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Design and Analysis of a stiffened-type prismatic pressure vessel

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Abstract - Pressure vessels are used for various purposes such as reservoirs and storage vessels of lethal or non-lethal fluids, nuclear reactor, heat exchanger etc. In general, cylindrical vessel is popular as pressure vessel, due to its simple and robust structure. However cylindrical pressure vessel has a limitation of low volume efficiency compared to prismatic pressure vessel. Volume efficiency of cylindrical pressure vessels is approximately 25 ~ 50% lower than that of prismatic pressure vessels. Even though, prismatic pressure vessels enjoy better volumetric efficiency, it is difficult to design, as the developed bending stress is much higher than membrane stress, whereas in cylindrical vessel membrane stress is predominant. In the present work, a pressure vessel made from stiffened panel is designed and analyzed analytically and numerically. An orthotropic plate approach is used to find the stiffness parameters of the stiffened panel. The loading on the plates is determined by the principle of superposition theory. Levy's method of plate theory is used to solve the governing equations of plates. Along with analytical method, Finite Element Method is also used to study the deformation behavior and stress levels of the prismatic vessels made from stiffened panel. By varying the stiffening properties of the stiffened panels, the response behavior of the prismatic vessel is investigated using both analytical and FEM method. A comparative study of the outcomes of the analytical and FEM method is performed.

Key Words: Prismatic pressure vessel, Levy Plate theory, Membrane stress, Bending stress

1.INTRODUCTION

Pressure vessel is very commonly used in different industries such as, refinery, petrochemical, nuclear power plant, thermal power plant to store lethal or non-lethal liquid or gases at a higher internal pressure. Further heat exchanger is also considered as unfired pressure vessel as it is subjected to high pressure. Pressure vessels are designed in different shapes such as cylindrical, spherical and prismatic. Out of the three, cylindrical vessel is most popular as it is easy to design. However, there is a recent trend of space optimization to reduce the cost of civil construction of the plant. This requirement is also applicable to Gas turbine and Compressor auxiliary heat exchangers which need to be mounted in a very confined space. To meet the minimum space requirement prismatic pressure vessel is most recommended, as the prismatic shape has higher volumetric efficiency compared to other shapes.

The major difficulty in designing a prismatic vessel, is to restrict the very high bending stress within the allowable stress limit of the material. Usage of isotropic plate leads to very high required thickness of the plate. In case of designing gigantic size prismatic vessel, the required plate thickness appears so high, that it becomes impractical for manufacturing. In such cases, the internal volume of the prismatic box is subdivided into smaller enclosures by placing plates welded to opposite walls of the vessels. Further, the configuration is analyzed using FEM to check the stress levels. Finally prototype testing is performed to obtain the exact deformation of such vessel. The whole process is very expensive and time consuming.

So, the possible solution to the difficulty in designing a prismatic pressure vessel, is to find a material or a structure which shall have high bending stiffness and shall also be light in weight. This kind of requirement i.e. high strength to weight ratio is of very much importance in aviation industry, ship building industry and space research domain. These industries are working on stiffened panels, which are basically combination of plate and grid structures. The grid structure can take very high point loads, and on the other hand plate structure can support the distributed load like fluid pressure. The stiffened panel bending stiffness is equal to the sum of plate bending stiffness and grillage bending stiffness.

In recent years research is going on different types of stiffened panels such as longitudinal stiffening, transverse stiffening, crisscross stiffening and curvilinear stiffening. In longitudinal stiffening the structure is strengthened only in longitudinal direction. In transverse stiffening the structure is strengthened only in transverse direction. In crisscross stiffening the structure is strengthened in two perpendicular directions. In curvilinear stiffening, the stiffeners are of curved shape and location and angle of placement is decided by iterative FEM analysis.

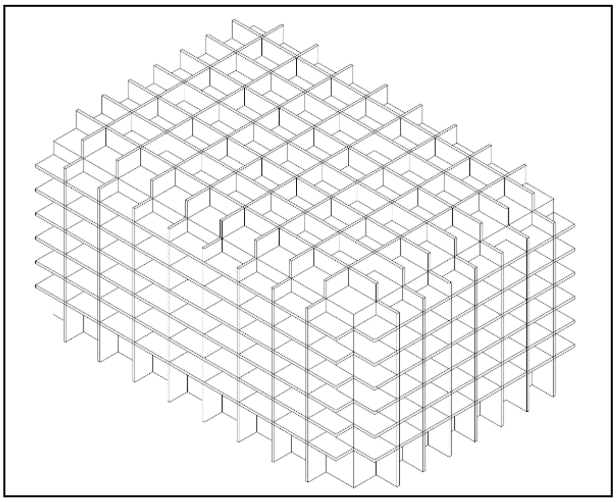


Fig -1: Comparison of the volume efficiency between cylindrical pressure vessel and prismatic pressure vessel.

As per literature, longitudinal stiffened panel, transverse stiffened panel and orthotropic stiffened panel are conceptualized as orthotropic plate. The orthotropic plate properties depend upon both the dimensions and material properties of both plate and stiffeners. In case of curvilinear stiffening, no analytical plate model has been established till date.

In view of the above, in this present work, crisscross stiffening panel is selected as wall of the prismatic pressure vessel as shown in Fig 1. Crisscross stiffening panel is conceptualized as orthotropic plates. A six plate analytical model of prismatic box, is applied to each stiffened panels to establish the boundary conditions. The governing equation of the orthotropic plate is solved using Levy's plate theory. Then, by using principal of superposition, the total deformation behavior of each wall is determined.

2. Analytical Method

2.1 Orthotropic plate approach of Stiffened Panel.

A stiffened panel is composed of longitudinal stiffeners attached to the plating. The stiffeners can be uni-directional or orthogonal. Stiffened plates are asymmetric with the neutral axis positioned usually outside the profile of the plate. A stiffened plates consists of a system of beams interacting

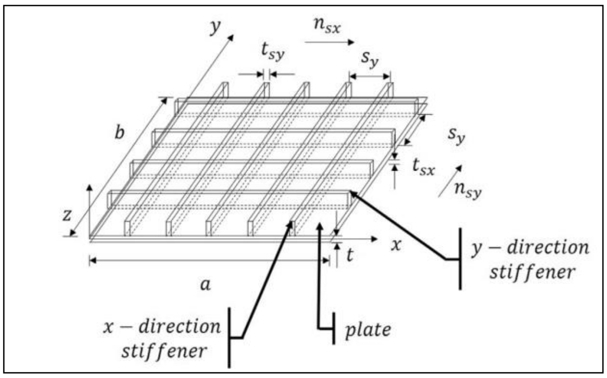


Fig -2: Stiffened Panel

with a uniform thickness plate. A typical stiffened plates is shown in Fig. 2.

As per Paik et al (7), for thin stiffeners welded to thin plates, the plate and the stiffener deforms together and the structure fails grossly. Only such stiffened panel can be analyzed as an equivalent orthotropic plate. As per Paik et al.(7) work, the geometric dissimilarity of Plate and Stiffener is converted into an orthotropic plate. Orthotropic plate properties such as Bending stiffness, torsional stiffness and Extensional stiffness are calculated, based on material property of plate and bar, and the geometric parameters such as length, height and thickness of stiffening bar.

2.2 Six plate analytical model for stiffened-type prismatic box.

As per Zheng Guo et el (5) a prismatic box can have three different plates interacting via three edges. In the present work, a symmetric prismatic box is considered made of stiffened panel, so that opposite stiffened panels are identical. The below figure (Fig 3) depicts the selected co-ordinate system for each plate, namely plate 1 i.e., the top plate, plate 2 i.e., the side plate and plate 3 i.e., the front plate. Plate 1 and Plate 3 are connected through edge A, Plate 1 and Plate 2 are connected through edge B and Plate 2 and Plate 3 are connected through edge C.

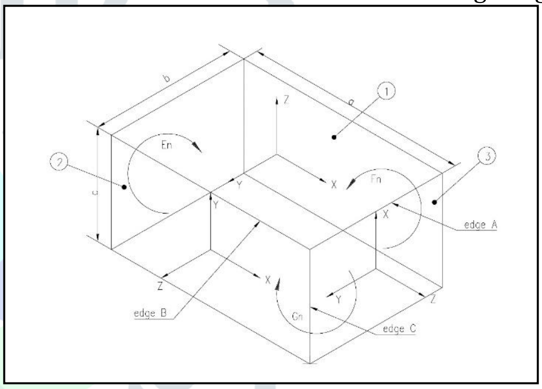


Fig -3: Prismatic pressure vessel and Co-ordinate system

As the vessel is subjected to internal pressure, the plates shall deform outwards, however, each plate shall experience a restoring moment acting at each edge. Since the moment is internal to the structure, it is not known. To find out the internal moment, each plate is considered separately. As shown in Figure 4, the loaded plate is considered as a combination of three different loading cases, namely, uniform transverse load (case A), uniform bending moment at the edge 1 (case B) and uniform bending moment at the edge 2 (case C) separately on a simply supported orthotropic plate. By the method of superposition the displacement can be added to get the final displacement function of the orthotropic plate.

In the present case, all the plates are stiffened with ribs, hence shall have different rigidity coefficients. Hence, the Bending Rigidities of stiffened panel D11, (D_12) and D22 , is calculated As per Paik et al (6).

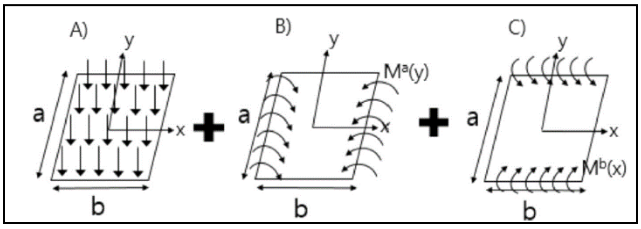


Fig. 4 Principle of superposition

Governing Equation of a rectangular orthotropic plate

$$D_{11}\frac{\partial^4 w_0}{\partial x^4} + 2\widetilde{D_{12}}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_0}{\partial y^4} = q \quad ..(1)$$

Where, $\widetilde{D_{12}}$ = (D12+2D66)

q= distributed load on the plate surface. The equation is solved using Levy solution.

Different load conditions for Plate 1 are as follows:

For loading Case-A & B, a Levy form shall be considered, i.e. plate is simply supported at the edges y=±b/2.

Hence we assume a solution

$$w_0(x,y) = \sum_{n=1,3,5,..}^{\infty} (-1)^{\frac{n-1}{2}} W(x) \cos\left(\frac{n\pi y}{b}\right) \quad ..(2)$$

Case-A, Simply supported plate subjected to uniform distributed load.

$w_0(x,y) = 0$ and $\frac{\partial^2 w(x,y)}{\partial x^2} = 0$ at x=±a/2 ..(3)

Case-B, Simply supported plate subjected to bending moment at the edges x = ±a/2 $w_0(x,y) = 0$ at x= a/2, -a/2 and

$$-D_{11}w_0\left(\frac{a}{2},y\right)_{,xx} - D_{12}w_0\left(\frac{a}{2},y\right)_{,yy} = M_{xx} \quad ..(4)$$

$$M_{xx} = \sum_{n=1,3,5,..}^{\infty} (-1)^{\frac{n-1}{2}} F_n \cos\left(\frac{n\pi y}{b}\right) \quad ..(5)$$

M_{xx} is the assumed bending moment at edge x = ±a/2

Case-C, a different Levy form shall be considered, i.e. plate is simply supported at the edges x = ±a/2.

Hence we assume a solution

$$w_0(x,y) = \sum_{m=1,3,5,..}^{\infty} (-1)^{\frac{m-1}{2}} W(y) \cos\left(\frac{m\pi x}{a}\right) \quad ..(6)$$

The boundary conditions shall be

$$-D_{12}w_0\left(x,\frac{b}{2}\right)_{,xx} - D_{22}w_0\left(x,\frac{b}{2}\right)_{,yy} = M_{yy} \quad ..(7)$$

$$M_{yy} = \sum_{m=1,3,5,..}^{\infty} (-1)^{\frac{m-1}{2}} E_m \cos\left(\frac{m\pi x}{a}\right) \quad ..(8)$$

M_{yy} is the assumed bending moment at edge y = ±b/2

After solving the differential equations,

We find the slope at x=±a/2 of Plate 1 for three different load cases

Case A

$$w_{0_q}\left(\frac{a}{2},y\right)_{,x} = \sum_{n=1,3,5,..}^{\infty} \frac{\hat{q}(-1)^{\frac{n-1}{2}}}{\lambda_1^2 - \lambda_2^2}\left(\lambda_1\lambda_2^2 \tanh\frac{\lambda_1 a}{2} - \lambda_1^2\lambda_2 \tanh\frac{\lambda_2 a}{2}\right)\cos\left(\frac{n\pi y}{b}\right) \quad ..(9)$$

Case B

$$w_{0_f}\left(\frac{a}{2},y\right)_{,x} = \sum_{n=1,3,5,..}^{\infty} \frac{F_n(-1)^{\frac{n-1}{2}}}{D_{11}(\lambda_1^2-\lambda_2^2)}\left(-\lambda_1 \tanh\frac{\lambda_1 a}{2} + \lambda_2 \tanh\frac{\lambda_2 a}{2}\right)\cos\left(\frac{n\pi y}{b}\right) \quad ..(10)$$

Case C

$$w_{0_e}\left(\frac{a}{2},y\right)_{,x} = -\sum_{m=1,3,5,..}^{\infty} \frac{4m\pi E_m}{D_{22}ab} \sum_{p=1,3,5,..}^{\infty} \frac{m(-1)^{\frac{m-1}{2}}\cos\left(\frac{p\pi y}{b}\right)}{(m^2+\lambda_1^2)(m^2+\lambda_2^2)} \quad ..(11)$$

λ_1 and λ_2 are the roots of the differential equation.

Similarly, we can calculate the slope at y=±b/2 of Plate 1 for three different load cases.

In similar way, slopes at all the edges are calculated for Plate 2 and Plate 3.

Total slope calculation at the edges is obtained by summing the slopes for three load cases A,B & C in line with principle of superposition.

For plate 1

The total slope at x = ±a/2 i.e, edge A of box as per fig 4.3 is

$$\theta_{x1} = w_{0_q}\left(\frac{a}{2},y\right)_{,x} + w_{0_f}\left(\frac{a}{2},y\right)_{,x} + w_{0_e}\left(\frac{a}{2},y\right)_{,x} \quad ..(12)$$

The total slope at y = ±b/2 i.e, edge B of box as per fig 4.3 is

$$\theta_{y1} = w_{0_q}\left(x,\frac{b}{2}\right)_{,y} + w_{0_f}\left(x,\frac{b}{2}\right)_{,y} + w_{0_e}\left(x,\frac{b}{2}\right)_{,y} \quad ..(13)$$

Similarly, for plate 2

Total slope at y = ±c/2 shall be i.e at edge B of box as per fig 4.3 is

$$\theta_{y2} = w_{0_q}\left(x,\frac{c}{2}\right)_{,y} + w_{0_f}\left(x,\frac{c}{2}\right)_{,y} + w_{0_e}\left(x,\frac{c}{2}\right)_{,y} \quad ..(14)$$

And total slope at x = ±a/2 shall be i.e at edge C of box as per fig 4.3 is

$$\theta_{x2} = w_{0_q}\left(\frac{a}{2}, y\right), x + w_{0_f}\left(\frac{a}{2}, y\right), x + w_{0_e}\left(\frac{a}{2}, y\right), x \quad ..(15)$$

Similarly, for plate 3

Total slope at y = ±b/2 shall be i.e at edge C of box as per fig 4.3 is

$$\theta_{y3} = w_{0_q}\left(x, \frac{b}{2}\right), y + w_{0_f}\left(x, \frac{b}{2}\right), y + w_{0_e}\left(x, \frac{b}{2}\right), y \quad ..(16)$$

And total slope at x = ±c/2 shall be i.e at edge A of box as per fig 4.3 is

$$\theta_{x3} = w_{0_q}\left(\frac{a}{2}, y\right), x + w_{0_f}\left(\frac{a}{2}, y\right), x + w_{0_e}\left(\frac{a}{2}, y\right), x \quad ..(17)$$

As per work of Zheng Guo [5] , the summation of total slope at one edge of adjacent plates shall be zero.

Hence, for edge A

$$\theta_{x1} + \theta_{x3} = 0 \quad (18)$$

For edge B

$$\theta_{y1} + \theta_{y2} = 0 \quad ..(19)$$

For edge C

$$\theta_{x2} + \theta_{y3} = 0 \quad ..(20)$$

From equations, 18 to 20 a System of linear equations is obtained.

By solving the system of linear equations, the co-efficient on unknown edge moments En, Fn and Gn are obtained.

The co-efficient of unknown moment edges are substituted that of deflection equations due to edges moments.

Adding all the three deflection equations corresponding to three load cases,

$$w = w_{0_q} + w_{0_f} + w_{0_e} \quad ..(21)$$

The total bending moment is calculated as follows

$$M_{xx} = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \quad ...(22)$$

$$M_{yy} = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} \quad ..(23)$$

Step 12.

The total shear load is calculated as follows

$$Q_x = -D_{11}\frac{\partial^3 w}{\partial x^3} - D_{12}\frac{\partial^3 w}{\partial y^3} \quad ..(24)$$

$$Q_y = -D_{12}\frac{\partial^3 w}{\partial x^3} - D_{22}\frac{\partial^3 w}{\partial y^3} \quad ...(25)$$

Bending Stress is calculated as follows

$$\sigma_b = \frac{6 \max(M_{xx}, M_{yy})}{t^2} \quad ...(26)$$

Membrane Stress is calculate as follows

$$\sigma_m = \frac{\max(Q_x, Q_y)}{t} \quad ...(27)$$

Total Stress

$$\sigma_T = \sigma_b + \sigma_b \quad ...(28)$$

2.3. Validation of Analytical Model

Material SA 516 Gr 70

Young's Modulus 202 GPa

Poisson Ratio 0.3

Internal pressure 1 Mpa

Plate thickness =50mm

Prismatic vessel Length varied from 2000mm to 10000mm

Prismatic vessel Breadth =1000mm

Prismatic vessel Height = 1000mm

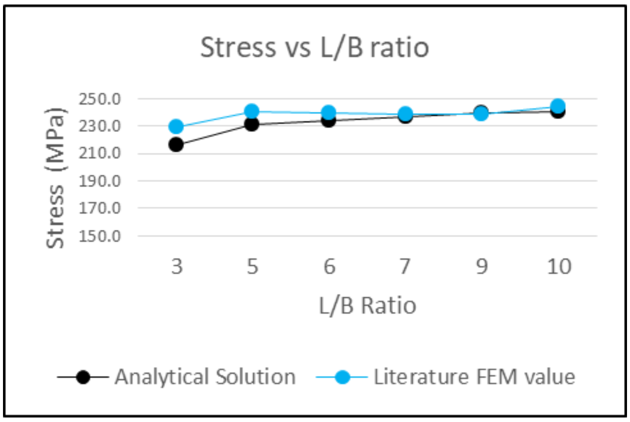
No of stiffeners in X and Y direction for each plate = 3

Stiffener Rib height =50mm

Stiffener Rib thickness 100mm

Table 1 Comparison of Equivalent Stress from Analytical Method and Literature FEM value

Equivalent Stress (Mpa)			
L/B	Analytical Solution	Sanjay et al FEM value	Error (%)
2	183.1	192.22	-4.98
3	215.9	228.79	-5.97
5	231.00	239.96	-3.88
6	234.00	239.49	-2.35
7	236.46	238.98	-1.07
9	239.68	238.09	0.66
10	240.80	243.67	-1.19



Graph 1 Comparison of Equivalent Stress from Analytical Method and Literature FEM value

3. Finite Element Analysis Method

In order to find the complete stress behavior of the actual prismatic pressure vessel made from stiffened panel, the 3D model of the vessel is developed and analyzed by FEM. The purpose of the FEM analysis is to find the localized stress behavior between plate and stiffener and plate to plate junction. The results are mentioned in table 2, table 3 and table 4. The FEM analysis shows that, the maximum stress is developed at the plate to plate junction. Stress at stiffener to plate junction is much less than the plate to plate junction stress.

Using ANSYS Software, FEM of the stiffened box has been performed.

Inputs

Material SA 516 Gr 70

Young's Modulus 201 GPa

Poisson Ratio 0.3

Internal Pr 0.45MPa

Model is prepared in APDL, ANSYS preprocessor. Meshing is performed using SHELL181 element. All the mid planes of the prismatic vessel is constrained, by restricting the in plane movement.

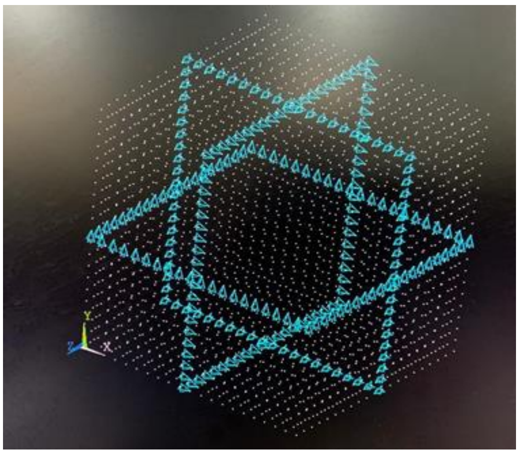


Fig-5 Boundary Condition of prismatic pressure vessel

4. Comparative study of outcome of Orthotropic six plate theory and FEM analysis.

Table.2 Comparison of Equivalent Stress with varying the no of ribs

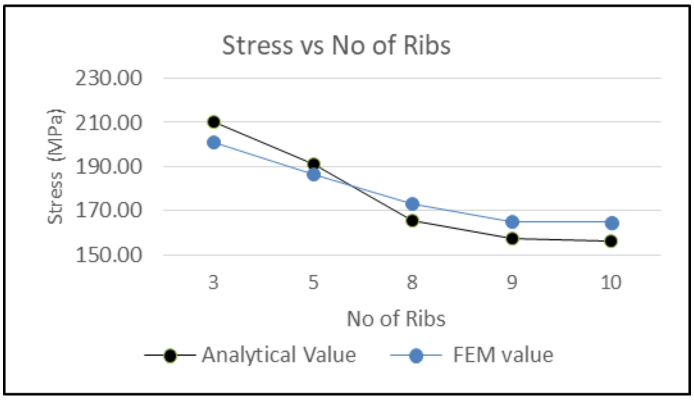
Sno	No of Stiffening Ribs	Orthotropic six plate theory Output (MPa)	FEM Output (MPa)	Error (%)
1	3	210.00	201.04	4.26
2	5	191.00	186.20	2.52
3	8	165.00	173.44	-5.12
4	9	157.00	165.00	-5.10
5	10	152.00	164.88	-8.47

Table.3 Comparison of Equivalent Stress with varying the rib height (mm)

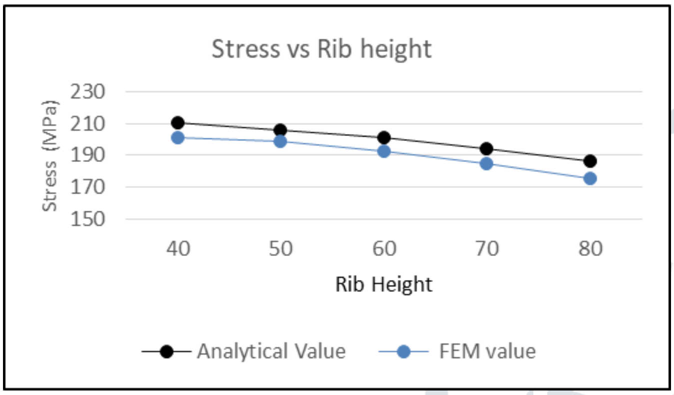
Sl.no	Stiffening rib height (mm)	Orthotropic six plate theory Output (MPa)	FEM Output (MPa)	Error (%)
1	40	210	201.04	4.26
2	50	214	198.64	7.18
3	60	209	192.45	7.92
4	70	201	184.67	8.12
5	80	190	175.25	7.76

Table.4 Comparison of Equivalent Stress with varying the rib thickness (mm)

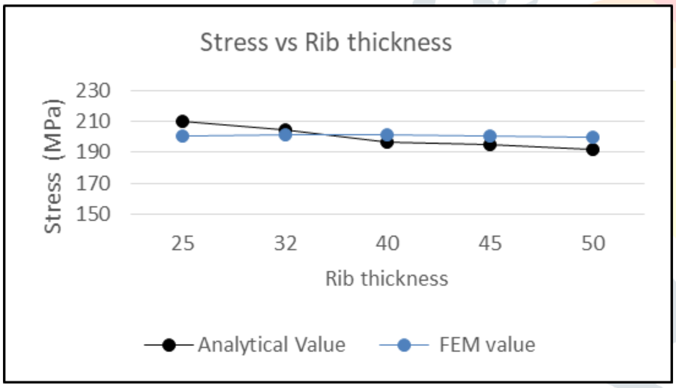
Sl. no	No of Stiffening Ribs	Orthotropic six plate theory Output (MPa)	FEM Output (MPa)	Error (%)
1	3	210	201.04	4.26
2	5	205	201.30	1.80
3	8	197	201.11	-2.09
4	9	191	200.60	-5.03
5	10	187	200.07	-6.99



Graph 2. Total Stress vs No of Ribs



Graph 3. Total Stress vs Rib Height



Graph 4. Total Stress vs Rib thickness

5. Results & Discussion

First, the prismatic pressure vessel is analyzed using the orthotropic plate theory and the results are mentioned in table 1. For validation of the analysis, the outcome is compared with literature of Sanjay et. al. The comparison of values are mentioned in table 1. It is found that errors are within 5%.
After validation, the input variables such as, no of stiffeners, bar thickness, bar height and width are varied. The results are mentioned in table 2, table 3 and table 4. The results indicate that, with more no of stiffeners, with thicker ribs and with increase in rib height the bending stress of the equivalent orthotropic plate model reduces.
Through the table 2, table 3 and table 4, the outcome from orthotropic six plate theory and FEM analysis are compared. The comparative results shows that both follows the similar trend. The orthotropic six plate theory conceptualize the stiffened panel as orthotropic plate and hence doesn't have the capability to obtain the stiffener to plate interaction behavior. However, the same got captured in FEM analysis. It is clear that, the stress value obtained via simplified analytical method is in very good agreement with the FEM analysis.
Through the graph 2, graph 3 and graph 4, the outcome from orthotropic six plate theory and FEM analysis are plotted. The trend of the graph shows that, by increasing the stiffening effect the total stress has reduces.

6. Conclusion

A six-plate analytical model based on orthotropic plate model is established, which is verified against the FEM values reported in literature Sanjay et. al. It is found that the errors are within 5%.
Using both the methodology, analytical and FEM it is found that, with increasing the no of stiffeners, the bending stress of the prismatic vessel reduces. Hence by increasing the no of stiffeners, the wall stiffness of the box increases.
Using both the methodology, analytical and FEM it is found that, with increasing the stiffener height, the bending stress reduces rapidly. Hence by increasing the height of the stiffener, the wall stiffness of the box increases rapidly.
Using both the methodology, analytical and FEM, it is also found that, by increasing the stiffener thickness, bending stress of the prismatic vessel reduces. However, the reduction of bending stress is very less.

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