# Śulbasūtras and Approximate Value of $\sqrt{2}$

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**Abstract :** Śulbasūtras was the oldest mathematical text found in India. The Śulbasūtras contain rich principles of mathematics, basically of ‘geometry’. The outstanding feature of Śulbasūtras is consistency and completeness of geometrical results and application of these results in actual construction shows that Śulbasūtras have deeper significance.

Following statement in Śulbasūtras has great importance because it covers the idea of properties of square. Further it covers the idea of irrational number $\sqrt{2}$.

चतुरस्त्रस्याक्ष्णयारज्जुर्द्विस्तावतीं भूमि करोति ।

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**Meaning**: - “The diagonal cord of a square makes double the area”.

Śulbasūtras stated the value of $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3\times4} - \frac{1}{3\times4\times34}$.

In this paper contain how Indian mathematicians obtain the value $\sqrt{2}$ and the geometrical proof for formula $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3\times4} - \frac{1}{3\times4\times34}$. Hence this value is practically useful and reliable for the geometrical constrictions.

Keywords:- Śulbasūtras, *Śulbasūtraskāras*, *Karaṇī*, Approximate value of $\sqrt{2}$

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## I. Introduction

Baudhāyana, Āpastamba and Kātyāyana Śulbasūtras stated the value of $\sqrt{2}$ in combining two squares of equal areas, in this case square has side$\sqrt{2}\,a$. The diagonal of a square of side a is $\sqrt{2}\,a$.

समस्य द्विकरणी । प्रमाणं तृतीयेन वर्धयेत् तच्चतुर्थेनात्म चतुसर्स्िसशोनेन सविशेषः

lеL; f0ndj.kh A प्रमाणं तृतीयेन वर्धयेत् तच्चतुर्थेनात्म चतुसर्स्िसशोनेन सविशेषः

The measure should be increased by one third of itself which again is increased by its one fourth and diminished by 1/34th of that (second) increment.

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3\times4} - \frac{1}{3\times4\times34}$$

By this method $\sqrt{2}$ = 1.414215 used in finding the diagonal of a square.

The Sanskrit term for the surd is *Karaṇī*. According to Śrīpatī (1039) “The number whose square-root cannot be obtained (exactly) is said to form an irrational quantity (*Karaṇī*)”. Literally *Karaṇī* means “making one” or “produce one”.

Indian used various methods to obtain approximate value of surds using algebra. In this paper we will discuss two methods of finding the approximate value of surds. All these methods show that the Indian Mathematicians knows the importance of finding the approximate value of surds.

In the second century of the Christian era, Umāsvāti (150 A.D.) treated the term “*mūla*” (root) and *Karaṇī* as synonymous.

## II. Approximate value of $\sqrt{2}$ in *Śulbasūtras*

In Indian mathematics the value of $\sqrt{2}$ was first observed in Baudhāyana and Āpastamba and *Śulbasūtras*. For the given square of side a, Baudhāyana and Āpastamba obtained the side d, of a square whose area is double the area of given square. This value of $\sqrt{2}$ was stated as “Increase the measurement (of which *dvi-*

*karaṇī* (side of a square whose area is double than given square) is to be found) by its third part, and again by the fourth part (of this third part) less by the thirty-fourth part of itself (i.e. of this fourth part). The value thus obtained is called saviśeṣa(approximate)".

Kātyāyana states that, If d be the *dvi- karaṇī* of a, that is , if d be the side of a square whose area is double that of the square whose side is a, then according to the rule, d is given by

$$d = a + \frac{a}{3} + \frac{a}{3\times4} - \frac{a}{3\times4\times34}$$

More clearly the diagonal of a square is its *dvi- karaṇī* which is ($\sqrt{2}$ a). This rule gives us the relation between side and the diagonal of a square. Thus we get

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3\times4} - \frac{1}{3\times4\times34}$$

Expressing in decimal fractions we obtain $\sqrt{2}$ = 1.4142156.... and the modern value is $\sqrt{2} =$ 1.414213….. correct up to five decimal places.

The question arises: **how did Baudhāyana, Āpastamba or Kātyāyana find this value of $\sqrt{2}$.** There was no method given in the *Śulbasūtras* but the formula $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3\times4} - \frac{1}{3\times4\times34}$ itself is self explanatory. We will discuss some methods that how this value is calculated in *Śulbasūtras*.

In *Śulbasūtras* many geometrical constructions are given. One of which is, to find a square whose area is equal to the sum (or difference) of areas of two unequal squares. Once square is constructed we can find the side of the square which gives approximate values of surds.

**There was no proof is given in finding the approximate value of surd. We will use best possible hypothesis in finding the value of surd. The *Śulbasūtras* contain geometric construction only. We find the value of surds $(\sqrt{2})$ using geometric construction.**

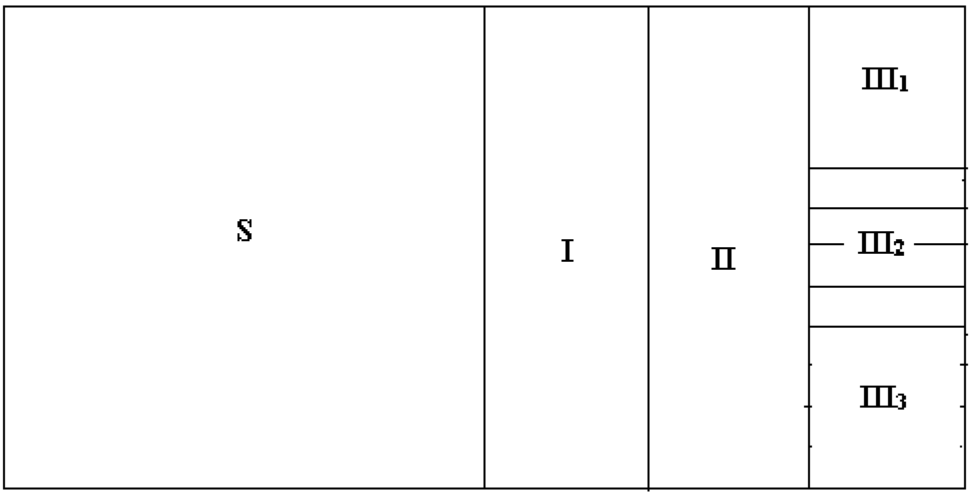
## III. METHOD 1 (OBTAINING A SQUARE FROM TWO SQUARES OF UNIT LENGTH)

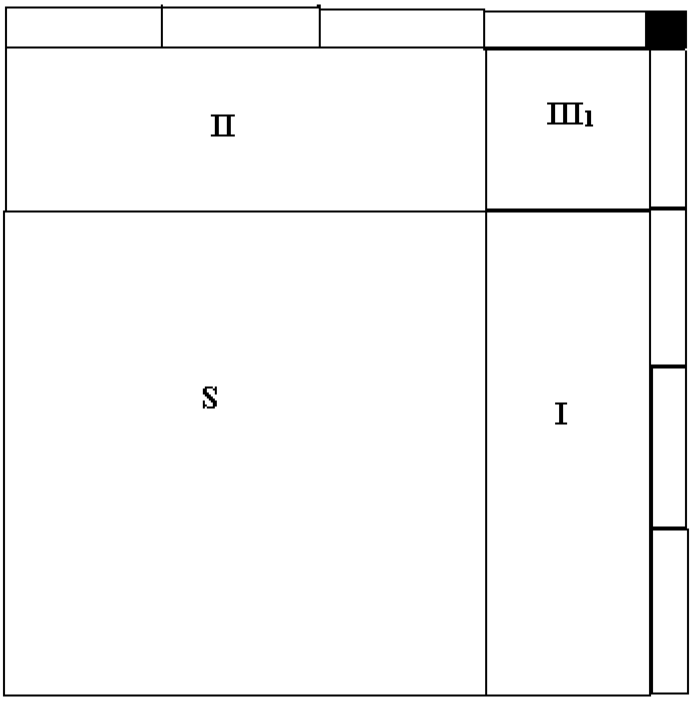
This is most simple and possible hypothesis. Take two squares whose sides are unit length. Divide the second square into three equal strips I, II and III of side (1/3) each. Sub-divide the last strip III into three small squares $III_1$, $III_2$ and $III_3$. (This is possible because we are obtaining nine squares from the original square). Place I along one side of square S and put II along the upper side of square S. Put $III_1$ in the corner as shown in the figure.

Now divide each of the portions $III_2$ and $III_3$ into four equal strips. Placing four and four of them about the square just formed along its east and north side. Now introduce a small square at the north east corner. The larger square will be formed whose side is equal to $\left(1 + \frac{1}{3} + \frac{1}{3\times4}\right)$. Now the square which we obtain is larger than two original squares by an amount $\left(\frac{1}{3\times4}\right)^2$ which is the area of the small square introduce at the corner. Now suppose x is the breadth of small strip then we must have

$2x\left(1 + \frac{1}{3} + \frac{1}{3\times4}\right) - x^2 = \left(\frac{1}{3\times4}\right)^2$ neglecting $x^2$ as too small we get

$x = \frac{1}{3\times4\times34}$ Thus finally we have $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3\times4} - \frac{1}{3\times4\times34}$






## IV. Method 2 (finding the length of a diagonal of a square of side 12)

1) Finding the value of $\sqrt{2}$ by finding the side of square whose area is $2a^2$ where 'a' is a side of a square whose diagonal is$\sqrt{2}a$.

All the authors of *Śulbasūtras* try to obtain a square whose side and diagonal are integers. They reached to a conclusion that such a square whose side and diagonal are integers cannot be obtain. Then what they have done.

They began by assuming two as the measure of a square's side. Then the square of the diagonal is 8. They tried to find the side of a square whose area is 8 which is$\sqrt{8}$. But all *Śulbasūtraskāras* were unsuccessful.

They (*Śulbasūtraskāras*) tried for the next number, taking side of a square 3. The square of diagonal is 18. But they did not obtain$\sqrt{18}$.

Now we find the square of a diagonal nearer to the real square number. For that we construct a table in which first column is side of a square (suppose a), second column is square of a diagonal of square of side a, the third column is nearest square number.

| a | $2a^2$ | Nearest Sq |
|---|---|---|
| 1 | 2 | 1 |
| **2** | **8** | **9** |
| 3 | 18 | 16 |
| 4 | 32 | 36 |
| **5** | **50** | **49** |
| 6 | 72 | 64 |
| 7 | 98 | 100 |
| 8 | 128 | 121 |
| 9 | 162 | 169 |
| 10 | 200 | 196 |

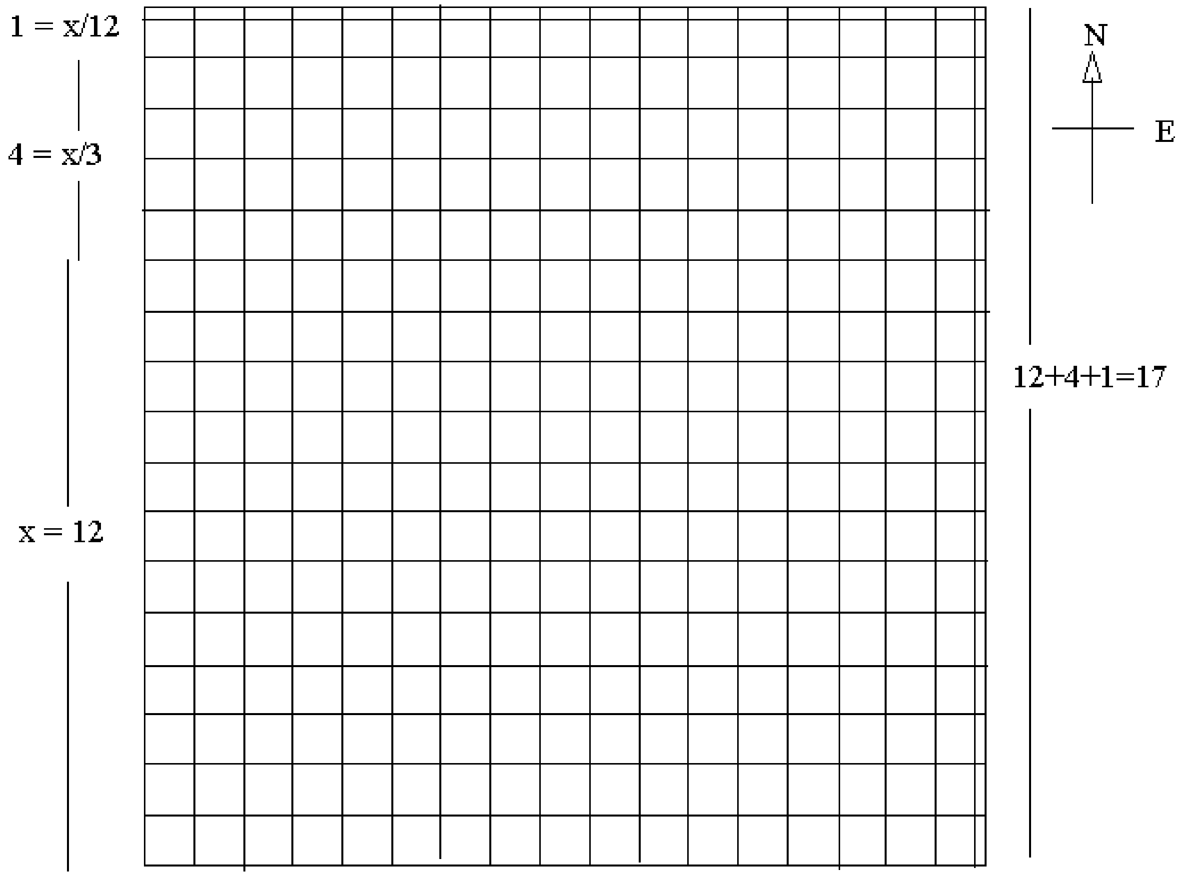
| a | $2a^2$ | Nearest Sq |
|---|---|---|
| 11 | 242 | 256 |
| **12** | **288** | **289** |
| 13 | 338 | 324 |
| 14 | 392 | 400 |
| 15 | 450 | 441 |
| 16 | 512 | 529 |
| 17 | 578 | 576 |
| 18 | 648 | 625 |
| 19 | 722 | 729 |
| 20 | 800 | 784 |

Here we have three cases in which the square of the diagonal of a square differs by 1 from a square of a number. These cases are 8-9, 50-49, 288-289. The last case is most suitable because it contain the largest number.

We can say that (for square of side 12 whose diagonal square is 288 which is nearer to 289 square of 17) the diagonal of a square of which side is 12 was very little shorter than seventeen. We try to reduce 17 in such a way that square of which is almost equal to 288.

Now suppose *Śulbasūtraskāras* drew a square whose side was 17 padas (Unit) long. Divide this square in to 17×17 =289 small squares. Then *Śulbasūtraskāras* shortened the side of square containing 289 small squares, so that its area contains not 289 but 288 small squares. The measure (length) of side of square containing 288 small squares, would be exact measure (length) of diagonal of square whose side is equal to 12 ($12^2+12^2=288$). Otherwise the measure (length) of the side of square containing 288 small squares would be exact square root of 288.

Now the question will arise that how *Śulbasūtraskāras* shorten the side of a square containing 289 small squares? They cut two strips from the two sides of a square (one strip from north side and other strip from east side, as shown in figure). The width of these two strips is so that the area of these two strips is equal to one of the 289 small squares.




Suppose x=12 (side of a square whose length of diagonal have to be found out). Then the length of side of one small square is (x/12). When two strips are cut resulting 34 small rectangles whose are equal to one small square. Here width of rectangle is $\frac{x}{12\times34}$. There is error in calculating the area, of a small square in the north-east corner. Neglecting this error we calculate the length of side of square containing 288 small squares. This side is $17-\frac{x}{12\times34}$. We write 17 as 12+4+1. Now the side of square containing 288 squares is $12\sqrt{2}$.

$12\sqrt{2}=12+4+1-\frac{1}{34}$ put x=12 we write it as $x\sqrt{2}=x+\frac{x}{3}+\frac{x}{12}-\frac{x}{12\times34}$ cancelling x we have $\sqrt{2}=1+\frac{1}{3}+\frac{1}{3\times4}-\frac{1}{3\times4\times34}$. This is the best possible value of $\sqrt{2}$ using geometry.

## V. Conclusion

The value of $\sqrt{2}=1+\frac{1}{3}+\frac{1}{3\times4}-\frac{1}{3\times4\times34}$ which is stated in *Śulbasūtras* was derived from geometrical construction. Hence this value is practically useful and reliable for the geometrical constrictions.

Śulbasūtras was the oldest mathematical text in India. The Śulbasūtras contain rich principles of mathematics, basically of ‘geometry’. The Śulbasūtras were considered as instruction manual for the construction of altars and fireplaces.

The important feature of Śulbasūtras is consistency and completeness of geometrical results. These results have application in actual construction shows that Śulbasūtras have deeper significance.

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