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Influence of Finite Plasma Conductance in Neutrino Magnetohydrodynamics on propagating modes

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Abstract : In the present study the dispersion relation for different modes of propagation in neutrino modified MHD plasma is derived. The neutrino modified MHD model proposed by Hass et al is taken and modified considering the finite conductance of plasma. It is found that purely transverse waves are not affected by the presence of neutrino beam in finitely conducting plasma, but the dynamics of other modes are greatly modified.

***Keywords* – Neutrinos, Neutrino-Magnetohydrodynamics, dispersion relation, waves**

1. INTRODUCTION

Neutrinos, akin to electrons but electrically neutral, interact via weak forces, significantly impacting astrophysics and particle physics. Their role extends to core-collapse supernovae and cosmic rays. During supernovae, neutrinos carry away substantial energy [1]. Realizing the importance of neutrino beams in astronomical plasma, we've investigated their role in various modes of waves in plasma by deriving a general dispersion relation for various propagation modes in Neutrino Magnetohydrodynamics (NMHD). By incorporating the finite conductance of plasma into Hass et al.'s NMHD model [2][3], we've revealed that this modification has profound implications, markedly altering the dispersion relation.

This work is organized as follows:

In the first section, the basic equation governing the dynamics of propagation waves has been presented for finitely conducting NMHD. In the second section, a general dispersion relation for the various modes of propagation has been derived. In the final section results has been discussed and concluded.

2. BASIC EQUATIONS

Consider a finitely conducting conducting plasma, strongly magnetized plasma system composed of electrons, ions and neutrinos embedded in a uniform magnetic field $\vec{B} = B_o\hat{z}$. Following the MHD given by Hass et al [3], the basic equations for finitely conducting plasma system are
The continuity equation for neutrinos:

$$\frac{\partial n_\nu}{\partial t} + \nabla\cdot(n_{\nu 1}\boldsymbol{v}_{\nu 1}) = 0 \tag{1}$$

Where n_ν, v_ν are neutrino number density and neutrino fluid velocity respectively.
In eq (1) First term $\left(\frac{\partial n_\nu}{\partial t}\right)$ represents the time derivative of the neutrino number density $n_\nu$and second term $(\nabla\cdot(\boldsymbol{v}_\nu n_\nu)$ represents the net flux of neutrinos into or out of the region.
This equation expresses the conservation of neutrino number, stating that the change in neutrino number density in a given region of space and time is equal to the net flux of neutrinos into or out of that region.
The momentum transfer equation for neutrinos:

$$\frac{\partial}{\partial t}(\boldsymbol{p}_\nu) + \boldsymbol{v}_\nu.\nabla(\boldsymbol{p}_\nu) = -\frac{\sqrt{2}G_F}{m_i}\nabla\rho_m \tag{2}$$

Here, G_F is fermi constant of weak interaction, $\boldsymbol{p_\nu} = \frac{\varepsilon_\nu \boldsymbol{v_\nu}}{c^2}$ is relativistic momentum of the neutrino with neutrino beam energy ε_ν. $\rho_m = (m_e n_e + m_i n_i)$ is plasma mass density where $n_{e,j}$ represents the number density of electron (ion) and $m_{e,i}$ is electron (ion) mass respectively.
The continuity and momentum equation for MHD fluid can be written as:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{V}) = 0 \tag{3}$$

$$\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}\right) = -\frac{C_s^2 \nabla \rho_m}{\rho_m} + \frac{1}{\mu_0} \frac{(\nabla \times B) \times \boldsymbol{B}}{\rho_m} + \frac{\boldsymbol{F_\nu}}{m_i} \tag{4}$$

Where, $\boldsymbol{V} = (m_e n_e \boldsymbol{v_e} + m_i n_i \boldsymbol{v}_i)/\rho_m$ is the plasma fluid velocity with $\boldsymbol{v_{e,i}}$ being the velocity of ion(electron) respectively.
In Eq (4), The first term $\frac{C_s^2 \nabla \rho_m}{\rho_m}$ represents the pressure force, second term represents the Lorentz force , the third represents the force acting on plasma due to neutrinos.

The force due to neutrinos can be written as

$$\boldsymbol{F_\nu} = \sqrt{2} G_F \left(\boldsymbol{E_\nu} + \left(\frac{m_i \nabla \times \boldsymbol{B}}{e \mu_0 \rho_m}\right) \times \boldsymbol{B}_\nu\right) \tag{5}$$

Where $\boldsymbol{E_\nu}$ and $\boldsymbol{B}_\nu$ are effective fields induced by the weak interactions.

$$\boldsymbol{E_\nu} = -\boldsymbol{\nabla} n_\nu - \frac{\mathbf{1}}{\boldsymbol{c^2}} \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}} (\boldsymbol{v}_\nu n_\nu)\, , \boldsymbol{B_\nu} = \frac{\mathbf{1}}{\boldsymbol{c^2}} \boldsymbol{\nabla} \times (\boldsymbol{v}_\nu n_\nu) \tag{6}$$

Finally, the Faraday law modified by electroweak force for finitely conducting plasma is

$$\frac{\partial B}{\partial t} = \nabla \times \left(\mathbf{V} \times \boldsymbol{B} - \frac{\boldsymbol{F_\nu}}{e}\right) + \eta \nabla^2 \boldsymbol{B} \tag{7}$$

Where η is resistivity of plasma.

3. GENERAL DISPERSION RELATION

The basic system of equation can be solved for dispersion relation using method of linearization [5] where we can separate the variables into two parts, equilibrium part indicated by a subscript 0 and perturbed part indicated by a subscript 1:

$$\begin{aligned} n_\nu &= n_{\nu 0} + n_{\nu 1}, & \boldsymbol{p}_\nu &= \boldsymbol{p}_{\nu 0} + \boldsymbol{p}_{\nu 1}, \\ \boldsymbol{v}_\nu &= \boldsymbol{v}_{\nu 0} + \boldsymbol{v}_{\nu 1}, & \mathbf{V} &= \mathbf{0} + \mathbf{V_1}, \\ \boldsymbol{B} &= \boldsymbol{B_0} + \boldsymbol{B_1}, & \rho_m &= \rho_{m0} + \rho_{m1} \end{aligned} \tag{8}$$

The equilibrium fluid velocity is taken as zero.

Using these, eq (1) –(7) becomes

$$\frac{\partial n_{\nu 1}}{\partial t} + n_{\nu 0} \nabla \cdot (\boldsymbol{v}_{\nu 1}) + \boldsymbol{v}_{\nu 0} \cdot \nabla (n_{\nu 1}) = 0 \tag{9}$$

$$\frac{\partial}{\partial t}(\boldsymbol{p}_{\nu 1}) + \boldsymbol{v}_{\nu 0}. \nabla(\boldsymbol{p}_{\nu 1}) = -\frac{\sqrt{2} G_F}{m_i} \nabla \rho_{m1} \tag{10}$$

$$\frac{\partial \rho_{m1}}{\partial t} + \rho_{m0} \nabla \cdot (\mathbf{V_1}) = 0 \tag{11}$$

$$\frac{\partial \mathbf{V_1}}{\partial t} = -\frac{C_s^2 \nabla \rho_{m1}}{\rho_{m0}} + \frac{1}{\mu_0}\frac{(\nabla \times \boldsymbol{B_1}) \times \boldsymbol{B_0}}{\rho_m} + \frac{\boldsymbol{F_{v1}}}{m_i} \tag{12}$$

$$\boldsymbol{F_{v1}} = \sqrt{2} G_F \left(-\boldsymbol{\nabla} n_{\nu 1} - \frac{1}{c^2}\frac{\partial}{\partial t}(\boldsymbol{v}_{\nu 1} n_{\nu 0} + \boldsymbol{v}_{\nu 0} n_{\nu 1}) \right) \tag{13}$$

$$\frac{\partial \boldsymbol{B_1}}{\partial t} = \nabla \times \left(\mathbf{V_1} \times \boldsymbol{B_0} - \frac{\boldsymbol{F_{v1}}}{e} \right) + \eta \nabla^2 \boldsymbol{B}_1 \tag{14}$$

Assuming the small amplitude wave with plane wave perturbation proportional to $exp[i\,(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)]$. Using eq (11) perturbed plasma mass density becomes

$$\rho_{m1} = \frac{\rho_{m0}\mathbf{k}\cdot\mathbf{V_1}}{\omega} \tag{15}$$

And using eq(15) in eq(10), the perturbed relative momentum of neutrino becomes:

$$\boldsymbol{p}_{\nu 1} = \frac{\sqrt{2}\rho_{m0} G_F}{m_i \omega}\,\frac{\mathbf{k}\,(\mathbf{k}\cdot\mathbf{V_1})}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})} \tag{16}$$

From relations

$$\boldsymbol{p_v} = \frac{\varepsilon_\nu \boldsymbol{v_v}}{c^2} \;\; and \;\; \varepsilon_\nu = (p_\nu^2 c^2 + m_\nu^2 c^4)^{\frac{1}{2}} \tag{17}$$

We have,

$$\varepsilon_{\nu 1} = \boldsymbol{p_{v0}}\cdot\boldsymbol{p_{v1}} c^2 \;\;, where\; \boldsymbol{p_{v0}} = \frac{\varepsilon_{\nu 0}\boldsymbol{v_{v0}}}{c^2} \tag{18}$$

And

$$\boldsymbol{p_{v1}} = \frac{\varepsilon_{\nu 1}\boldsymbol{v_{v0}}}{c^2} + \frac{\varepsilon_{\nu 0}\boldsymbol{v_{v1}}}{c^2} \tag{19}$$

From eq(16)-eq(19) , we have

$$\boldsymbol{v_{v1}} = \frac{1}{\varepsilon_{\nu 0}}\left(c^2 \boldsymbol{p}_{\mathbf{v1}} - \boldsymbol{v_{v0}}(\boldsymbol{v_{v0}}\cdot \boldsymbol{p}_{\mathbf{v1}})\right)$$

$$= \frac{\sqrt{2}\rho_{m0} G_F}{m_i \omega \varepsilon_{\nu 0}}\,\frac{\left(c^2\mathbf{k} - \boldsymbol{v_{v0}}(\boldsymbol{v_{v0}}\cdot\mathbf{k})\right)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})}(\mathbf{k}\cdot\mathbf{V_1}) \tag{20}$$

The perturbed neutrino density can be obtained using eq (20) in eq(9) as

$$n_{\nu 1} = \frac{\sqrt{2}\rho_{m0} G_F n_{\nu 0}}{m_i \omega \varepsilon_{\nu 0}}\,\frac{(c^2\mathrm{k}^2 - (\boldsymbol{v_{v0}}\cdot\mathbf{k})^2)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2}(\mathbf{k}\cdot\mathbf{V_1}) \tag{21}$$

Now perturbed neutrino force can be obtained using eq (20) and eq(21) in eq(13) as

$$\boldsymbol{F_{v1}} = n_{\nu 0} G_F^2 \frac{2i\rho_{m0}}{m_i \omega \varepsilon_{\nu 0}} \begin{pmatrix} ((\boldsymbol{v_{v0}}\cdot\mathbf{k})^2 - c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{v0}}\cdot\mathbf{k}) + \omega^2)\mathbf{k} \\ + \frac{\omega}{c^2}\left(c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{v0}}\cdot\mathbf{k})\right)\boldsymbol{v_{v0}} \end{pmatrix} \frac{(\mathbf{k}\cdot\mathbf{V_1})}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} \tag{22}$$

Using characteristic neutrino plasma speed as

$$V_n = \left(\frac{2\rho_{m0} n_{\nu 0} G_F^2}{m_i^2 \varepsilon_{\nu 0}}\right)^{1/2}$$

We get:

$$\boldsymbol{F_{v1}} = \frac{i m_i V_n^2}{\omega} \begin{pmatrix} ((\boldsymbol{v_{v0}}\cdot\mathbf{k})^2 - c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{v0}}\cdot\mathbf{k}) + \omega^2)\mathbf{k} \\ + \frac{\omega}{c^2}\left(c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{v0}}\cdot\mathbf{k})\right)\boldsymbol{v_{v0}} \end{pmatrix} \frac{(\mathbf{k}\cdot\mathbf{V_1})}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} \tag{23}$$

From eq (14) , perturbed magnetic field can be written as

$$(i\eta \mathrm{k}^2 - \omega)\boldsymbol{B_1} = \mathbf{k} \times (\mathbf{V_1} \times \boldsymbol{B_0}) - \frac{im_i V_n^2}{c^2 e} \frac{\left(c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{\nu 0}} \cdot \mathbf{k})\right)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} (\mathbf{k} \cdot \mathbf{V_1})(\mathbf{k} \times \boldsymbol{v_{\nu 0}}) \qquad \mathbf{(24)}$$

Using eqs (15) –(24) in eq (13) , the dispersion relation for finitely conducting neutrino plasma can be written as:

$$\begin{aligned} \omega^2\mathbf{V_1} = {} & C_s^2(\mathbf{k} \cdot \mathbf{V_1})\mathbf{k} - \frac{\omega}{(i\eta\mathrm{k}^2 - \omega)} \{\mathbf{k} \times [\mathbf{k} \times (\mathbf{V_1} \times \boldsymbol{V_A})]\} \times \boldsymbol{V_A} \\ & + \frac{iV_n^2 V_A \omega}{(i\eta\mathrm{k}^2 - \omega)c^2\Omega_i} \frac{\left(c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{\nu 0}} \cdot \mathbf{k})\right)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} (\mathbf{k} \cdot \mathbf{V_1})\left(\mathbf{k} \times (\mathbf{k} \times \boldsymbol{v_{\nu 0}})\right) \times \boldsymbol{V_A} \\ & - V_n^2 \begin{pmatrix} \left((\boldsymbol{v_{\nu 0}} \cdot \mathbf{k})^2 - c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{\nu 0}} \cdot \mathbf{k}) + \omega^2 \right)\mathbf{k} \\ + \frac{\omega}{c^2}\left(c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{\nu 0}} \cdot \mathbf{k})\right)\boldsymbol{v_{\nu 0}} \end{pmatrix} \frac{(\mathbf{k} \cdot \mathbf{V_1})}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} \end{aligned} \qquad (25)$$

Where vector Alfven velocity and ion cyclotron frequency is given by

$$\boldsymbol{V_A} = \frac{\boldsymbol{B_0}}{\sqrt{\mu_0 \rho_m}}, \qquad \Omega_i = \frac{eB_0}{m_i} \qquad (26)$$

4. DISCUSSION AND CONCLUSION

As evident from equation (25), the dispersion relation undergoes modification due to the influence of finite conductivity of plasma in presence of neutrino beam. It becomes evident from the general dispersion relation that purely transverse waves with wave vectors perpendicular to both the background magnetic field and the perturbed velocity ((**k⊥B₀** and **k⊥V1**) remain unaffected by the presence of neutrino beams in a finitely conductive plasma. In such cases, the dispersion relation simplifies to $\omega^2 = \mathrm{k}^2 V_A$, indicating that Alfven waves are not influenced by neutrino beams. However, magnetosonic waves are rendered unstable in the presence of neutrino beams within finitely conductive plasma. The angles between the wave vector and the background magnetic field, as well as the angle between the wave vector and the perturbed plasma fluid velocity, play crucial roles in determining the various propagation modes of these waves

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