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TWO-DIMENSIONAL FOURIER-LAPLACE TRANSFORM AND ITS REPRESENTATION

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Abstract : Due to the wide applications of Fourier transform and Laplace transform we can develop a new integral transform by combining these transform we get an elegant integral transform that is Two Dimensional Fourier-Laplace Transform which will also be used in several fields of mathematics physics and engineering

This paper presented the generalization of Two-Dimensional Fourier-Laplace Transform and discuss the Representation theorem. The work may be useful for solving higher order ordinary and partial differential equations as well as integral equations.

KEYWORD: Fourier Transform, Laplace Transform, Fourier-Laplace Transform, Generalized function, Testing function space.

I. INTRODUCTION

The integral transforms play important role in the various field of optics and signal processing. Integral transforms are important to solve real problems. It helps to convert differential equations as well as integral equations into terms of an algebraic equation that can be solved easily. It provides a powerful technique for solving initial and boundary value problems arising in applied mathematics, mathematical physics and engineering. There use is still predominant in advanced study and research. Historically, the origin of the integral transformation was found in 1937, Actually the field of integral transformation was flourished when the French mathematician Pierre Simon Laplace introduced his transformation, now known as Laplace transform and French mathematician Jean Baptiste Joseph Fourier (1768–1830) introduced the transformation known as Fourier Transform

The Fourier Transform is a mathematical procedure which transforms a function from the time domain to the frequency domain. The Fourier transform has many applications, in fact any field of physical science that uses sinusoidal signals, such as engineering, physics, applied mathematics, and chemistry, will make use of Fourier series and Fourier transforms. It would be impossible to give examples of all the areas where the Fourier transform is involved, but here are some examples from physics, engineering, and signal processing. Fourier transform can be used to convert from the series of numbers to sound [1]. Fourier transform is also used in signal processing, cell phones, in the measurement of heart rate variability (HRV), image processing. It is also of fundamental importance in quantum mechanics, communication, geology, optics and many more.

The Laplace transform is a mathematical tool based on integration that has a number of applications. It is used to convert complex differential equations to a simpler form having polynomials. It is used to convert derivatives into multiple domain variables and then convert the polynomials back to the differential equation using Inverse Laplace transform. Laplace Transform methods have a key role to play in the modern approach to the analysis and design of engineering system. The concepts of Laplace Transforms are applied in the area of science and technology such as Electric circuit analysis, Communication engineering, Control engineering and nuclear physics etc. Apart from this it is also used in all modern buildings and constructions.

Due to wide spread applicability of these integral transforms, as a powerful tool in solving ordinary and partial differential equations involving distributional boundary conditions, the extension of number of integral transforms into generalized functions appeared. Zemanian [2], [3], Pathak [4], Zayed [6], [7] encountered the generalization of the most commonly used integral transforms e.g., Fourier, Laplace, Mellin, Hankel, etc. The contribution by Mc Bride, Saxena, Tiwari [8], [9], Bhosale [10], [11] is worth mentioning. Some mathematicians also extended double integral transforms to the spaces of generalized functions and studied them e.g. Tiwari worked on Laplace -Hankel, Bhosale [11] on Fourier -Hankel, Sharma [12] on Fourier-Mellin, Gudadhe [13] on Mellin-Whittaker etc. Inspiring by these two disciplines that is integral transform and generalized functions and vast applications in engineering and applied mathematics this research paper aims to generalize Two Dimensional Fourier-Laplace transform in the distributional sense.

This paper is summarized as follows: Definitions are defined in section 2, Testing function spaces are described in section 3, Section 4, gives the idea about definition of Distributional generalized Two-dimensional Fourier-Laplace Transform. Section 5 is mainly focuses on Representation theorem and lastly conclude the paper.

The notations and Terminologies are as per Zemanian [2], [3].

2. DEFINITIONS

The Two-Dimensional Fourier Transform with the parameters s, u of function $f(t, z)$ denoted by $F[f(t,z)] = F(s,u)$ and is given by $F[f(t,z)] = F(s,u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(st+uz)} f(t,z)\,dtdz$ (2.1)

The Two-Dimensional Laplace Transform with the parameters p, v of function $f(x, y)$ denoted by $L[f(x,y)] = F(p,v)$ and is given by $L[f(x,y)] = F(p,v) = \int_0^{\infty}\int_0^{\infty} e^{-px-qy} f(x,y)\,dxdy$ (2.2)

The Two-Dimensional Fourier-Laplace Transform with parameters s, u, p, v of function $f(t,z,x,y)$ is defined as,

$$FL\{f(t,z,x,y)\} = F(s,u,p,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} f(t,z,x,y)\, e^{-i\{(st+uz)-i(px+vy)\}} dt\, dz\, dx\, dy \tag{2.3}$$

Where the kernel $K(s,u,p,v) = e^{-i\{(st+uz)-i(px+vy)\}}$

The Two-Dimensional Inverse Fourier Transform is defined as

$$f(t,z) = F^{-1}[F(s,u)] = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(st+uz)} F(s,u)\, ds\, du \tag{2.4}$$

The Two-dimensional Inverse Laplace transform is defined as,

$$f(x,y) = L^{-1}[F(p,v)] = -\frac{1}{4\pi^2}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{px+vy} F(p,v)\, dp\, dv \tag{2.5}$$

The Two-Dimensional Inverse Fourier-Laplace Transform is defined as,

$$f(t,z,x,y) = FL^{-1}[F(s,u,p,v)] = -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{i\{(st+uz)-i(px+vy)\}} F(s,u,p,v)\, ds\, du\, dp\, dv \tag{2.6}$$

3. VARIOUS TESTING FUNCTION SPACES: -

3.1. THE SPACE $FL_{a,b,c,d,\alpha}$ (S_α-TYPE SPACE):

Let I be the open set in $R_+ \times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I, the space $FL_{a,b,c,d,\alpha}$ is given by

$$FL_{a,b,c,d,\alpha} = \{\phi : \phi \in E_+ / \gamma_{a,b,c,d,k,r,q,m,l,n}[\phi(t,z,x,y)]$$

$$= \sup_{I_1} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m [\phi(t,z,x,y)] \right| \le C_{l,q,n,m} A^k k^{k\alpha} B^r r^{r\alpha} \}$$

where the constants A, B and $C_{l,q,n,m}$ depend on the testing function ϕ.

3.2. THE SPACE $FL^{\beta}_{a,b,c,d}$ (S^{β}-TYPE SPACE):

This space is given by,

$$FL^{\beta}_{a,b,c,d} = \{\phi : \phi \in E_+ / \sigma_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m\left[\phi(t,z,x,y)\right]\right| \leq C_{k,r,q,m} G^l l^{l\beta} H^n n^{n\beta}\}$$

where the constants G, H and $C_{k,r,q,m}$ depend on the testing function ϕ.

3.3. THE SPACE $FL^{\beta}_{a,b,c,d,\alpha}$ (S^{β}_{α}-TYPE SPACE):

This space is formed by combining the conditions (3.1) and (3.2)

$$FL^{\beta}_{a,b,c,d,\alpha} = \{\phi : \phi \in E_+ / \rho_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m\left[\phi(t,z,x,y)\right]\right| \leq C A^k k^{k\alpha} B^r r^{r\alpha} G^l l^{l\beta} H^n n^{n\beta}\}$$

where the constants A, B, G, H and C depend on the testing function ϕ.

4. DISTRIBUTIONAL GENERALIZED TWO-DIMENSIONAL FOURIER-LAPLACE TRANSFORM (2DFLT): -

For $f(t,z,x,y) \in FL^{*}_{a,b,c,d,\alpha}$, where $FL^{*}_{a,b,c,d,\alpha}$ is the dual space of $FL_{a,b,c,d,\alpha}$. It contains all distributions of compact support. The distributional Two-Dimensional Fourier-Laplace transform is a function of $f(t,z,x,y)$ is defined as,

$$FL\{f(t,z,x,y)\} = F(s,u,p,v) = \langle f(t,z,x,y), \phi(t,z,x,y,s,u,p,v)\rangle, \tag{4.1}$$

where $\phi(t,z,x,y,s,u,p,v) = e^{-i\{(st+uz)-i(px+vy)\}}$ and for each fixed $t(0<t<\infty)$, $z(0<z<\infty)$, $x(0<x<\infty)$ and $y(0<y<\infty)$. Also $s>0$, $u>0$, $p>0$ and $v>0$. The right-hand side of (4.1) has a sense as an application of $f(t,z,x,y) \in FL^{*}_{a,b,c,d,\alpha}$ to $\phi(t,z,x,y,s,u,p,v) \in FL_{a,b,c,d,\alpha}$.

5. REPRESENTATION THEOREM

Let $f(t, z. x, y)$ be an arbitrary element of $FL^{*\beta}_{a,b,c,d,\alpha}$ and $\phi(t, z, x, y)$ be an element of $D(I)$, the space of infinitely differentiable function with compact support on I. Then there exists a bounded measurable function $g_{b,d,f,w}(t, z, x, y)$ defined over I such that

$$\langle f, \phi\rangle = \left\langle \sum_{b=0}^{e+1}\sum_{d=0}^{h+1}\sum_{f=0}^{i+1}\sum_{w=0}^{j+1}(-1)^{b+d+f+w} t^k z^r e^{ax} e^{cy} \frac{\partial^{b+d+f+w}}{\partial t^b \partial z^d \partial x^f \partial y^w} g_{b,d,f,w}(t, z, x, y), \quad \phi(t, z, x, y)\right\rangle$$

where k and r are fixed real numbers and e, h, i and j are appropriate non-negative integers satisfying $b \leq e+1$, $d \leq h+1$, $f \leq i+1$ and $w \leq j+1$.

Proof: - Let $\left\{\gamma_{a,c,k,r,l,q,n,m}\right\}_{l,q,n,m=0}^{\infty}$ be the sequence of seminorms. Let $f(t, z. x, y)$ and $\phi(t, z, x, y)$ be arbitrary elements of $FL^{*\beta}_{a,b,c,d,\alpha}$ and $D(I)$ respectively. Then by boundedness property of generalized function by Zemanian [3], pp.52, we have for an appropriate constant C and a non-negative integers e, h, i and j satisfying $|l| \leq e$, $|q| \leq i$, $|n| \leq h$, and $|m| \leq j$

$$\left|\langle f,\ \phi\rangle\right| \le C \max_{\substack{|l|\le e\\ |q|\le i\\ |n|\le h\\ |m|\le j}} \gamma_{a,c,k,r,l,q,n,m}\ \phi(t,z,x,y) \le C \max_{\substack{|l|\le e\\ |q|\le i\\ |n|\le h\\ |m|\le j}} \sup_{\substack{0<t<\infty\\ 0<z<\infty\\ 0<x<\infty\\ 0<y<\infty}} t^k\ z^r\ e^{ax}\ e^{cy} \left|D_t^l D_x^q D_z^n D_y^m \phi(t,z,x,y)\right|$$

$$\le C \max_{\substack{|l|\le e\\ |q|\le i\\ |n|\le h\\ |m|\le j}} \sup_{\substack{0<t<\infty\\ 0<z<\infty\\ 0<x<\infty\\ 0<y<\infty}} t^k\ z^r\ e^{ax}\ e^{cy} \left|\sum_{b=0}^{l}\sum_{d=0}^{n}\sum_{f=0}^{q}\sum_{w=0}^{m} B_n \frac{\partial^{b+d+f+w}}{\partial t^b \partial z^d \partial x^f \partial y^w}\phi(t,z,x,y)\right|$$

$$\le C' \max_{\substack{|l|\le e\\ |q|\le i\\ |n|\le h\\ |m|\le j}} \sup_{\substack{0<t<\infty\\ 0<z<\infty\\ 0<x<\infty\\ 0<y<\infty}} t^k\ z^r\ e^{ax}\ e^{cy} \max_{\substack{b\le l\\ d\le n\\ f\le q\\ w\le m}} \frac{\partial^{b+d+f+w}}{\partial t^b \partial z^d \partial x^f \partial y^w}\phi(t,z,x,y)$$

where C' is a constant which depends only on b, d, f, w and hence on l, q, n, m, so

$$\left|\langle f,\ \phi\rangle\right| \le C'' \max_{\substack{|b|\le e\\ |d|\le h\\ |f|\le i\\ |w|\le j}} \sup_{\substack{0<t<\infty\\ 0<x<\infty\\ 0<z<\infty\\ 0<y<\infty}} t^k\ z^r\ e^{ax}\ e^{cy} \frac{\partial^{b+d+f+w}}{\partial t^b \partial z^d \partial x^f \partial y^w}\phi(t,z,x,y) \qquad (5.1)$$

Now let us set

$$\phi_{e,h,i,j}(t,z,x,y) = t^k\ z^r\ e^{ax}\ e^{cy}\ \phi(t,z,x,y),\ \ b\le e,\ \ d\le h,\ f\le i \text{ and } w\le j$$

Then clearly $\phi_{e,h,i,j}(t,z,x,y)\in D(I)$.

Also $\phi(t,z,x,y) = t^{-k}\ z^{-r}\ e^{-ax}\ e^{-cy}\ \phi_{e,h,i,j}(t,z,x,y)$ (5.2)

On differentiating (5.2) partially with respect to $t,\ z,\ x$ and y successively we get,

$$\frac{\partial\phi}{\partial t} = z^{-r}e^{-ax}e^{-cy}\left\{t^{-k}\frac{\partial\phi_{e,h,i,j}}{\partial t} + (-k)t^{-k-1}\phi_{e,h,i,j}\right\}$$

$$\frac{\partial\phi}{\partial t} = e^{-ax}e^{-cy}\ z^{-r}t^{-k}\frac{\partial\phi_{e,h,i,j}}{\partial t} - k\ e^{-ax}e^{-cy}\ z^{-r}t^{-k-1}\ \phi_{e,h,i,j}$$

$$\frac{\partial^2\phi}{\partial t\partial z} = e^{-ax}e^{-cy}\ t^{-k}\left\{z^{-r}\frac{\partial^2\phi_{e,h,i,j}}{\partial t\partial z} + (-r)\ z^{-r-1}\frac{\partial\phi_{e,h,i,j}}{\partial t}\right\}$$

$$-k\ e^{-ax}e^{-cy}t^{-k-1}\left\{z^{-r}\frac{\partial\phi_{e,h,i,j}}{\partial z} + (-r)\ z^{-r-1}\phi_{e,h,i,j}\right\}$$

$$= e^{-ax}e^{-cy}\ t^{-k}z^{-r}\frac{\partial^2\phi_{e,h,i,j}}{\partial t\partial z} - r\ z^{-r-1}e^{-ax}e^{-cy}\ t^{-k}\frac{\partial\phi_{e,h,i,j}}{\partial t}$$

$$-k\ e^{-ax}e^{-cy}t^{-k-1}z^{-r}\frac{\partial\phi_{e,h,i,j}}{\partial z} + k\ r\ e^{-ax}e^{-cy}t^{-k-1}z^{-r-1}\phi_{e,h,i,j}$$

$$\frac{\partial^3\phi}{\partial t\partial z\partial x} = e^{-cy}\ t^{-k}z^{-r}\left\{e^{-ax}\frac{\partial^3\phi_{e,h,i,j}}{\partial t\partial z\partial x} + (-a)\ e^{-ax}\frac{\partial^2\phi_{e,h,i,j}}{\partial t\partial z}\right\} - r\ z^{-r-1}e^{-cy}\ t^{-k}\left\{e^{-ax}\frac{\partial^2\phi_{e,h,i,j}}{\partial t\partial x} + (-a)\ e^{-ax}\frac{\partial\phi_{e,h,i,j}}{\partial t}\right\}$$

$$-k\, e^{-cy} t^{-k-1} z^{-r} \left\{ e^{-ax} \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial x} + (-a)\, e^{-ax} \frac{\partial \phi_{e,h,i,j}}{\partial z} \right\} + k\, r\, e^{-cy} t^{-k-1} z^{-r-1} \left\{ e^{-ax} \frac{\partial \phi_{e,h,i,j}}{\partial x} + (-a)\, e^{-ax} \phi_{e,h,i,j} \right\}$$

$$= e^{-cy}\, t^{-k} z^{-r} e^{-ax} \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial z \partial x} - a\, e^{-ax} e^{-cy}\, t^{-k} z^{-r} \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial z} - r\, z^{-r-1} e^{-cy}\, t^{-k} e^{-ax} \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial x} + r\, a\, z^{-r-1} e^{-cy}\, t^{-k} e^{-ax} \frac{\partial \phi_{e,h,i,j}}{\partial t}$$

$$-k\, e^{-ax} e^{-cy}\, t^{-k-1} z^{-r} \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial x} + k\, a\, e^{-ax} e^{-cy}\, t^{-k-1} z^{-r} \frac{\partial \phi_{e,h,i,j}}{\partial z} + k\, r\, e^{-cy}\, t^{-k-1} z^{-r-1} e^{-ax} \frac{\partial \phi_{e,h,i,j}}{\partial x} - akr\, z^{-r-1} e^{-cy}\, t^{-k-1} e^{-ax} \phi_{e,h,i,j}$$

$$\frac{\partial^4 \phi}{\partial t \partial z \partial x \partial y} = t^{-k} z^{-r} e^{-ax} \left\{ e^{-cy} \frac{\partial^4 \phi_{e,h,i,j}}{\partial t \partial z \partial x \partial y} + (-c)\, e^{-cy} \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial z \partial x} \right\} - a\, e^{-ax}\, t^{-k} z^{-r} \left\{ e^{-cy} \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial z \partial y} + (-c)\, e^{-cy} \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial z} \right\}$$

$$-r\, z^{-r-1} t^{-k} e^{-ax} \left\{ e^{-cy} \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial x \partial y} + (-c)\, e^{-cy} \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial x} \right\} + r\, a\, z^{-r-1} t^{-k} e^{-ax} \left\{ e^{-cy} \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial y} + (-c)\, e^{-cy} \frac{\partial \phi_{e,h,i,j}}{\partial t} \right\}$$

$$-k\, e^{-ax} t^{-k-1} z^{-r} \left\{ e^{-cy} \frac{\partial^3 \phi_{e,h,i,j}}{\partial z \partial x \partial y} + (-c)\, e^{-cy} \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial x} \right\} + k\, a\, e^{-ax} t^{-k-1} z^{-r} \left\{ e^{-cy} \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial y} + (-c)\, e^{-cy} \frac{\partial \phi_{e,h,i,j}}{\partial z} \right\}$$

$$+ k\, r\, t^{-k-1} z^{-r-1} e^{-ax} \left\{ e^{-cy} \frac{\partial^2 \phi_{e,h,i,j}}{\partial x \partial y} + (-c)\, e^{-cy} \frac{\partial \phi_{e,h,i,j}}{\partial x} \right\} - akr\, e^{-ax} t^{-k-1} z^{-r-1} \left\{ e^{-cy} \frac{\partial \phi_{e,h,i,j}}{\partial y} + (-c)\, e^{-cy} \phi_{e,h,i,j} \right\}$$

$$\therefore \quad \frac{\partial^4 \phi}{\partial t \partial z \partial x \partial y} = t^{-k} z^{-r} e^{-ax} e^{-cy} \left[\frac{\partial^4 \phi_{e,h,i,j}}{\partial t \partial z \partial x \partial y} - c \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial z \partial x} - a \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial z \partial y} + ac \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial z} - \frac{r}{z} \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial x \partial y} \right.$$

$$+ \frac{r\,c}{z} \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial x} + \frac{r\,a}{z} \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial y} - \frac{r\,a\,c}{z} \frac{\partial \phi_{e,h,i,j}}{\partial t} - \frac{k}{t} \frac{\partial^3 \phi_{e,h,i,j}}{\partial z \partial x \partial y} + \frac{c\,k}{t} \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial x}$$

$$\left. + \frac{ak}{t} \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial y} - \frac{cak}{t} \frac{\partial \phi_{e,h,i,j}}{\partial z} + \frac{kr}{tz} \frac{\partial^2 \phi_{e,h,i,j}}{\partial x \partial y} - \frac{ckr}{tz} \frac{\partial \phi_{e,h,i,j}}{\partial x} - \frac{akr}{tz} \frac{\partial \phi_{e,h,i,j}}{\partial y} + \frac{cakr}{tz} \phi_{e,h,i,j} \right]$$

Let us suppose that in I , $sup\ \phi = sup\ \phi_{e,h,i,j} = [A, B, C, E]$. Then since $t^{-k}\ z^{-r}\ e^{-ax} e^{-cy} > 0$

$$\left| \frac{\partial^4 \phi}{\partial t \partial z \partial x \partial y} \right| \le t^{-k} z^{-r} e^{-ax} e^{-cy} \left[\frac{|ackr|}{AB} \left| \phi_{e,h,i,j} \right| + \frac{|rac|}{B} \left| \frac{\partial \phi_{e,h,i,j}}{\partial t} \right| + \frac{|ack|}{A} \left| \frac{\partial \phi_{e,h,i,j}}{\partial z} \right| + \frac{|ckr|}{AB} \left| \frac{\partial \phi_{e,h,i,j}}{\partial x} \right| \right.$$

$$+ \frac{|akr|}{AB} \left| \frac{\partial \phi_{e,h,i,j}}{\partial y} \right| + ac \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial z} \right| + \frac{|rc|}{B} \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial x} \right| + \frac{|ra|}{B} \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial y} \right| + \frac{|ck|}{A} \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial x} \right| + \frac{|ak|}{A} \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial y} \right|$$

$$\left. + \frac{|kr|}{AB} \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial x \partial y} \right| + c \left| \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial z \partial x} \right| + a \left| \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial z \partial y} \right| + \frac{|r|}{B} \left| \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial x \partial y} \right| + \frac{|k|}{A} \left| \frac{\partial^3 \phi_{e,h,i,j}}{\partial z \partial x \partial y} \right| + \left| \frac{\partial^4 \phi_{e,h,i,j}}{\partial t \partial z \partial x \partial y} \right| \right]$$

$$\le C''' t^{-k} z^{-r} e^{-ax} e^{-cy} \left[\left| \phi_{e,h,i,j} \right| + \left| \frac{\partial \phi_{e,h,i,j}}{\partial t} \right| + \left| \frac{\partial \phi_{e,h,i,j}}{\partial z} \right| + \left| \frac{\partial \phi_{e,h,i,j}}{\partial x} \right| + \left| \frac{\partial \phi_{e,h,i,j}}{\partial y} \right| + \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial z} \right| \right.$$

$$+ \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial x} \right| + \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial t \partial y} \right| + \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial x} \right| + \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial z \partial y} \right| + \left| \frac{\partial^2 \phi_{e,h,i,j}}{\partial x \partial y} \right| + \left| \frac{\partial^3 \phi_{e,h,i,j}}{\partial t \partial z \partial x} \right|$$

$$+\left|\frac{\partial^3\phi_{e,h,i,j}}{\partial t\partial z\partial y}\right|+\left|\frac{\partial^3\phi_{e,h,i,j}}{\partial t\partial x\partial y}\right|+\left|\frac{\partial^3\phi_{e,h,i,j}}{\partial z\partial x\partial y}\right|+\left|\frac{\partial^4\phi_{e,h,i,j}}{\partial t\partial z\partial x\partial y}\right|\Bigg]$$

where $C''' = \max\left[\frac{|ackr|}{AB},\frac{|rac|}{B},\frac{|ack|}{A},\frac{|ckr|}{AB},\frac{|akr|}{AB},ac,\frac{|rc|}{B},\frac{|ra|}{B},\frac{|ck|}{A},\frac{|ak|}{A},\frac{|kr|}{AB},c,a,\frac{|r|}{B},\frac{|k|}{A},1\right]$

If C^{iv} is a constant which depends on a, c, k, r then

$$\left|\frac{\partial^4\phi}{\partial t\partial z\partial x\partial y}\right| \le C^{iv}\, t^{-k}z^{-r}e^{-ax}e^{-cy}\left|\frac{\partial^4\phi_{e,h,i,j}}{\partial t\partial z\partial x\partial y}\right|$$

Hence by induction we prove that in I, for obvious constant C^{v}.

$$\left|\frac{\partial^{b+d+f+w}\phi}{\partial t^b\partial z^d\partial x^f\partial y^w}\right| \le C^{v}\, t^{-k}z^{-r}e^{-ax}e^{-cy}\sum_{\substack{\alpha\le b\\ \beta\le d\\ \eta\le f\\ \delta\le w}}\left|\frac{\partial^{\alpha+\beta+\eta+\delta}\phi}{\partial t^\alpha\partial z^\beta\partial x^\eta\partial y^\delta}\right|\phi_{e,h,i,j}$$

Substituting this into (5.1)

$$\left|\langle f,\ \phi\rangle\right| \le C^{vi}\max_{\substack{|b|\le e\\ |d|\le h\\ |f|\le i\\ |w|\le j}}\ \sup_{\substack{0<t<\infty\\ 0<x<\infty\\ 0<z<\infty\\ 0<y<\infty}}\left|\frac{\partial^{\alpha+\beta+\eta+\delta}}{\partial t^\alpha\partial z^\beta\partial x^\eta\partial y^\delta}\phi_{e,h,i,j}(t,z,x,y)\right| \quad (5.3)$$

where $\alpha\le b,\ \beta\le d,\ \eta\le f$ and $\delta\le w$

Now we can write

$$\sup_{\substack{0<t<\infty\\ 0<x<\infty\\ 0<z<\infty\\ 0<y<\infty}}|\phi(t,z,x,y)| \le \sup_{\substack{0<t<\infty\\ 0<x<\infty\\ 0<z<\infty\\ 0<y<\infty}}\left|\int_t^\infty\int_z^\infty\int_x^\infty\int_y^\infty\frac{\partial^4}{\partial t\partial z\partial x\partial y}\phi(t,z,x,y)\,dt\,dz\,dx\,dy\right| \le \left\|\frac{\partial^4}{\partial t\partial z\partial x\partial y}\phi(t,z,x,y)\right\|_{L'\,X\,L'} \quad (5.4)$$

Hence from (5.3)

$$\left|\langle f,\ \phi\rangle\right| \le C^{vi}\max_{\substack{|b|\le e+1\\ |d|\le h+1\\ |f|\le i+1\\ |w|\le j+1}}\ \sup_{\substack{0<t<\infty\\ 0<x<\infty\\ 0<z<\infty\\ 0<y<\infty}}\left\|\frac{\partial^{b+d+f+w}}{\partial t^b\partial z^d\partial x^f\partial y^w}\phi_{e,h,i,j}(t,z,x,y)\right\|_{L'\,X\,L'}$$

Let the product space $L'\times L'$ be denoted by $(L')^2$. We consider the linear one-to-one mapping $\tau:\phi\to\left\{\frac{\partial^{b+d+f+w}}{\partial t^b\partial z^d\partial x^f\partial y^w}\phi\right\}_{\substack{b\le e+1\\ d\le h+1\\ f\le i+1\\ w\le j+1}}$ of $D(I)$ into $(L')^2$. In view of (5.4) we see that the linear functional $\tau:\phi_{e,h,i,j}\to\langle f,\phi\rangle$ is continuous on $\tau D(I)$ for the topology induced by (L'). Hence by Hahn-Banach theorem, it can be a continuous linear functional in the whole of $(L')^2$. But the dual of $(L')^2$ is isomorphic with $(L^\infty)^2$ [16] pp.214 and 259, therefore there exist two L^∞ functions $g_{b,d,f,w}$ $(b\le e+1,\ d\le h+1,\ f\le i+1,\ w\le j+1)$ such that,

$$\langle f,\ \phi\rangle = \sum_{\substack{b\le e+1\\ d\le h+1\\ f\le i+1\\ w\le j+1}} \left\langle g_{b,d,f,w}\,,\ \ \frac{\partial^{b+d+f+w}}{\partial t^b \partial z^d \partial x^f \partial y^w}\,\phi_{e,h,i,j}(t,z,x,y)\right\rangle$$

By (5.2), we have

$$\langle f,\ \phi\rangle = \sum_{\substack{b\le e+1\\ d\le h+1\\ f\le i+1\\ w\le j+1}} \left\langle g_{b,d,f,w}\,,\ \ \frac{\partial^{b+d+f+w}}{\partial t^b \partial z^d \partial x^f \partial y^w}\ t^k\ z^r\ e^{ax}\ e^{cy}\phi(t,z,x,y)\right\rangle$$

Now by using property of differentiation of a distribution and property of multiplication of a distribution by an infinitely smooth function,

$$\langle f,\ \phi\rangle = \sum_{\substack{b\le e+1\\ d\le h+1\\ f\le i+1\\ w\le j+1}} \left\langle (-1)^{b+d+f+w}\, t^k\ z^r\ e^{ax}\ e^{cy}\ \frac{\partial^{b+d+f+w}}{\partial t^b \partial z^d \partial x^f \partial y^w}\, g_{b,d,f,w}\,,\quad \phi(t,z,x,y)\right\rangle$$

where $g_{b,d,f,w}(t,z,x,y)$ are bounded measurable functions defined over $I=(0,\infty)$. Therefore

$$f(t,z,x,y) = \sum_{\substack{b\le e+1\\ d\le h+1\\ f\le i+1\\ w\le j+1}} (-1)^{b+d+f+w}\, t^k\ z^r\ e^{ax}\ e^{cy}\ \frac{\partial^{b+d+f+w}}{\partial t^b \partial z^d \partial x^f \partial y^w}\, g_{b,d,f,w}(t,z,x,y)$$

6. CONCLUSION

In this paper, we developed Two-Dimensional Fourier-Laplace Transform in the distributional sense. We also defined various testing function spaces formed by this transform. Here we also proved Representation theorem. This work may be useful for solving higher order ordinary and partial differential equations as well as integral equations.

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