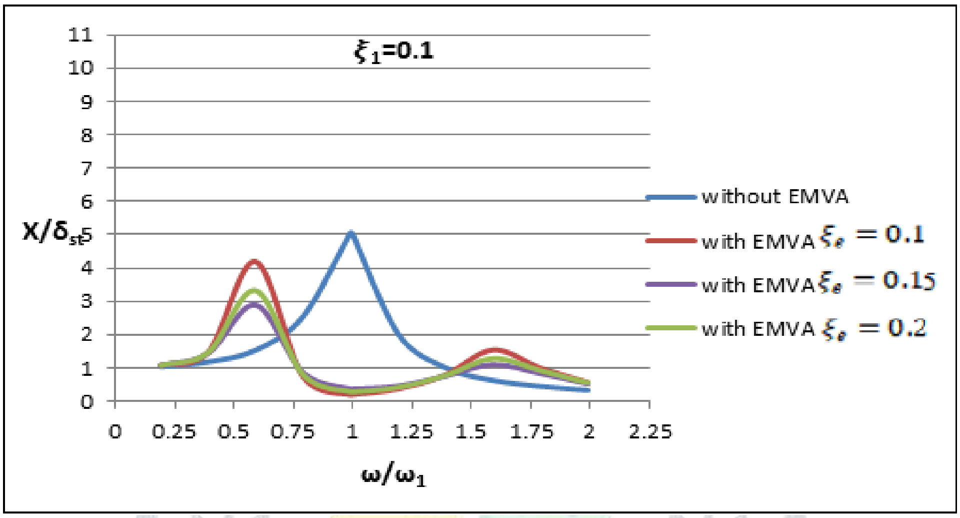

Fig. 3 Motion transmissibility (X/δ_{st}) vs. frequency ratio (ω/ω_1)

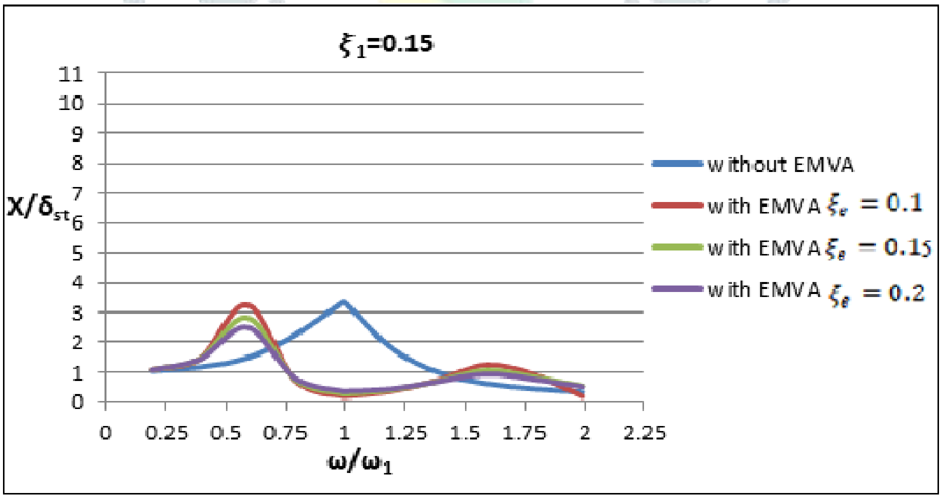

Fig. 4 Motion transmissibility (X/δ_{st}) vs. frequency ratio (ω/ω_1)

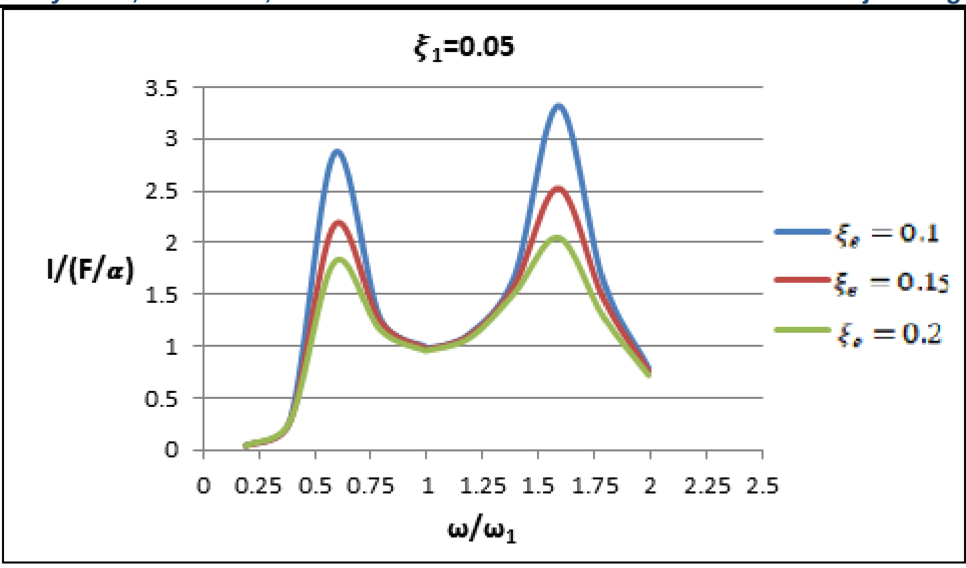

Fig.5 Current transmissibility (I/(F/α)) vs. frequency ratio (ω/ω$_1$)

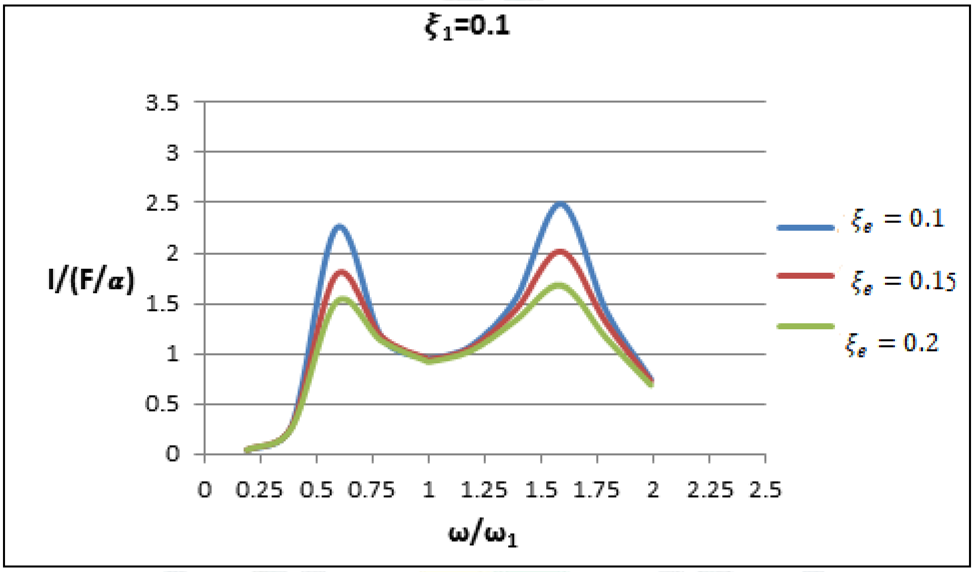

Fig. 6 Current transmissibility (I/(F/α)) vs. frequency ratio (ω/ω$_1$)

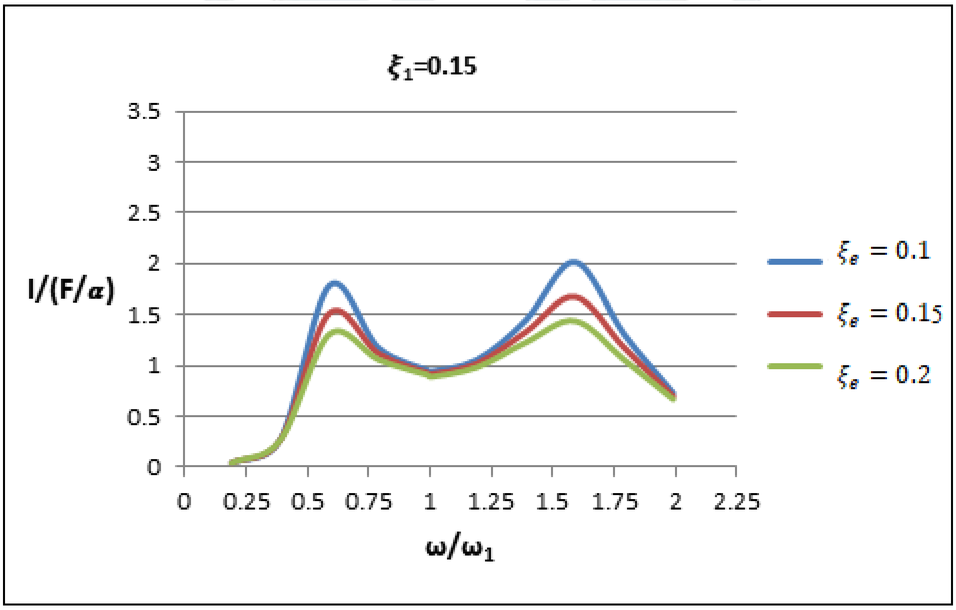

Fig.7 Current transmissibility (I/(F/α)) vs. frequency ratio (ω/ω$_1$)

2.5 Discussion on results of frequency response

From figs. 2 to 4, it is observed that as the mechanical damping ratio $\xi_1$increases, the resonant response of the primary mass system decreases, in a wideband of frequency ratio for a given value of electrical damping ratio ξ_e. As the value of $\xi_e$is increased, substantial reduction in the resonance response of the primary system is achieved. Thus it is seen that, the EMVA is highly effective in controlling the vibrations of primary system with increased bandwidth of operation.

From figs. 5 to 7, it is seen that the resonant values of current in the coil are drastically reduced when the value of ξ_e is increased for a given value of ξ_1, resulting in less electrical energy loss in the electrical circuit in the EMVA.

CONCLUSIONS

i. Compared to conventional TVA, the designed EMVA is more effective in reduction of the resonant amplitude of vibration of the primary mass system at the same time it reduces the resonant amplitude of vibration of absorber system.

ii. The parameter values in electrical resonant RLC circuit containing the value of electrical damping ξ_e play a vital role in the reduction of vibrations of primary mass system. Thus the designed EMVA may result in less weight and space as compared to that of conventional TVA.

iii. From the frequency response analysis, it is observed that there is a reduction in the resonant vibration amplitude of primary mass system in a wide band of frequency ratio for the given value of mechanical damping ratio ξ_1 and electrical damping ratio ξ_e. There is further reduction in the vibration amplitude of primary mass system upto 70% with increase in the values of mechanical and electrical damping ratio.

iv. Also, with increase in the electrical damping ratio ξ_e there is a large reduction upto 60% in the resonant values of current in the electromagnetic coil, resulting less electrical energy loss in the electrical RLC resonant circuit.

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Appendix I : Expression for transduction parameter ‘α’

The expression for transduction parameter α is derived as follows, [7]

The magnetic flux $\emptyset_m$ is given as,

$$\emptyset_m = \iint_A B dA \qquad (I-1)$$

Where, A= area enclosed by the wire loop

The induced voltage (emf) e is given as,

$$e = -\frac{d}{dt}\left(\iint_A B dA\right) \qquad i.e. \qquad e = -\left(B\frac{dA}{dt} + A\frac{dB}{dt}\right) \qquad (I-2)$$

In most applications and for magnet-coil configurations B is constant i.e. $\frac{dB}{dt} = 0$

Using this condition in equation (I-2), we get

$$e = -B\frac{dA}{dt} \qquad (I-3)$$

In case of cylindrical coil-magnet system with n number of turns equation (I-3) becomes,

$$e = -B.n\frac{dA}{dt} = -B.n\frac{d(l' \times x)}{dt} = -B.n.l'\frac{dx}{dt} = -B.n.l'.\dot{x} = -\alpha\dot{x}$$

Where, x= relative displacement between coil and magnet, l'= perimeter of coil = 2π.r, r = the mean radius of the coil

Hence the transduction parameter is given as,

$$\alpha = 2\pi nrB \qquad (I-4)$$