On Radio D-distance in harmonic mean labelling of some basic graphs

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Abstract:

A radio harmonic mean D-distance labelling of a connect graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $d^D(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq diam^D(G) + 1$. The radio D-distance harmonic mean number of $f, rh^D n(f)$ is the maximum number assigned to any vertex of G.

Key words: D-distance, Radio harmonic mean number, Radio D-distance in harmonic mean number

Introduction:

A graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Graph labelling was introduced by Alexander Rosa in 1967. Radio mean labelling was introduced by S.Somasundaram and R.Ponraj in 2004. Harmonic mean labelling was introduced by S.Somasundaram and S.S.Sandhya in 2012.

The concept of D-distance was introduced by D.Reddy Babu et al. The concept of radio D-distance was introduced by T.Nicholas and K.John Bosco in 2017.

Radio labelling (multi-level distance labelling)can be regarded as an extension of distance-two labelling whi8ch is motivated by the channel assignment problem introduced by Hale[6]. Chartrand et al. [2] introduced the concept of radio labelling of graph. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [10]. However Chartrand et al.[2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of cycle. Liu [9] gave the lower bound for the radio number of tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. M. M. Rivera et al. [20]. Gave the radio number of $C_n \times C_n$ the Cartesian product of C_n. In [4] C. Fernandez et al. [20] gave the radio number for star graph, wheel graph, helm graph.

The concept of D-distance was introduced by D. Reddy Babu et al. [17,18,19]. If u, v are vertices of connected graph G, the D-length of a connected $u - v$ path s is defined as $l^D(s) = l(s) + \deg(v) + \deg(u) + \sum \deg(w)$ where the sum runs over all international vertices w of s and $l(s)$ is the length of the path. The D-distance $d^D(u,v)$ between two vertices u, v of a connected graph. G is defined a $d^D(u,v) = \min\{l^D(s)\}$ where the minimum is taken over all $u - v$ path s in G. In other words, $d^D(u,v) = \min\{l(s) + \deg(v) + \deg(u) + \sum \deg(w)$ where the sum runs over all intermediate vertices w in s and minimum is taken over all $u - v$ path s in G.

In this paper we introduce the concept the radio D-distance in harmonic mean number. A radio D-distance in harmonic labelling is a one to one mapping f from $V(G)$ to $\mathbb{N}$ satisfying condition.

$$d^D(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq diam^D(G) + 1.$$

For every $u, v \epsilon V(G)$. The span of a labelling f is the maximum integer that f maps to a vertex of G. The radio D-distance harmonic mean number of G, $rh^D n(G)$ is the lowest span taken over all radio D-distance mean labelling of the graph G. The condition is called radio D-distance mean condition. In this paper we determine the radio harmonic mean number of some well-known graphs. The function $f: V(G) \rightarrow \mathbb{N}$ always represents injective map unless otherwise stated.

Theorem 1.1

The radio D-distance in harmonic mean number of a star $rh^D n\left(K_{1,n}\right) = n + 5, n \geq 3.$

Proof:

Let $v\left(K_{1,n}\right) = \{v_0, v_1, \dots, v_n\}$ be the vertex set, where v_0 is the central vertex and $E\left(K_{1,n}\right) = \{v_0 v_i \ / i = 1,2, \dots, n\}$ be the edge set. It is obvious that $diam^D\left(K_{1,n}\right) = n + 4.$

Hence the radio D-distance in harmonic mean number of a star is $n + 5,\ n \geq 3.$

Theorem 1.2:

The radio D-distance in harmonic mean number of a path $rh^D n = \begin{cases} \frac{5n}{2} - 3 & if\ n\ is\ 2{,}4 \\ \frac{7n}{2} - 6 & if\ n\ is\ even\ and\ n \geq 6 \\ \frac{7(n-1)}{2} - 3 & if\ n\ is\ odd \end{cases}$.

Proof:

Let $V(P_n) = \{v_0, v_1, \dots, v_n\}$ be the vertex set and $E(P_n) = \{v_i v_{i+1} / \ i = 1,2,..,n-1\}$ be the edge set. Obviously $diam^D(P_n) = 3n - 3$.Let $f: v(P_n) \rightarrow \mathbb{N}$ be a radio D-distance harmonic mean labelling. Where $v \in v(G)$.

Hence the radio D-distance in harmonic mean number of a path is $rh^D n = \begin{cases} \frac{5n}{2} - 3 & if\ n\ is\ 2{,}4 \\ \frac{7n}{2} - 6 & if\ n\ is\ even\ and\ n \geq 6 \\ \frac{7(n-1)}{2} - 3 & if\ n\ is\ odd \end{cases}$.

Theorem 1.3:

The radio D-distance in harmonic mean number of a friendship graph $rh^D n\left(C_n^{(3)}\right) \leq \begin{cases} \frac{7(n-1)}{2} + 4 & if\ n\ is\ odd \\ \frac{7n}{2} & id\ n\ is\ even \end{cases}$

Proof:

Let $V\left(C_n^{(3)}\right) = \{v_0, v_1, \dots, v_n\}$ be the vertex set, where v_1 is the central vertex and $E\left(C_n^{(3)}\right) = \{v_i v_i, v_i v_{i+1} \ / \ i = 1,2, \dots, 2n-1\}$ be the edge set. Then $diam^D\left(C_n^{(3)}\right) = 2n + 6$. Let $f: v_1(C_n^{(3)}) \rightarrow \mathbb{N}$ be a radio D-distance in harmonic mean labelling.

Hence the radio D-distance in harmonic mean number of a friendship graph is $rh^D n\left(C_n^{(3)}\right) \leq \begin{cases} \frac{7(n-1)}{2}+4 & if\ n\ is\ odd \\ \frac{7n}{2} & id\ n\ is\ even \end{cases}$.

Theorem 1.4:

The radio D-distance in harmonic mean number of a wheel graph $rh^D n(W_n) \leq \begin{cases} n+5 & if\ 3 \leq n \leq 6 \\ 2n-1 & id\ n \geq 7 \end{cases}$.

Proof:

Let $V(W_n) = \{v_0, v_1, \ldots, v_n\}$ be the vertex set, where v_1 is the central vertex and $E(W_n) = \{v_i v_i, v_i v_{i+1} \ /\ i = 1,2, \ldots, 2n-1\}$ be the edge set. It is obvious that $diam^D(W_n) = n+8$.

Hence $rh^D n(W_n) \leq \begin{cases} n+5 & if\ 3 \leq n \leq 6 \\ 2n-1 & id\ n \geq 7 \end{cases}$.

Theorem 1.5:

The radio D-distance in harmonic mean number of a helm graph $rh^D n(H_n) \leq \begin{cases} \frac{5(n-1)}{2}+10 & if\ n\ is\ odd \\ \frac{5n}{2}+8 & id\ n\ is\ even \end{cases}$.

Proof:

Let $\{v_0, v_1, \ldots, v_n\}$ and $\{u_1, \ldots, u_n\}$ are the vertex set v_1 where is the central vertex and also $\{u_1, \ldots, u_n\}$ are the pendent vertices and $E(H_n) = \{v_1 v_i, v_i u_i \ /\ i = 1,2, \ldots, n\}$ be the edge set. It is obvious that $diam^D(H_n) = n + 14$.

Hence $rh^D n(H_n) \leq \begin{cases} \frac{5(n-1)}{2}+10 & if\ n\ is\ odd \\ \frac{5n}{2}+8 & id\ n\ is\ even \end{cases}$.

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