A Study on Time-Frequency analysis and its Representation

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Abstract : Time-frequency analysis has been intensively investigated and developed within the last 20 years. In signal process, time–frequency analysis includes those techniques that study a signal in both the time and frequency domains at the same time, mistreatment numerous time–frequency representations.The practical motivation for time–frequency analysis is that classical harmonic analysis assumes that signals square measure infinite in time or periodic, whereas several signals in follow square measure of short length, and alter considerably over their length. As an example, ancient musical instruments don't manufacture infinite length sinusoids, however instead begin with associate degree attack, then step by step decay. this is often poorly drawn by ancient ways, that motivates time–frequency analysis.A time–frequency illustration (TFR) is usually complex-valued fields over time and frequency, wherever the modulus of the spectrum represents either amplitude or "energy density" (the concentration of the foundation mean sq. over time and frequency).

Keywords:- Signal processing, Time-Frequency analysis, Time-Frequency Representation, Modulus.

1.INTRODUCTION

A signal is defined as any physical quantity that varies with time,space or any independent variable or variables. Mathematically, we describe a sign as a function of 1 or more independent variables.
For example, the function
x1(t)=5t;
describes that varies linearly with the independent variable t(time).
As another example, consider the function
s(x,y)=3x+2xy+10y;
This function describes a signal of two independent variables x and y that could represents the two spatial coordinates in a plane.
the frequency domain refers to the analysis of mathematical functions or signals with reference to frequency, instead of time.Put simply, a time-domain graph shows how a sign changes over time, whereas a frequency-domain graph shows what proportion of the signal lies within each given waveband over a variety of frequencies. A frequency-domain representation also can include information on the phase shift that has got to be applied to every sinusoid so as to be ready to recombine the frequency components to recover the original time signal.
The Fourier transform converts the function's time-domain representation to the function's frequency-domain representation.The component frequencies, spread across the frequency spectrum, are represented as peaks within the frequency domain.
A given function or signal are often converted between the time and frequency domains with a pair of mathematical operators called transforms. An example is that the Fourier transform, which converts a time function into a sum or integral of sine waves of various frequencies, each of which represents a frequency component. The "spectrum" of frequency components is that the frequency-domain representation of the signal. The inverse Fourier transform converts the frequency-domain function back to the time-domain function. A spectrum analyzer may be a tool commonly wont to visualize electronic signals within the frequency domain.
The following figure shows how the time domain vs frequency domain representation as follows:-

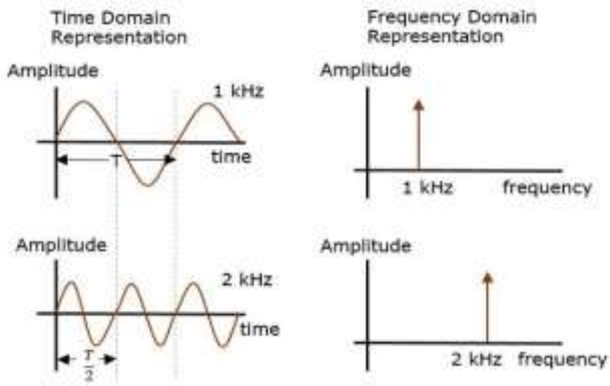

FIG.1:- Time domain Vs frequency domain Representation

In time domain representation of signal consists x-axis as time and y- axis as amplitude whereas in the frequency domain representation of same signal consists x-axis as frequency and y- axis as amplitude however both representations gives of same information in different domains.The main reason to migrate from one domain to another domain is to perform the task at hand in an easier manner.

2.TIME FREQUENCY ANALYSIS

The Fourier transform provides spectral content of a signal. It has been a valuable tool in various applications. However, for nonstationary signals the Fourier transform cannot give satisfactory results since the information about frequency components variations in time is required.

Furthermore, it can happen that two different signals have the same spectral contents, as illustrated in Figures 2.1(a) and 2.2(b). Based on Figure 2.3(c), however, we can conclude that the time-frequency representations of the two signals are quite different. This example is a simple illustration of the importance of time-frequency analysis for signals whose spectral contents vary with time. Various time-frequency distributions are used for this purpose. The ideal time-frequency representation can be described as

where a signal of the form

is considered. This representation provides the signal local energy distribution, as well.

$$TF(t,\omega) = 2\pi A^2 \delta(\omega - \Phi'(t))$$

$$\boxed{x(t) = Ae^{j\Phi(t)}}$$

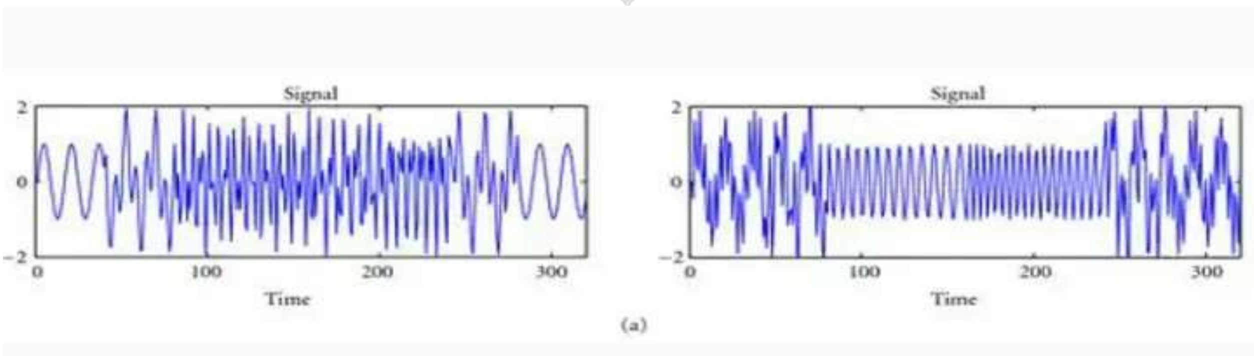

FIG. 2.1 Nonstationary signals. (a) Time domain representations.

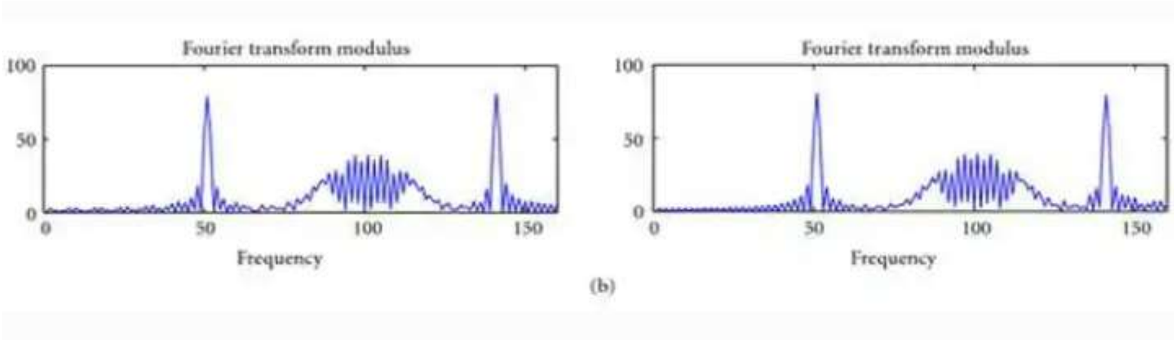

FIG.2.2 (b) Fourier domain representations.

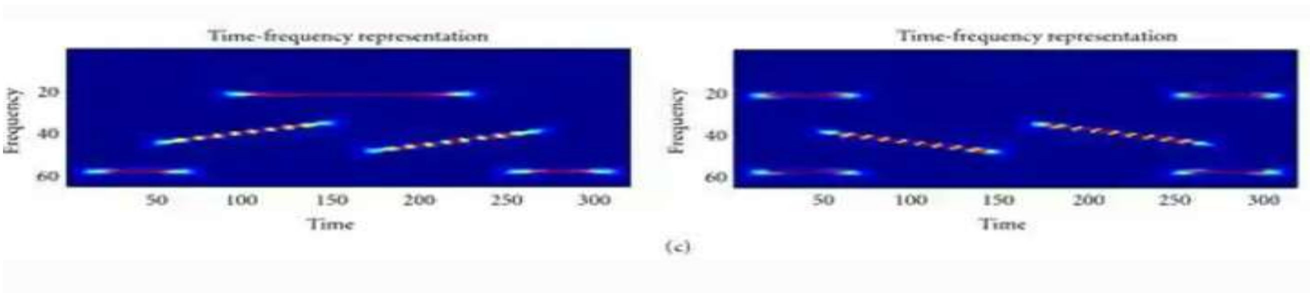

FIG.2.3 (c) Spectral components' variations along the time axes.

3. The Short-Time Fourier Transform

The simplest and most commonly used time-frequency representation is obtained by using the short-time Fourier transform (STFT) defined as

Thus, it is a windowed version of the Fourier transform. The sliding window function is denoted by , where τ is the lag coordinate. An illustration of the STFT calculation is shown in Figure 3.1.

$$STFT(t,\omega) = \int_{-\infty}^{\infty} x(t+\tau)w(\tau)e^{-j\omega\tau}d\tau$$

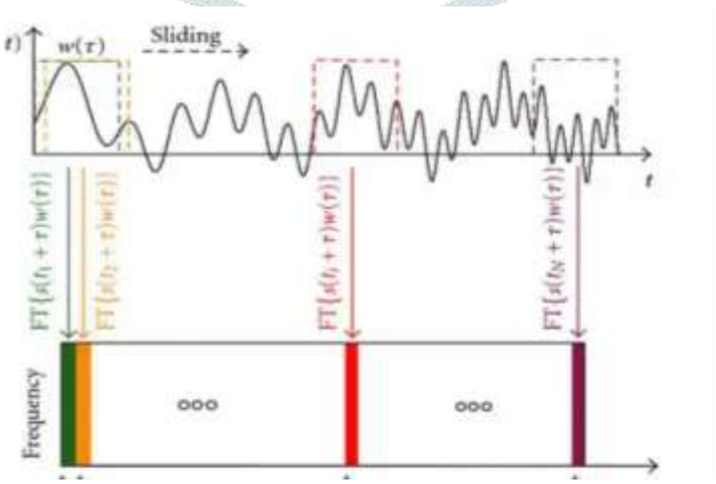

FIG. 3.1 Illustration of STFT Calculation.

Note that the spectral content is calculated for each windowed part of the signal. The central point of the sliding window is the time instant for which the spectrum is calculated .

The energetic version of the STFT is known as the spectrogram. It can be written as

The spectrogram satisfies the marginal properties

$$\begin{aligned} \mathrm{SPEC}(t,\omega) &= |\mathrm{STFT}(t,\omega)|^2 \\ &= 2\pi A^2 W(\omega - \Phi'(t)) *_{\omega} \mathrm{FT}\{e^{j(Q(t,\tau))}\}, \end{aligned}$$

The total energy is obtained as

$$\int_t \mathrm{SPEC}(t,\omega)dt = |X(\omega)|^2,$$

$$\frac{1}{2\pi}\int_{\omega} \mathrm{SPEC}(t,\omega)d\omega = |x(t)|^2.$$

$$Ex = \frac{1}{2\pi}\int_t\int_{\omega} \mathrm{SPEC}(t,\omega)dt\,d\omega$$

4.RESULTS

From the above a signal can be represented in both time domain and frequency domain without changing the information present in the signal.Here we analysed a signal by using short time Fourier analysis(STFA) in these we applied a window of fixed resolution which is sliding over the original signal that makes the signal of small enough segments which is represented by time vs frequency.The energy can also be calculated by using the spectrogram.

5.LIMITATIONS

- A question that naturally arises at this point is whether there exist a single representation that would be ideal for any signal at hand. The answer is no, hence a number of time-frequency distributions have been introduced.
- A trade-off is need between size of the window and resolution.
- Once we choose a particular size for the time window then it will be same for all frequencies of the signal.

6.CONCLUSION & DISCUSSION

In this paper we can conclude that a signal can be analysed in time and frequency by using many representations.Here ,we analysed a signal by using short-time Fourier transform of fixed resolution window which is not possible to change over the time and it gives a compromise between time-based and frequency based views of a signal both time and frequency are represented in limited precision.The precision is determined by the size of the window.Wavelet transform overcomes the preset resolution problem of the short time Fourier transform by using a variable length window.

7.REFERENCES

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