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Intuitionistic Fuzzy Soft α-open Sets

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Abstract

In this paper, we present a new class of generalized closed set known as intuitionistic fuzzy soft α- closed set has been introduced and their topological structure has been studied. Also, intuitionistic fuzzy soft α- continuous mapping is defined and some basic properties have been derived.

Keywords: intuitionistic fuzzy soft open set, intuitionistic fuzzy soft α –open set, intuitionistic fuzzy soft α-interior, intuitionistic fuzzy soft α-closure.

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1 INTRODUCTION

1.1. Preliminaries

Definition 1.1: An intuitionistic fuzzy set A in X is defined as an object of the following form A = {(x, $\mu_A(x), \gamma_A(x)$): x ∈E}, where X is a non-empty set, the functions μ_A: X → [0, 1] and γ_A: X → [0, 1] define the degree of membership and the degree of nonmembership of the element x ∈X , respectively, and for every x ∈X; $0 \le \mu_A(x) + \gamma_A(x) \le 1$. In this paper, IF(U) denotes the family of all IF sets in U .

Definition 1.2: A pair (f,A) is called a soft set over U , where F is a mapping given by f : A → P(U) . In other words, a soft set over U is a parameterized family of the subsets of the universe U . For e ∈A, f(e) may be considered as the set of e -approximate elements of the soft set (f,A) .

Definition 1.3: Let A ⊆E . A pair (f,A) is called an IF soft set over U , where f is a mapping given by f : A → IF(U) . We denote (f,A) (resp. $\mu_{f(e)}, \gamma_{f(e)}$) by f_A (resp.f_e, f^e) .

In other words, an IF soft set $f_A$over U is a parameterized family of IF sets in the universe U, and $\mu_{f(e)} = f_e \in F(U)$, $\gamma_{f(e)} = f^e \in F(U)$, $f(e) = (f_{e.} f^e) \in IF(U)$ and $f(e)(x) = (f_e(x), f^e(x)) \in J$ for any e $\in$A and x $\in$U .where J = {(a, b) $\in$[0, 1] × [0, 1] : a + b ≤ 1}. Let A $\subseteq$E. Denote $IFS(U)_A$ = {f_A: f_A is an IF Soft set over U}; IFS(U) = {f_A= f_A: is an IF soft set over U and A$\subseteq$E}.

Obviously, IFS(U) = $\bigcup_{A\subseteq E}$ IFS(U)A .

Definition 1.4: Let f_A,$g_B$$\in$ IFS (U) .

1. $f_A$is called IF soft subset of g_B,, if A $\subseteq$B and f(e) $\subseteq$g(e) for any e $\in$A. We write $f_A \tilde{\subseteq} g_B$,
2. f_A and g_B are called IF soft equal, if $f_A \tilde{\subseteq} g_B$ and $g_B \tilde{\subseteq} f_A$ We write$f_A = g_B$. Obviously, f_A= $g_B$if and only if A = B and f(e) = g(e) for any e $\in$A.

Definition 1.5: Let , $f_A g_B$ $\in$IFS (U).

1. The intersection of $f_A$and g_B is the IF soft set h_C where C = A ∩ B, and h (e) = f (e) ∩ g(e) for any e $\in$C . We write $f_A \cap g_B = h_C$
2. The union f_A and is g_B the IF soft set h_C , where C = A∪B, and for any e $\in$C ,

$$h(e) = \begin{cases} f(e), \text{ if } e \in A \\ g(e), \text{ if } e \in B \\ f(e) \cup g(e) \text{ if } e \in A \cap B \end{cases}$$

We write $f_A \cup g_B = h_C$

Definition 1.6: The relative complement of an IF soft set $f_E$is denoted by f'_E and is defined By $(f_E)^c = f^c_E$; where f^c: E → IF(U) is a mapping given by $f^c(e) = (f(e))^c$ for each e $\in$E .

Proposition 1.7: Let f_E, g_E IFS $(U)_E$. Then

$$\bigcap_{i\in I} f_i(E)^C = (\bigcup_{i\in I} f_i(E))^C \text{ and } \bigcup_{i\in I} f_i(E)^C = (\bigcap_{i\in I} f_i(E))^C$$

Definition 1.8: Let f_E $\in$IFS $(U)_E$.

1. f_E is called absolute IF soft over U , if f(e) = $\tilde{1}$ for any e $\in$E . We denoted it by U_E.
2. f_E is called relative null IF soft over U , if f(e) =$\tilde{0}$ for any e $\in$E . We denoted it by ϕ_E. Obviously, $\phi_E = U^C_E$ and $U_E = \phi^C_E$.

Theorem 1.9: Let (f, E) (or f_E) $\in$IFS $(U)_E$. Then,

1. $(f_E \tilde{\cup} f_E) = f_E$,
2. $(f_E \tilde{\cap}) f_E = f_E$,
3. $(f_E \tilde{\cup} \phi_E = f_E$,
4. $(f_E \tilde{\cap} \phi_E) = \phi_E$,
5. $(f_E \tilde{\cup} U_E) = U_E$,
6. $(f_E \tilde{\cap} U_E) = f_E$,

Definition 1.10: Let $\tau \subseteq$IFS $(U)_E$ and $\tau^C = \{f_E : f^C_E \in \tau\}$

Then τ is called an IF soft topology on U if the following conditions are satisfied:

1. U_E ,ϕ_E $\in\tau$,
2. f_E, $g_E$$\in\tau$ implies $f_E \cap g_E \in \tau$,

3. $\{(f_{\alpha})_{E} : \alpha \in \Gamma\} \subseteq \tau$ impli g_E es $\bigcup_{\alpha\in\Gamma}(f_{\alpha})_E \subseteq \tau$. The triple (U, τ, E) is called an IF soft topological space over U. Every member of τ is called an IF soft open set in U. f_E is called an IF soft closed set in U if $f_E \in \tau^C$.

Definition 1.11: Let (X, τ, E), (Y, σ , E) be two intuitionistic fuzzy soft topological spaces, f :X → Y be a mapping and $G_E$be an intuitionistic fuzzy soft set over X. Then the image of f_E Under the mapping f denoted by $f(G_E) = (f(G_e), f(G^e))$is an intuitionistic fuzzy soft set

Over Y defined by $f(G_E)(e)(y) = \left(\bigcup_{y=f(x)} G_e(x), \bigcap_{y=f(x)} G^e(x)\right)$ for each $e \in E$ and $y \in Y$.

Definition 1.12: Let (X, τ, E), (Y, σ ,E) be two intuitionistic fuzzy soft topological spaces, g : X → Y be a mapping and $G_E$be an intuitionistic fuzzy soft set over Y. Then the pre-image of (f_E) under the mapping g , denoted by $g-1 (f_E) = (g-1(F_e), g-1(F^e))$ is an intuitionistic fuzzy soft set over X defined by $(g-1(F_e), g-1(F^e))(x) = (F_e(g(x)) , F^e(g(x))$
for each $e\in E$ and $x \in X$.

Proposition 1.13: Let A_E, Q_E be two intuitionistic fuzzy soft sets over X and Y respectively, and f : X → Y be a mapping. Then
1. $(A_E) \subseteq f^{-1} (f(A_E))$
2. $f(f^{-1}(Q_E)) \subseteq (Q_E)$.

Proposition 1.14: Let $\{P_{iE}: i \in I\}$ be a family of intuitionistic fuzzy soft sets over Y.
Then

1. $f^{-1}\left(\bigcup_i P_{iE}\right) = \bigcup_i f^{-1}(P_{iE})$
2. $f^{-1}\left(\bigcap_i P_{iE}\right) = \bigcap_i f^{-1}(P_{iE})$
3. $f^{-1}(f^{-1}(P_E))^C = f^{-1}(P_E^C)$

2.Intuitionistic fuzzy soft α-open sets

Definition 2.1: Let (U, τ , E) be an intuitionistic fuzzy soft topological space over U . An intuitionistic fuzzy soft set $A_E$over U is said to be an intuitionistic fuzzy soft α-open (briefly IFS α− open) if $F_E \subseteq$int (cl (int (F_E))) . F_E^C is known as intuitionistic fuzzy soft α-closed (briefly IFS α− closed) set in IFS topological space (U, τ , E) over U . Also, IFSαO(U, τ, E) (resp. IFSαC(U, τ ,E)) denote the set of all intuitionistic fuzzy soft α-open (resp. α-closed) sets in IFS topological space (U, τ ,E) over U respectively.

Remark 2.2: The following example ensures the existence of IFS α-open set in IFS topological space (U, τ ,E) over U.
Example 2.3: Consider the universe set U = $\{x_1,x_2\}$ and the set of parameters E = $\{e_1, e_2, e_3\}$. Define F : E → IF(U) as follows.

	$(\mu(x_1),\gamma(x_1))$	$(\mu(x_2,\gamma(x_2))$
$F(e_1)$	(0.6,0.3)	(0.7,0.2)
$F(e_2)$	(0.9,0.01)	(0.8,0.03)

Now F_E= {F(e₁), F(e₂)} is an intuitionistic fuzzy soft set over U. The collection τ ={ϕ_E, U_E, F_E} defines an IFS topology over U. Also, $F_E$is an intuitionistic fuzzy soft α-opensets in the IFSTS (U, τ ,E) over U.

Theorem 2.4: In an IFS topological space (U, τ ,E) , an intuitionistic fuzzy soft set F_E is saidto be an intuitionistic fuzzy soft α-closed in IFSTS (U, τ ,E) iff cl (int (cl (F_E)))⊆F_E.

Proof: It follows from definition and [8]

Proposition 2.5: Every intuitionistic fuzzy soft open set is intuitionistic fuzzy soft α-open setin an IFSTS (U, τ, E) over U .

Proof: Let (U, τ , E) be an IFSTS over U . Le F_E be any intuitionistic fuzzy soft open set inIFSTS (U, τ, E) over U. By [6], int (F_E) = F_E ⊆c l(F_E) . Therefore, F_E⊆ int (cl (int (F_E))).
By definition, $F_E$is an intuitionistic fuzzy soft α-open set in IFSTS (U, τ , E) over U .

Remark 2.6: The following example establishes that the converse of the above proposition is not true in general. It is shown that there are sets which can be an intuitionistic fuzzy soft α-open set but not a intuitionistic fuzzy soft open set.

Example 2.7: Consider a universe set U = {u₁, u₂} and let E = {e₁, e₂} be a set of parameters.
Define mappingsα_E : E → IF (U),β_E : E → IF (U), :γ_EE → IF (U), and ϵ_E : E→ IF (U) as follows. The second table refers its complementγ_E

IF(U)	u_1	u_2
$\alpha_E(e_1)$	(0.2,0.3)	(0.5,0.4)
$\alpha_E(e_2)$	(0.3,0.4)	(0.2,0.4)
$\beta_E(e_1)$	(0.6,0.1)	(0.4,0.2)
$\beta_E(e_2)$	(0.4,0.6)	(0.3,0.5)
$\gamma_E(e_1)$	(0.6,0.1)	(0.5,0.2)
$\gamma_E(e_2)$	(0.4,0.4)	(0.3,0.4)
$\epsilon_E(e_1)$	(0.2,0.3)	(0.4,0.4)
$\epsilon_E(e_2)$	(0.3,0.6)	(0.2,0.5)
$\delta_E(e_1)$	(0.7,0.01)	(0.6,0.1)
$\delta_E(e_2)$	(0.6,0.3)	(0.3,0.3)

IF(U)	u_1	u_2
$\alpha_E^c(e_1)$	(0.3,0.2)	(0.4,0.5)
$\alpha_E^c(e_2)$	(0.4,0.3)	(0.4,0.2)
$\beta_E^c(e_1)$	(0.1,0.6)	(0.2,0.4)

$\beta_E^C(e_2)$	(0.6,0.4)	(0.5,0.3)
$\gamma_E^C(e_1)$	(0.1,0.6)	(0.2,0.5)
$\gamma_E^C(e_2)$	(0.4,0.4)	(0.4,0.3)
$\epsilon_E^C(e_1)$	(0.3,0.2)	(0.4,0.4)
$\epsilon_E^C(e_2)$	(0.6,0.3)	(0.5,0.2)
$\delta_E^C(e_1)$	(0.01,0.7)	(0.1,0.6)
$\delta_E^C(e_2)$	(0.3,0.6)	(0.3,0.3)

$\delta_E = \{\delta_E(e_1), \delta_E(e_2)\}$, $\epsilon_E = \{\epsilon_E(e_1), \epsilon_E(e_2)\}$ are intuitionistic fuzzy soft sets over U . The collection $\tau = \{\phi_E, U_E, \alpha_E, \beta_E, \gamma_E, \epsilon_E\}$ defines an intuitionistic fuzzy soft topology over U. Also, δ_E is an intuitionistic fuzzy soft set is an intuitionistic fuzzy soft α-open set on (U, τ , E) but not in τ.

Theorem 2.8: Let (U, τ, E) be an intuitionistic fuzzy soft topological space over U . Then the Following properties hold.
1. ϕ_E, U_E are intuitionistic fuzzy soft α-open sets in (U, τ, E).
2. Arbitrary union of intuitionistic fuzzy soft α-open sets is an intuitionistic fuzzy soft α- open set over (U, τ, E) .
3. Finite intersection of intuitionistic fuzzy soft α-open sets is an intuitionistic fuzzy soft α-open set over (U, τ, E) .

Remark 2.9: [6] The above theorem yields that the family of all intuitionistic fuzzy soft α-open sets form a topology on (U, τ, E) . It is denoted by IFSαO (U, τ, E). Always $\tau \subseteq$ IFSαO (U, τ, E).

Theorem 2.10: Let (U, τ, E) be an intuitionistic fuzzy soft topological space over U. Then then following properties hold.
1. ϕ_E, U_E are intuitionistic fuzzy soft α-closed sets over (U, τ, E) .
2. Arbitrary intersection of intuitionistic fuzzy soft α-closed sets is an intuitionistic fuzzy soft α-closed set over (U, τ, E) .
3. Finite union of intuitionistic fuzzy soft α-closed sets is an intuitionistic fuzzy soft α-closed set over (U, τ, E) .

Proof:
1. $(\phi_E)^C = U_E$ and $(U_E)^C = \phi_E$. By the definition of intuitionistic fuzzy soft α-closed set, it is arrived.
2. By De-morgan's law, $\left(\bigcap_i F_{iE}\right)^C = \bigcup_i (F_{iE})^C$. Since arbitrary union of intuitionistic fuzzy soft α-open sets is an intuitionistic fuzzy soft α-open, $\left(\bigcap_i F_{iE}\right)^C$ is intuitionistic fuzzysoft α-open. Soft, it's complement $\bigcap_i F_{iE}$ is an intuitionistic fuzzy soft α-closed.
3. By De-morgan's law, $\left(\bigcup_i F_{iE}\right)^C = \bigcap_i (F_{iE})^C$. Since finite union of intuitionistic fuzzy soft α-open sets is an intuitionistic fuzzy soft α-open, $\left(\bigcup_i F_{iE}\right)^C$ is intuitionistic fuzzy soft α-open. So, its complement $\bigcup_i F_{iE}$ is intuitionistic fuzzy soft α-closed.

Definition 2.11: Let (U, τ, E) be an IFSTS over U . For any intuitionistic fuzzy soft set , F_E intuitionistic fuzzy soft α-interior and intuitionistic fuzzy soft α-closure are denoted by $\alpha int(F_E)$ and $\alpha cl(F_E)$ respectively.

They are defined as
$\alpha int(F_E) = \bigcup\{F_E \in IFS\alpha O(U, \tau, E) : G_E \tilde{\subseteq} F_E\}$ and
$\alpha cl(F_E) = \bigcap\{ F_E \in IFS\alpha C(U, \tau, E): F_E \tilde{\subseteq} G_E\}$

Proposition 2.12: In an intuitionistic fuzzy soft topological space (U, τ, E)
then following hold for any $F_E \in IFS(U)_E$.
1. $\alpha int(F_E) \in IFS\alpha O(U, \tau, E)$ (res.$\alpha cl(F_E) \in IFS\alpha C(U, \tau, E)$)
2. $\alpha int(F_E) \tilde{\subseteq} F_E$ (res. $F_E \tilde{\subseteq} cl(F_E)$
3. $\alpha int(F_E)$) is the largest intuitionistic fuzzy soft α-open set such that $\alpha int(F_E) \tilde{\subseteq} F_E$.
(res.$\alpha cl(F_E)$ is the smallest intuitionistic fuzzy soft α-closed set such that $F_E \tilde{\subseteq} \alpha cl(F_E)$.
Proof:
1. It follows from the fact that arbitrary union (rep.intersection) of intuitionistic fuzzy softα-open (res.α-closed) set is intuitionistic fuzzy soft α-open (res.α-closed).

2. By the definition of intuitionistic fuzzy soft α-interior (res.α-closure), it is true.

3. Let G_E be any IFS α-open (res.α-closed) set in (X, τ, E) such that $G_E \subseteq F_E$ (resp. $F_E \subseteq G_E$) By definition, $G_E \subseteq \alpha int(F_E)$.(resp.$\alpha cl(F_E) \subseteq G_E$)
By 2, $G_E \subseteq \alpha int(F_E) \subseteq F_E$ (resp.$F_E \subseteq \alpha cl(F_E) \subseteq G_E$) .

Proposition 2.13: In an IFSTS (U, τ, E) over U , $int(F_E) \subseteq \alpha int(F_E)$ for any $F_E \in IFS(U)$.
Proof: It becomes true from the proposition3.5 ....

Theorem 2.14: Let (U, τ, E) be an intuitionistic fuzzy soft topological space over U. Then the Following properties hold.
1. $F_E \tilde{\subseteq} G_E \Rightarrow \alpha int(F_E) \tilde{\subseteq} \alpha int(G_E)$for any $F_E, G_E \in IFS(U)$
2. F_E is an intuitionistic fuzzy soft α open set iff $\alpha int(F_E) = F_E$.
3. $\alpha int(\alpha int(F_E)) = \alpha int(F_E)$.
4. $\alpha int(F_E) \tilde{\cap} \alpha int(G_E) = \alpha int(F_E \tilde{\cap} G_E)$.
5. $\alpha int(F_E) \tilde{\cup} \alpha int(G_E) \tilde{\subseteq} \alpha int(F_E \tilde{\cup} G_E)$.
6. $(\alpha int(F_E))^C = \alpha cl(F_E)^C$.
Proof:
1. By definition of IFSα-interior, $\alpha int(F_E) \tilde{\subseteq} F_E \tilde{\subseteq} G_E$. But $\alpha int(G_E)$ is the largest IFSα-open set such that $\alpha int(G_E) \tilde{\subseteq} G_E$. Therefore, $\alpha int(F_E) \tilde{\subseteq} \alpha int(G_E)$.

2. By definition IFSα-interior $\alpha int(F_E) \tilde{\subseteq} F_E$holds always. If $F_E$is IFSα-open, then reversible implication becomes true and hence $\alpha int(F_E) = F_E$.
Conversely, assume that $\alpha int(F_E) = F_E$. By proportion 3.12, $\alpha int(F_E) \in IFS\alpha O(U, \tau, E)$.
So,$F_E \in IFS\alpha O(U, \tau, E)$.

3.By proportion 3.12, $\alpha int(F_E) \in IFS\alpha O(U, \tau, E)$. By 2, $\alpha int(\alpha int(F_E)) = \alpha int F_E$.

4. Always, $F_E \tilde{\cap} G_E \tilde{\subseteq} F_E$. By thm3.14, $\alpha int(F_E \tilde{\cap} G_E) \tilde{\subseteq} \alpha int(F_E)$.
Similarly, $\alpha int(F_E \tilde{\cap} G_E) \tilde{\subseteq} \alpha int(G_E)$ .So , $\alpha int(F_E) \tilde{\cap} \alpha int(G_E) \tilde{\subseteq} \alpha int(F_E \tilde{\cap} G_E)$.
On the other hand , $\alpha int(F_E) \tilde{\subseteq} F_E$and $\alpha int(G_E) \tilde{\subseteq} G_E$gives $\alpha int(F_E) \tilde{\cap} \alpha int(G_E) \tilde{\subseteq} (F_E \cap G_E)$. Since intersection of any two IFSα-open sets is again an IFSα-open, $\alpha int(F_E) \tilde{\cap} \alpha int(G_E)$ is an IFSα-open But, $\alpha int(F_E \tilde{\cap} G_E)$ is the

largest IFSα-open such that $\alpha int(F_E \tilde{\cap} G_E) \tilde{\subseteq} (F_E \cap G_E)$. Therefore, $\alpha int(F_E) \tilde{\cap} \alpha int(G_E) = \alpha int(F_E \tilde{\cap} G_E)$. Hence,the equality holds.

5. By definition, $\alpha int(F_E) \tilde{\subseteq} F_E$, $\alpha int(G_E) \tilde{\subseteq} G_E$. then $\alpha int(F_E \tilde{\cup} G_E) \tilde{\subseteq} (F_E \tilde{\cup} G_E)$. .
But, is the larges $\alpha int(F_E \tilde{\cap} G_E)$ is the largest IFSα-open set such that $\alpha int(F_E) \tilde{\cup} \alpha int(G_E) \tilde{\subseteq} (F_E \tilde{\cup} G_E)$. Therefore, $\alpha int(F_E) \tilde{\cup} \alpha int(G_E) \tilde{\subseteq} \alpha int(F_E \tilde{\cup} G_E)$.

6. $\alpha int\ (F_E) = \bigcup \{ G_E \in IFS\alpha O(U, \tau, E) : G_E \tilde{\subseteq} F_E \}$
$\Rightarrow (\alpha int\ (F_E))^C = (\bigcup \{ G_E \in IFS\alpha O(U, \tau, E) : G_E \tilde{\subseteq} F_E \})^C$
$\Rightarrow (\alpha int\ (F_E))^C = \bigcap \{ ((G_E)^C \in IFS\alpha C(U, \tau, E): (F_E)^C \tilde{\subseteq} (G_E)^C \}$
$\Rightarrow (\alpha int(F_E))^C = \alpha cl(F_E)^C$.

Theorem 2.15:Let (U, τ, E) be an intuitionistic fuzzy soft topological space over U . and let
$F_E, G_E \in$ IFS $(U)_E$. Then the following properties hold.

1. $F_E \tilde{\subseteq} G_E \Rightarrow \alpha cl(F_E) \tilde{\subseteq} \alpha cl(G_E)$.
2. F_E is an intuitionistic fuzzy soft α-closed set iff $\alpha cl(F_E) = F_E$.
3. 3. $\alpha cl(\alpha cl(F_E)) = \alpha cl(F_E)$.

Proof:1. By proposition 3.12, $F_E \tilde{\subseteq} G_E \tilde{\subseteq} \alpha\ cl(G_E)$. . Now,$\alpha cl(G_E)$ is an IFSα-closed set containing F_E. By proposition $\alpha cl(F_E)$ is the smallest IFSα-closed set containing F_E
and hence $\alpha cl(F_E) \tilde{\subseteq} \alpha cl(G_E)$.

2. By definition of IFSα-closure, $F_E \tilde{\subseteq} \alpha\ cl(F_E)$ is always true. If F_E is an IFSα-closed set,
The $\alpha cl(F_E) \tilde{\subseteq} F_E$ Thus,$(\alpha cl(F_E) = F_E$
3. If $\alpha cl(F_E)$ is an IFSα-closed set , then By 2, we have $\alpha cl\ (\alpha cl(F_E)) = \alpha cl(F_E)$

Theorem 2.16. Let (U, τ,E) be an intuitionistic fuzzy soft topological space over U . Let
F_E, $G_E \in$ IFS(U). Then the following properties hold.
1. $\alpha cl(F_E) \tilde{\cup} \alpha cl\ (G_E) = \alpha cl\ (F_E \tilde{\cup} G_E)$
2. $\alpha cl(F_E) \tilde{\cap} \alpha cl(G_E) \tilde{\subseteq} \alpha cl(F_E \tilde{\cap} G_E)$.
3. $\alpha cl(F_E))^C = \alpha int(F_E)^C)$

Proof.1. Always, $F_E \tilde{\subseteq} F_E \tilde{\cup} G_E$. By proposition3.12, , $\alpha cl\ (F_E) \tilde{\subseteq} \alpha cl\ (F_E \tilde{\cup} G_E)$.
Similarly,$\alpha cl\ (G_E) \tilde{\subseteq} \alpha cl\ (F_E \tilde{\cup} G_E)$. Therefore,$\alpha cl(F_E) \tilde{\cup} \alpha cl\ (G_E) \tilde{\subseteq} \alpha cl\ (F_E \tilde{\cup} G_E)$. On the other hand, $F_E \tilde{\subseteq} \alpha\ cl(F_E)$ and $G_E \tilde{\subseteq} \alpha\ cl(G_E)$ Hence,$F_E \tilde{\cup} G_E \tilde{\subseteq} \alpha cl(F_E) \tilde{\cup} \alpha cl\ (G_E)$. Since
union of two IFSα-closed set is IFSα-closed, $\alpha cl(F_E) \tilde{\cup} \alpha cl\ (G_E)$ is IFSα-closed set. By
proposition $\alpha cl\ (F_E \tilde{\cup} G_E)$ is the smallest IFSα-closed set containing $(F_E \tilde{\cup} G_E)$. So,
$\alpha cl(F_E \tilde{\cup} G_E) \tilde{\subseteq} \alpha cl(F_E) \tilde{\cup} \alpha cl\ (G_E)$. Therefore, $\alpha cl(F_E) \tilde{\cup} \alpha cl\ (G_E) = \alpha cl\ (F_E \tilde{\cup} G_E)$.
2. Now, $F_E \tilde{\cap} G_E \tilde{\subseteq} F_E$. By proposition 2.12, $F_E \tilde{\cap} G_E \tilde{\subseteq} \alpha cl\ (F_E)$Similarly, $F_E \tilde{\cap} G_E \tilde{\subseteq} \alpha cl\ (G_E)$
Now, $\alpha cl(F_E) \tilde{\cap} \alpha cl(G_E)$ is an IFSα-closed set such that $F_E \tilde{\cap} G_E \tilde{\subseteq} \alpha cl(F_E) \tilde{\cap} \alpha cl(G_E)$. By
proposition 2.12 , $\alpha cl(F_E \tilde{\cap} G_E)$. is the smallest IFSα-closed set containing $(F_E \tilde{\cap} G_E)$. Therefore, $\alpha cl(F_E) \tilde{\cap} \alpha cl(G_E) \tilde{\subseteq} \alpha cl(F_E \tilde{\cap} G_E)$.
3. It holds from the definitions of IFSα-interior and IFSα-closure and from De-Morgan's law.

4 Intuitionistic fuzzy soft α-continuous mappings

Definition 4.1. Let (X, τ, E) and (Y, σ, E) be two intuitionistic fuzzy soft topological spaces.
A mapping f : (X, τ, E) → (Y, σ, E) is said to be intuitionistic fuzzy soft α-continuous if for

each $F_E \in \sigma$, $f^{-1}(F_E) \in$ IFSαO(U, τ,E) .

Example 4.2. Consider (U, τ, E) an intuitionistic fuzzy soft topological space over U as in the example... and let V = {v_1, v_2, v_3}. Define a mapping P_E: E → IF(V)

	V_1	V_2	V_3
$P_E(e_1)$	(0.4,0.4)	(0.3,0.1)	(0.2,0.3)
$P_E(e_2)$	(0.2,0.5)	(0.5,0.2)	(0.3,0.6)

Now, (Y, σ, E) is an intuitionistic fuzzy soft topological space over V . Define a map f : X → Y by f(x_1) = y_3, f(x_2) = y_1. Then $f^{-1}(P_E) \in$ IFSαO(U, τ, E) and $f^{-1}(P_E)=\epsilon_E \in \tau$. Therefore, f is an intuitionistic fuzzy soft α-continuous.

Theorem 4.3: Let (U, τ, E) and (V, σ, E) be two intuitionistic fuzzy soft topological spaces. A mapping f : (U, τ, E) → (V, σ, E) is IFSα-continuous iff $f^{-1}(G_E) \in$ IFSαC (U, τ, E) for all $G_E \in \sigma^C$.

Proof. Assume that f : (U, τ, E) → (V, σ, E) is IFSα-continuous. Let $G_E \in \sigma^C$. By definition,

. $G_E \in \sigma^C$ By hypothesis, $f^{-1}((G_E)^C) \in$ IFSαO(U, τ, E).

since $f^{-1}((G_E)^C) = (f^{-1}(G_E))^C$

$(f^{-1}(G_E))^C \in$ IFSαO(U, τ,E). Again by definition, $f^{-1}(G_E) \in$ IFSαC(U, τ, E)

Conversely, suppose that inverse image of any intuitionistic fuzzy soft closed set in (V, σ, E) is intuitionistic fuzzy soft α-closed set in (U, τ, E) . Let $F_E \in \sigma$.

Then, $(F_E)^C \in \sigma^C$. By hypothesis, $f^{-1}(F_E^C) = (f^{-1}(F_E))^C$.

Now, is an intuitionistic fuzzy soft α-open set in (U, τ, E) . Therefore, f is IFSα-continuous.

Theorem 4.4. Let (U, τ, E) and (V, σ, E) be two intuitionistic fuzzy soft topological spaces. and f : (U, τ, E) → (V, σ, E) is an IFSα-continuous iff $f^{-1}(\mathrm{int}(F_E)) \tilde{\subseteq} \alpha\,\mathrm{int}(f^{-1}(F_E)$ for each $F_E \in$ IFS(V $)_E$.

Proof. Assume that f : (U, τ, E) → (V, σ, E) is an IFSα-continuous. Let $F_E \in$ IFS(V $)_E$

By[6]..., int(F_E) $\in \sigma$. By hypothesis, $f^{-1}(\mathrm{int}(F_E))$ is an intuitionistic fuzzy soft α-open set in (U, τ, E) . By [6], $\mathrm{int}(F_E) \tilde{\subseteq} F_E$ that gives $f^{-1}(\mathrm{int}(F_E)) \tilde{\subseteq} f^{-1}(F_E)$.

By proportion , $\alpha\,\mathrm{int}(f^{-1}(F_E))$

is the largest intuitionistic fuzzy soft α-open set such that $\alpha\,\mathrm{int}(f^{-1}(F_E)) \tilde{\subseteq} f^{-1}(F_E)$

Therefore, $f^{-1}(\mathrm{int}(F_E)) \tilde{\subseteq} \alpha\,\mathrm{int}(f^{-1}(F_E)$.

Conversely, suppose that $f^{-1}(\mathrm{int}(F_E)) \tilde{\subseteq} \alpha\,\mathrm{int}(f^{-1}(F_E)$ for any $F_E \in$ IFS(V $)_E$.

Let $G_E \in \sigma$.

By [6], G_E= int(G_E). Now, by proposition 3.13, $f^{-1}(G_E) = f^{-1}(\mathrm{int}(G_E)) \tilde{\subseteq} .\alpha\,\mathrm{int}(f^{-1}(F_E))$

By proposition. $\alpha \operatorname{int}(f^{-1}(F_E)) \tilde{\subseteq} f^{-1}(F_E)$ Therefore $\alpha \operatorname{int}(f^{-1}(G_E)) = f^{-1}(G_E)$. By proportion2.12, $f^{-1}(G_E)$ is an intuitionistic fuzzy soft α-open set in (U, τ,E) . Thus, f is intuitionistic fuzzy soft α-continuous. The above theorem gives the following result.

Theorem 4.5. Let (U, τ, E) and (V, σ, E) be two intuitionistic fuzzy soft topological spaces. Then f : (U, τ, E) → (V, σ, E) is an IFSα-continuous iff

$f^{-1}(\alpha cl(F_E)) \tilde{\subseteq} cl(f^{-1}(F_E))$ for each $G_E \in$ IFS(V)$_E$

Proof. It follows by taking complements in ....

Theorem 4.6. Every intuitionistic fuzzy soft continuous function is IFSα-continuous.
Proof. Assume that f : (U, τ, E) → (V, σ, E) is any intuitionistic fuzzy soft continuous function. Let $G_E \in \sigma$ .By [6],. By proposition. $f^{-1}(G_E) \in \tau$. $f^{-1}(G_E)$ is an intuitionistic fuzzy soft α-open set in (U, τ, E) . Thus, f is IFSα-continuous.

Remark 4.7. It is discussed in the following example that IFSα-continuous map need not be an intuitionistic fuzzy soft continuous.

Example 4.8. Consider (U, τ, E) , an intuitionistic fuzzy soft topological space over U as in the example 3.7,. and let V = {v_1, v_2, v_3}. Define a mapping δ_E: E → IF(V) by

Map	v_1	v_2	v_3
$\delta_E(e_1)$	(0.6,0.1)	(0.8,0.1)	(0.7,0.01)
$\delta_E(e_2)$	(0.3,0.3)	(0.5,0.2)	(0.6,0.3)

Now, (Y, σ, E) is an intuitionistic fuzzy soft topological space over V . Define a map f : X → Y by f(x_1) = y_3, f(x_2) = y_1. Then $f^{-1}(\delta_E) \in$ IFSαO(U, τ, E) and $f^{-1}(\delta_E)$ not inτ . Therefore, f is an intuitionistic fuzzy soft α-continuous but not intuitionistic fuzzy soft continuous.

Remark 4.9. The following example shows that the composition of two IFSα-continuous mappings is not an IFSα-continuous in general.

Example 4.10. Consider (U, τ, E) as in example ...,. Let V = {y_1, y_2, y_3},W = {z_1, z_2, z_3}. Define mappings P_E: E → IF(V) for all x ∈ V and Q_E: E → IF(W) for all x ∈ W as follows.

Map	y_1	Y_2	y_3
$P_E(e_1)$	(0.4,0.4)	(0.3,0.1)	(0.2,0.3)
$P_E(e_2)$	(0.2,0.5)	(0.5,0.2)	(0.3,0.6)

I

Map	z_1	z_2	z_3
$Q_E(e_1)$	(0.4,0.1)	(0.3,0.3)	(0.5,0.3)
$Q_E(e_2)$	(0.6,0.2)	(0.4,0.5)	(0.3,0.4)

Here, $P_E = \{ P_E(e_1), P_E(e_2)\}$ and $Q_E = \{Q_E(e_1), Q_E(e_2)\}$ are IFS sets over V and W

respectively. Also,the collection $\sigma = \{\phi_E, P_E, V_E\}$ and $\eta = \{\phi_E, Q_E, W_E\}$ form IFS topology over V and W respectively and (V, σ, E), (W, η, E) are IFS topological spaces. Define f : $U \rightarrow V$ and $g : V \rightarrow W$ by $f(x_1) = y_3$, $f(x_2) = y_1$, $g(y_1) = z_3$, $g(y_2) = z_1$, $g(y_3) = z_2$.
Then $g \circ f : U \rightarrow W$ is defined as $(g \circ f)(x) = g(f(x))$ for all $x \in U$. Now, f and g are IFSα-continuous but $g \circ f$ is not.

Theorem 4.11. Let (U, τ, E), (V, σ, E) and (W, η, E) be any three intuitionistic fuzzy soft topological spaces over U, V and W respectively. If f : (U, τ, E) → (V, σ, K) is IFSα-continuous and g : (V, σ, E) → (W, η, E) is intuitionistic fuzzy soft continuous, then their composition
$f \circ g : (U, \tau, E) \rightarrow (W, \eta, E)$ defined by $(g \circ f)(FE) = g(f(FE))$ for all FE 2 (U, τ,E) is IFSα-continuous.
Proof. Let $G_E \in \eta$ be arbitrary. Since g is intuitionistic fuzzy soft continuous, $g^{-1}(G_E) \in \sigma$.
Again by hypothesis, $(f^{-1}(g^{-1}(G_E)) = (g \circ f)^{-1}(G_E)) \in$ IFSαO(U, τ, E).
Therefore, $g \circ f$ is IFSα-continuous.

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