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Perturbation modes in neutrino modified self-gravitating MHD plasma

Monika Malik,
Associate Professor,
Department of Physics,
Dronacharya Government college, Gurugram, Haryana

Abstract : In the present study the dispersion relation for different modes of propagation in neutrino modified MHD self-gravitating plasma is derived. The neutrino modified MHD model proposed by Hass et al is taken and modified considering the self-gravitating force of plasma. It is found that purely transverse waves do not affect the neutrino beam in self-gravitating plasma, but the dynamics of other modes are greatly modified.

***Keywords* – Neutrinos, Magnetohydrodynamics, dispersion relation, waves**

I. INTRODUCTION

Neutrinos are very similar to electrons but have no electric charge. They interact with matter via weak interactions. Neutrinos play a significant role in astrophysics, particle physics, and our understanding of the universe, especially in processes like, understanding core-collapse supernovae and cosmic ray interactions. The emission of a massive burst of neutrinos during a supernova event carries away a significant portion of the star's energy. Neutrinos influence the surrounding plasma through the weak force's charged current interaction, involving electrons and electron-neutrinos via the exchange of charged bosons known as W^- and W^+[3].

A novel Neutrino Magnetohydrodynamics (NMHD) model is proposed by Hass et al[2] considering the influence of the charged weak current on electron-ion ideal magnetohydrodynamic fluid. In our present investigation we modified his proposed model by taking into account the self-gravitating force of plasma. The behavior of self-gravitational perturbation modes is investigated by Asaduzzamanthe et al [6]. in super dense degenerate quantum plasmas, characterized by the presence of heavy nuclei or elements and degenerate electrons. Our investigation is valid for all perturbation modes such as magnetosonic waves, self-gravitational modes etc. in NMHD.

II. BASIC EQUATIONS

Consider a self-gravitating, highly conducting plasma, strongly magnetized plasma system composed of electrons, ions and neutrinos embedded in a uniform magnetic field $\vec{B} = B_o\hat{z}$. Following the MHD given by Hass et al [3], the basic equations for self-gravitating plasma system are
The continuity equation for neutrinos:

$$\frac{\partial n_\nu}{\partial t} + \nabla \cdot (n_{\nu 1}\boldsymbol{v}_{\nu 1}) = 0 \qquad (1)$$

Where n_ν, v_ν are neutrino number density and neutrino fluid velocity respectively.
In eq (1) First term $\left(\frac{\partial n_\nu}{\partial t}\right)$ represents the time derivative of the neutrino number density n_ν and second term $(\nabla \cdot (\boldsymbol{v}_\nu n_\nu)$ represents the net flux of neutrinos into or out of the region.
This equation expresses the conservation of neutrino number, stating that the change in neutrino number density in a given region of space and time is equal to the net flux of neutrinos into or out of that region.
The momentum transfer equation for neutrinos:

$$\frac{\partial}{\partial t}(\boldsymbol{p}_{\boldsymbol{\nu}}) + \boldsymbol{v}_{\boldsymbol{\nu}}.\nabla(\boldsymbol{p}_{\boldsymbol{\nu}}) = -\frac{\sqrt{2}G_F}{m_i}\nabla\rho_m \qquad (2)$$

Here, G_F is fermi constant of weak interaction, $\boldsymbol{p}_{\boldsymbol{\nu}} = \frac{\varepsilon_\nu \boldsymbol{v}_{\boldsymbol{\nu}}}{c^2}$ is relativistic momentum of the neutrino with neutrino beam energy ε_ν. $\rho_m = (m_e n_e + m_i n_i)$ is plasma mass density where $n_{e,j}$ represents the number density of electron (ion) and $m_{e,i}$ is electron (ion) mass respectively.
The continuity and momentum equation for MHD fluid can be written as:

$$\frac{\partial\rho_m}{\partial t} + \nabla\cdot(\rho_m\mathbf{V}) = 0 \qquad (3)$$

$$\left(\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right) = -\frac{C_s^2\nabla\rho_m}{\rho_m} + \frac{1}{\mu_0}\frac{(\nabla\times B)\times\boldsymbol{B}}{\rho_m} + \frac{\boldsymbol{F}_{\boldsymbol{\nu}}}{m_i} + \nabla\Phi \qquad (4)$$

Where, $\boldsymbol{V} = (m_e n_e \boldsymbol{v}_{\boldsymbol{e}} + m_i n_i \boldsymbol{v}_i)/\rho_m$ is the plasma fluid velocity with $\boldsymbol{v}_{\boldsymbol{e},\boldsymbol{i}}$ being the velocity of ion(electron) respectively. Φ is the gravitational potential.
In Eq (4), The first term $\frac{C_s^2\nabla\rho_m}{\rho_m}$ represents the pressure force, second term represents the Lorentz force , the third represents the force acting on plasma due to neutrinos. The last term represents the contribution due to gravitational force which can be obtain from passion equation.

$$\nabla^2\Phi = -4\pi G\rho_m \qquad (5)$$

The force due to neutrions can be written as

$$\boldsymbol{F}_{\boldsymbol{\nu}} = \sqrt{2}G_F\left(\boldsymbol{E}_{\boldsymbol{\nu}} + \left(\frac{m_i\nabla\times\boldsymbol{B}}{e\mu_0\rho_m}\right)\times\boldsymbol{B}_\nu\right) \qquad (\mathbf{6})$$

Where $\boldsymbol{E}_{\boldsymbol{\nu}}$ and $\boldsymbol{B}_\nu$ are effective fields induced by the weak interactions.

$$\boldsymbol{E}_{\boldsymbol{\nu}} = -\boldsymbol{\nabla}n_\nu - \frac{\mathbf{1}}{\boldsymbol{c^2}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}(\boldsymbol{v}_\nu n_\nu)\,, \boldsymbol{B}_{\boldsymbol{\nu}} = \frac{\mathbf{1}}{\boldsymbol{c^2}}\boldsymbol{\nabla}\times(\boldsymbol{v}_\nu n_\nu) \qquad (\mathbf{7})$$

Finally, the Faraday law modified by electroweak force is

$$\frac{\partial B}{\partial t} = \nabla\times\left(\mathbf{V}\times\boldsymbol{B} - \frac{\boldsymbol{F}_{\boldsymbol{\nu}}}{e}\right) \qquad (8)$$

III. GENERAL DISPERSION RELATION

The basic system of equation can be solved for dispersion relation using method of linearization [5] where we can separate the variables into two parts, equilibrium part indicated by a subscript 0 and perturbed part indicated by a subscript 1:

$$\begin{gathered} n_\nu = n_{\nu 0} + n_{\nu 1},\ \boldsymbol{p}_\nu = \boldsymbol{p}_{\nu 0} + \boldsymbol{p}_{\nu 1}, \\ \boldsymbol{v}_\nu = \boldsymbol{v}_{\nu 0} + \boldsymbol{v}_{\nu 1}, \mathbf{V} = \mathbf{0} + \mathbf{V_1}, \\ \boldsymbol{B} = \boldsymbol{B_0} + \boldsymbol{B_1}, \rho_m = \rho_{m0} + \rho_{m1}, \Phi = 0 + \Phi_1 \end{gathered} \qquad (9)$$

The equilibrium fluid velocity and equilibrium gravitational potential are taken as zero.
Using these, eq (1) –(8) becomes

$$\frac{\partial n_{\nu 1}}{\partial t} + n_{\nu 0}\nabla\cdot(\boldsymbol{v}_{\nu 1}) + \boldsymbol{v}_{\nu 0}\cdot\nabla(n_{\nu 1}) = 0 \qquad (10)$$

$$\frac{\partial}{\partial t}(\boldsymbol{p}_{\nu 1}) + \boldsymbol{v}_{\nu 0}.\nabla(\boldsymbol{p}_{\nu 1}) = -\frac{\sqrt{2}G_F}{m_i}\nabla\rho_{m1} \qquad (11)$$

$$\frac{\partial \rho_{m1}}{\partial t} + \rho_{m0}\nabla\cdot(\mathbf{V_1}) = 0 \tag{12}$$

$$\frac{\partial \mathbf{V_1}}{\partial t} = -\frac{C_s^2\nabla\rho_{m1}}{\rho_{m0}} + \frac{1}{\mu_0}\frac{(\nabla\times\boldsymbol{B_1})\times\boldsymbol{B_0}}{\rho_m} + \frac{\boldsymbol{F_{v1}}}{m_i} + \nabla\Phi_1 \tag{13}$$

$$\nabla^2\Phi_1 = -4\pi G\rho_{m1} \tag{14}$$

$$\boldsymbol{F_{v1}} = \sqrt{2}G_F\left(-\boldsymbol{\nabla} n_{\nu1} - \frac{1}{c^2}\frac{\partial}{\partial t}(\boldsymbol{v}_{\nu1}n_{\nu0} + \boldsymbol{v}_{\nu0}n_{\nu1})\right) \tag{15}$$

$$\frac{\partial \boldsymbol{B_1}}{\partial t} = \nabla\times\left(\mathbf{V_1}\times\boldsymbol{B_0} - \frac{\boldsymbol{F_{v1}}}{e}\right) \tag{16}$$

Assuming the small amplitude wave with plane wave perturbation proportional to $exp[i\,(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$. Using eq (12) perturbed plasma mass density becomes

$$\rho_{m1} = \frac{\rho_{m0}\mathbf{k}\cdot\mathbf{V_1}}{\omega} \tag{17}$$

And using eq(17) in eq(11), the perturbed relative momentum of neutrino becomes:

$$\boldsymbol{p}_{\nu1} = \frac{\sqrt{2}\rho_{m0}G_F}{m_i\omega}\,\frac{\mathbf{k}\,(\mathbf{k}\cdot\mathbf{V_1})}{(\omega - \boldsymbol{v}_{\nu0}.\mathbf{k})} \tag{18}$$

From relations:

$$\boldsymbol{p_v} = \frac{\varepsilon_\nu \boldsymbol{v_v}}{c^2}\ and\ \varepsilon_\nu = (p_\nu^2c^2 + m_\nu^2c^4)^{\frac{1}{2}} \tag{19}$$

We have ,

$$\varepsilon_{\nu1} = \boldsymbol{p_{v0}}\cdot\boldsymbol{p_{v1}}c^2\ \ where\ \boldsymbol{p_{v0}} = \frac{\varepsilon_{\nu0}\boldsymbol{v_{v0}}}{c^2} \tag{20}$$

And

$$\boldsymbol{p_{v1}} = \frac{\varepsilon_{\nu1}\boldsymbol{v_{v0}}}{c^2} + \frac{\varepsilon_{\nu0}\boldsymbol{v_{v1}}}{c^2} \tag{21}$$

On solving for $\boldsymbol{v_{v1}}$ using eq(18)-eq(21) , we have

$$\boldsymbol{v_{v1}} = \frac{1}{\varepsilon_{\nu0}}\left(c^2\boldsymbol{p}_{\ \mathbf{v1}} - \boldsymbol{v_{v0}}(\boldsymbol{v_{v0}}\cdot\boldsymbol{p}_{\ \mathbf{v1}})\right)$$

$$= \frac{\sqrt{2}\rho_{m0}G_F}{m_i\omega\varepsilon_{\nu0}}\,\frac{\left(c^2\mathbf{k} - \boldsymbol{v_{v0}}(\boldsymbol{v_{v0}}\cdot\mathbf{k})\right)}{(\omega - \boldsymbol{v}_{\nu0}.\mathbf{k})}\,(\mathbf{k}\cdot\mathbf{V_1}) \tag{22}$$

The perturbed neutrino density can be obtained using eq (22) in eq(10) as

$$n_{\nu1} = \frac{\sqrt{2}\rho_{m0}G_Fn_{\nu0}}{m_i\omega\varepsilon_{\nu0}}\,\frac{(c^2\mathrm{k}^2 - (\boldsymbol{v_{v0}}\cdot\mathbf{k})^2)}{(\omega - \boldsymbol{v}_{\nu0}.\mathbf{k})^2}\,(\mathbf{k}\cdot\mathbf{V_1}) \tag{23}$$

Now perturbed neutrino force can be obtained using eq (22) and eq(23) in eq(15) as

$$\boldsymbol{F_{v1}} = n_{\nu0}G_F^2\frac{2i\rho_{m0}}{m_i\omega\varepsilon_{\nu0}}\begin{pmatrix}((\boldsymbol{v_{v0}}\cdot\mathbf{k})^2 - c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{v0}}\cdot\mathbf{k}) + \omega^2)\mathbf{k}\\ +\frac{\omega}{c^2}(c^2\mathrm{k}^2 - \omega(\boldsymbol{v_{v0}}\cdot\mathbf{k}))\boldsymbol{v_{v0}}\end{pmatrix}\frac{(\mathbf{k}\cdot\mathbf{V_1})}{(\omega - \boldsymbol{v}_{\nu0}.\mathbf{k})^2} \tag{24}$$

Using characteristic neutrino plasma speed as

$$V_n = \left(\frac{2\rho_{m0} n_{\nu 0} G_F^2}{m_i^2 \varepsilon_{\nu 0}}\right)^{1/2}$$

We get

$$\boldsymbol{F}_{\nu 1} = \frac{i m_i V_n^2}{\omega}\left(\begin{matrix}((\boldsymbol{v}_{\nu 0}\cdot\mathbf{k})^2 - c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}) + \omega^2)\mathbf{k} \\ +\frac{\omega}{c^2}(c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}))\boldsymbol{v}_{\nu 0}\end{matrix}\right)\frac{(\mathbf{k}\cdot\mathbf{V}_1)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} \qquad (25)$$

From eq 16 , perturbed magnetic field can be written as

$$\boldsymbol{B}_1 = \frac{\mathbf{k}\times(\mathbf{V}_1\times\boldsymbol{B}_0)}{-\omega} + \frac{i m_i V_n^2}{\omega c^2 e}\left((c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}))(\mathbf{k}\times\boldsymbol{v}_{\nu 0})\right)\frac{(\mathbf{k}\cdot\mathbf{V}_1)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} \qquad (26)$$

The value of perturbed gravitational potential is:

$$\Phi_1 = 4\pi G\frac{\rho_{m0}\mathbf{k}\cdot\mathbf{V}_1}{\omega k^2} = \frac{\omega_j^2}{\omega k^2}\,\mathbf{k}\cdot\mathbf{V}_1 \qquad (27)$$

Where $\omega_j = \sqrt{4\pi G\rho_{m0}}$ is jean frequency.
Using eqs (17) –(27) in eq (13) , the dispersion relation for self-gravitating neutrino plasma can be obtained as follows :

$$\omega^2\mathbf{V}_1 = C_s^2(\mathbf{k}\cdot\mathbf{V}_1)\mathbf{k} + \{\mathbf{k}\times[\mathbf{k}\times(\mathbf{V}_1\times\boldsymbol{B}_0)]\}\times\frac{\boldsymbol{B}_0}{\mu_0\rho_m}$$
$$-\left[\frac{i m_i V_n^2}{c^2 e}\left((c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}))(\mathbf{k}\times(\mathbf{k}\times\boldsymbol{v}_{\nu 0}))\right)\frac{(\mathbf{k}\cdot\mathbf{V}_1)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2}\right]\times\frac{\boldsymbol{B}_0}{\mu_0\rho_m}$$
$$- V_n^2\left(\begin{matrix}((\boldsymbol{v}_{\nu 0}\cdot\mathbf{k})^2 - c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}) + \omega^2)\mathbf{k} \\ +\frac{\omega}{c^2}(c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}))\boldsymbol{v}_{\nu 0}\end{matrix}\right)\frac{(\mathbf{k}\cdot\mathbf{V}_1)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} - \frac{\omega_j^2}{k^2}(\mathbf{k}\cdot\mathbf{V}_1)\mathbf{k}$$

or

$$\omega^2\mathbf{V}_1 = \left(C_s^2 - \frac{\omega_j^2}{k^2}\right)(\mathbf{k}\cdot\mathbf{V}_1)\mathbf{k} + \{\mathbf{k}\times[\mathbf{k}\times(\mathbf{V}_1\times\boldsymbol{V}_A)]\}\times\boldsymbol{V}_A$$
$$-\left[\frac{i V_n^2 V_A}{c^2\Omega_i}(c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}))\frac{(\mathbf{k}\cdot\mathbf{V}_1)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2}\right](\mathbf{k}\times(\mathbf{k}\times\boldsymbol{v}_{\nu 0}))\times\boldsymbol{V}_A$$
$$- V_n^2\left(\begin{matrix}((\boldsymbol{v}_{\nu 0}\cdot\mathbf{k})^2 - c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}) + \omega^2)\mathbf{k} \\ +\frac{\omega}{c^2}(c^2\mathrm{k}^2 - \omega(\boldsymbol{v}_{\nu 0}\cdot\mathbf{k}))\boldsymbol{v}_{\nu 0}\end{matrix}\right)\frac{(\mathbf{k}\cdot\mathbf{V}_1)}{(\omega - \boldsymbol{v}_{\nu 0}.\mathbf{k})^2} \qquad (27)$$

Where vector Alfven velocity and ion cyclotron frequency is given by

$$\boldsymbol{V}_A = \frac{\boldsymbol{B}_0}{\sqrt{\mu_0\rho_m}}, \qquad \Omega_i = \frac{eB_0}{m_i} \qquad (28)$$

IV. DISCUSSION AND CONCLUSION

As seen in eq (27) , the dispersion relation is modified due to self-gravitational effect. From general dispersion relation it can realized that purely transverse wave with $\mathbf{k}\perp\mathbf{B}_0$ and $\mathbf{k}\perp\mathbf{V}_1$ are not affected by neutrino beam in a self-gravitating plasma. The dispersion relation in this case reduced to

$$\omega^2 = \mathrm{k}^2 V_A$$

Hence Alfven waves are not affected by neutrino beam. However, magnetosonic waves are destabilized due to the presence of neutrino beam in self-gravitating plasma. The angle between wave-vector and background magnetic field and well as angle between wave-vector and perturbed plasma fluid velocity affects the various modes of propagation of waves.

V. REFERENCES

[1] Raffelt GG. Stars as laboratories for fundamental physics: The astrophysics of neutrinos, axions, and other weakly interacting particles. University of Chicago press; 1996.
[2] Haas F, Pascoal KA, Mendonça JT. Neutrino magnetohydrodynamics. Physics of Plasmas. 2016 Jan 1;23(1).
[3] Haas F, Pascoal KA, Mendonça JT. Collisional effects, ion-acoustic waves, and neutrino oscillations. Physics of Plasmas. 2017 May 1;24(5).
[4] Haas F, Pascoal KA. Instabilities and propagation of neutrino magnetohydrodynamic waves in arbitrary direction. Physics of Plasmas. 2017 Sep 1;24(9).
[5] Chen FF. Introduction to plasma physics. Springer Science & Business Media; 2012 Dec 6.
[6] Asaduzzaman M, Mannan A, Mamun AA. Self-gravitational perturbation in super dense degenerate quantum plasmas. Physics of Plasmas. 2017 May 1;24(5).