



# MAGNETOHYDRODYNAMICS FREE CONVECTIVE FLOW OF A VISCO-ELASTIC FLUID THROUGH A POROUS MEDIUM BOUNDED BY AN OSCILLATING POROUS FLAT PLATE

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## INTRODUCTION

The study of flow and heat transfer in slip flow regime is of great interest due to its application in high speed flight in the upper atmosphere.

Sharma and Yadav [2006] have reported steady MHD boundary layer flow and heat transfer between two long vertical wavy walls.

Dash [2007] has studied the effects of radiation and chemical reaction in MHD flow past a stretched vertical permeable surface through a porous medium with constant suction.

Sharma *et al.* [2007] have analysed the steady MHD flow and heat transfer between two rotating porous disk.

Das and Panda [2009] have reported the effect of heat source and variable magnetic field on unsteady hydromagnetic flow of a viscous stratified fluid past a porous flat moving plate in the slip flow.

Panda and Das[2010] have analyzed the MHD free convection flow of a particulate suspension past an infinite porous inclined flat plate with heat absorption.

The aim of the present problem is to study and bring out the effect of rarefaction parameter  $R$ , Hartman number  $M$ , Schmidt number  $Sc$  and Grashof number  $Gr$ , on velocity, concentration and skin friction of the MHD free convective flow of visco-elastic fluid (Walters B $\square$ Model) through a porous medium bounded by an oscillating porous flat plate in slip flow regime.

### FORMULATION OF THE PROBLEM

The physical configuration consists of an unsteady flow of an electrically conducting and incompressible elasticoviscous liquid of Walters B $\square$  model near an oscillating infinite porous flat plate in slip flow regime under the influence of a transverse magnetic field of uniform strength.

The  $x$ -axis is taken along the flat plate in horizontal direction and  $y$ -axis is perpendicular to it. A uniform magnetic field of strength  $B_0$  is applied in the direction of  $y$  – axis.

The pressure  $P$  in the fluid is assumed to be constant. The  $V_0$  represents suction velocity. Initially the plate and fluid are at rest.

Then the plate is set to an oscillatory motion.

The plate is at constant temperature  $T_w$  and concentration  $C_w$ .

- With the following assumptions the present problem has been studied.

The molecular transport properties are assumed to be constant.

The density variation due to temperature and concentration difference is approximated by Boussinesq approximation.

Mass fraction of diffusing species is low compared to the other species in the binary mixture.

The viscous dissipation in energy equation is negligible.

No chemical reaction takes place in the fluid.

The permeability of the medium is uniform.

- The governing equations for visco-elastic liquid of Walters B model is given by

$$\frac{\partial v}{\partial y} = 0 \quad v = \text{constant} (= -V_0 \text{ at } y=0) \tag{1}$$

$$\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} = g(T - T_\infty) + \frac{\partial}{\partial y} \left( \frac{2\mu}{3} \frac{\partial u}{\partial y} - \frac{B_0}{2} \frac{\partial^2 u}{\partial y^2} + g_0 * (C - C_\infty) t \right) \tag{2}$$

$$\frac{\partial T}{\partial t} - V_0 \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \frac{K_0}{3} \frac{\partial T}{\partial y} - \frac{K_0}{3} \frac{\partial^2 T}{\partial y^2} \right) \tag{3}$$

First order velocity slip boundary condition of the problem when the plate executes linear harmonic oscillation in its own plane are given by

$$u = U_0 e^{int} + L_1 \frac{\partial u}{\partial y}, \quad T = T_\infty, \quad C = C_\infty \quad \text{at } y = 0$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \tag{5}$$

- On introducing the following nondimensional quantities

$$y^* = \frac{y}{U_0}, \quad u^* = \frac{u}{U_0}, \quad \theta^* = \frac{T - T_\infty}{T_0 - T_\infty}$$

$$t^* = \frac{t}{U_0 L}, \quad V_0^* = \frac{V_0}{U_0}, \quad C^* = \frac{C - C_\infty}{C_0 - C_\infty}, \quad n^* = \frac{n}{n_0}$$

$$R = \frac{U_0 L}{\nu}$$

(Rarefaction parameter),

$$Rc = \frac{\mu}{\rho U_0 \lambda}$$

(Elasticity parameter)

$$Sc = \frac{U_0 L}{D} \quad (\text{Schmidt number}), \quad D$$

$$Pr = \frac{\mu c_p}{k} \quad (\text{Prandtl number})$$

$$B_0 = \frac{1}{2}$$

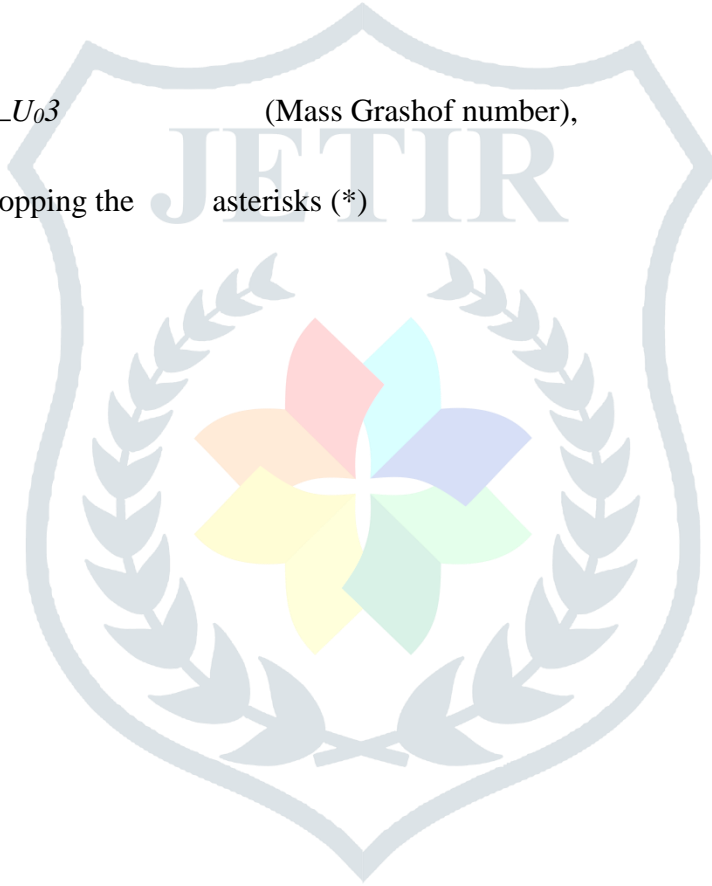
$$M = \frac{U_0}{c} \quad (\text{Hartman number}), \quad c$$

$$U_0$$

$$Gr = \frac{g \beta (T - T_\infty) L^3}{\nu^2} \quad \text{(Thermal Grashof number)}$$

$$Gm = \frac{g \beta^* (C_\infty - C_s) L^3}{U_0^2} \quad \text{(Mass Grashof number),}$$

in equation (2), (3) and (4), after dropping the asterisks (\*)



$$\frac{\partial^2 u}{\partial t^2} - V \frac{\partial u}{\partial y} = Gr^2 + Gm^2 + \frac{-M^2 u - V R c}{y^2} \quad (6)$$

$$\frac{\partial^2 u}{\partial t^2} - V \frac{\partial u}{\partial y} = \frac{2u}{y^2} \quad (7)$$

$$\frac{\partial^2 u}{\partial t^2} - V \frac{\partial u}{\partial y} = \frac{2u}{y^2} \quad (8)$$

with the boundary conditions

$$u = e^{int} + R u, \quad \frac{\partial u}{\partial y} = 1, \quad \frac{\partial u}{\partial t} = 1 \text{ at } y = 0$$

$$u \rightarrow 0, \quad \frac{\partial u}{\partial t} \rightarrow 0, \quad \frac{\partial u}{\partial y} \rightarrow 0 \text{ as } y \rightarrow \infty$$

(9)



**METHOD OF SOLUTION**

Equation (6) is the third order and two boundary conditions are available. Due to inadequate condition a perturbation method has been applied with  $Rc < 1$  as the perturbation parameter. Let

$$u = u_0 + Rc u_1 + Rc^2 u_2 + \dots$$

$$\phi = \phi_0 + Rc \phi_1 + Rc^2 \phi_2 + \dots \quad \psi = \psi_0 + Rc \psi_1 + Rc^2 \psi_2 + \dots$$

Substituting in equation(6)-(8) and equating the powers of  $Rc$  we get the following zeroth order and first order equations with the boundary conditions.

**Zeroth order**

$$\frac{\partial^3 u_0}{\partial t^3} - V_0 \frac{\partial^2 u_0}{\partial y^2} = Gr_0 + Gm_0 + \frac{1}{2} \frac{\partial^2 u_0}{\partial y^2} - M^2 u_0 \tag{10}$$

$$Pr \frac{\partial^2 \phi_0}{\partial t^2} - V_0 \frac{\partial \phi_0}{\partial y} = \frac{\partial^2 \phi_0}{\partial y^2} \tag{11}$$

$$Sc \frac{\partial^2 \psi_0}{\partial t^2} - V_0 \frac{\partial \psi_0}{\partial y} = \frac{\partial^2 \psi_0}{\partial y^2} \tag{12}$$

**First order**

$$\begin{aligned}
 & \frac{\partial u_1}{\partial t} - \frac{\partial^2 u_1}{\partial y^2} = Gr \frac{Gm}{1 + \frac{2u_1}{y^2}} - M \frac{2u_1}{y^2} - \frac{V \frac{\partial^3 u_1}{\partial y^3} - \frac{\partial^3 u_0}{\partial y^3}}{1 - \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial^2 u_0}{\partial y^2}} \quad (13) \\
 & u_0 = e^{int} + R, u_1 = 0, \frac{\partial u_0}{\partial y} = 0
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\partial^2 u_1}{\partial y^2} = \frac{V \frac{\partial^3 u_1}{\partial y^3} - \frac{\partial^3 u_0}{\partial y^3}}{1 - \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial^2 u_0}{\partial y^2}} \quad (14) \\
 & \text{Pr} \frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\partial^2 u_1}{\partial y^2} = \frac{Sc \frac{\partial^3 \theta_1}{\partial y^3} - \frac{\partial^3 \theta_0}{\partial y^3}}{1 - \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial^2 u_0}{\partial y^2}} \quad (15) \\
 & \theta_0 = \frac{\partial^2 u_1}{\partial y^2}
 \end{aligned}$$

with boundary conditions

$$\begin{aligned}
 & u_0 = 1, \frac{\partial u_0}{\partial y} = 1, \frac{\partial u_0}{\partial y} = 1, \frac{\partial u_0}{\partial y} = 0 \text{ at } y = 0 \\
 & u_0 \rightarrow 0, u_1 \rightarrow 0, \frac{\partial u_0}{\partial y} \rightarrow 0, \frac{\partial u_1}{\partial y} \rightarrow 0, \frac{\partial u_0}{\partial y} \rightarrow 0, \frac{\partial u_1}{\partial y} \rightarrow 0 \text{ as } y \rightarrow \infty \quad (16)
 \end{aligned}$$

Further we introduce



$$u_0 = u_{00} + u_{01} e^{int} \quad (17) \quad u_1 = u_{10} + u_{11} e^{int} \quad (18)$$

$$\phi_0 = \phi_{00} + \phi_{01} e^{int} \quad (19)$$

$$\phi_1 = \phi_{10} + \phi_{11} e^{int} \quad (20)$$

$$\phi_0 = \phi_{00} + \phi_{01} e^{int} \quad (21) \quad \phi_1 = \phi_{10} + \phi_{11} e^{int} \quad (22)$$

in zeroth order and first order equations harmonic and non-harmonic considerations separately and obtain the the following equations .

$$u_{00} \square \square + V_0 u_{00} \square - M^2 u_{00} = -Gr \square_{00} - Gm \square \quad (23)$$

$$u_{01} \square \square + V_0 u_{01} \square - (M^2 + in) u_{01} = -Gr \square_{01} - Gm \square_{01} \quad (24)$$

$$u_{10} \square \square + V_0 u_{10} \square - M^2 u_{10} = -Gr \square_{10} - Gm \square_{10} + V_0 u_{00} \square \square \quad (25)$$

$$u_{11} \square \square + V_0 u_{11} \square - (M^2 + in) u_{11} = -Gr \square_{11} - Gm \square_{11} + V_0 u_{01} \square \square + in u_{01} \square \square \quad (26)$$

$$\phi_{00} \square \square + V_0 Pr \phi_{00} \square = 0 \quad (27)$$

$$\phi_{01} + V_0 \text{Pr} \phi_{01} - in \text{Pr} \phi_{01} = 0 \tag{28}$$

(28)

$$\begin{aligned} \phi_{10} + V_0 \text{Pr} \phi_{10} &= 0 \\ \phi_{11} + V_0 \text{Pr} \phi_{11} - in \text{Pr} \phi_{11} &= 0 \end{aligned} \tag{29}$$

(29)

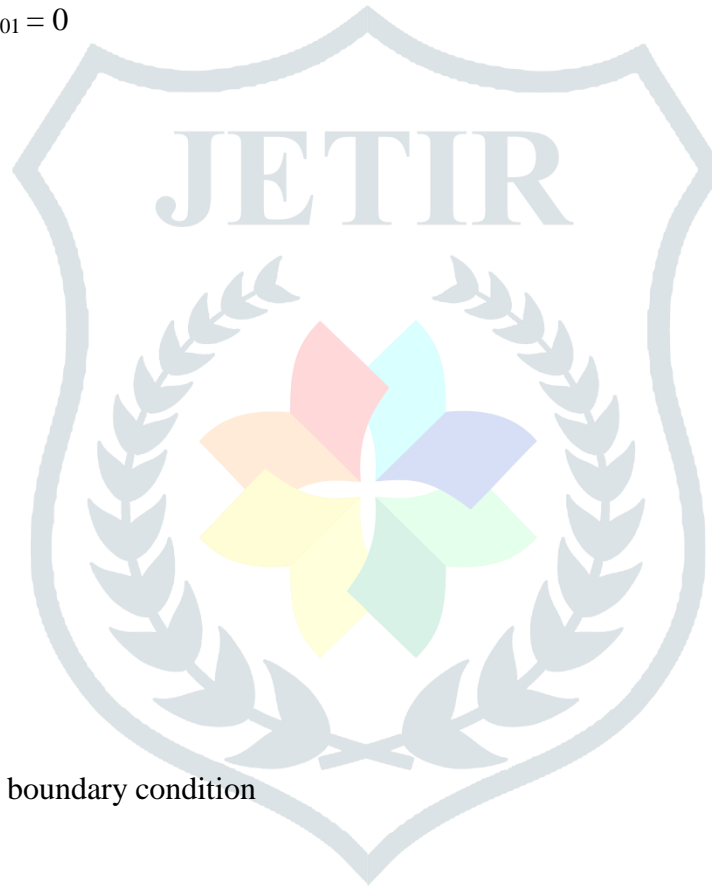
$$\phi_{00} + V_0 \text{Sc} \phi_{00} = 0$$

$$\phi_{01} + V_0 \text{Sc} \phi_{01} - in \text{Sc} \phi_{01} = 0$$

$$\phi_{10} + V_0 \text{Sc} \phi_{10} = 0$$

$$\phi_{11} + V_0 \text{Sc} \phi_{11} - in \text{Sc} \phi_{11} = 0 \text{ with boundary condition}$$

$$u_{00} = R \frac{\phi_{u00}}{\phi_{u01}}, u_{01} = I + R \frac{\phi_{u01}}{\phi_{u00}}, u_{10} = 0, u_{11} = 0$$



$\square y$

$\square y$

$$\square 00 = 1, \square 01 = 0, \square 10 = 0, \square 11 = 0, \square_{00} = 1, \square_{01}$$

$$= 0, \square_{10}$$

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(30) (31)

(32)

(33)

(34)

$$= 0, \square_{11}$$

= 0

as  $y=0$ 

$$u_{00} \rightarrow 0, u_{01} \rightarrow 0, u_{10} \rightarrow 0, u_{11} \rightarrow 0, \theta_{00} \text{ as } y \rightarrow \infty$$

(35)

$$\theta_{10} \rightarrow 0, \theta_{11} \rightarrow 0, \theta_{00} \rightarrow 0, \theta_{01} \rightarrow 0, \theta_{10} \rightarrow 0, \theta_{11} \rightarrow 0 \\ \rightarrow 0, \theta_{01} \rightarrow 0$$

Hence the velocity, temperature and concentration field can be expressed in terms of the fluctuating part as

$$u = -A_1 e^{-PrV_0 y} - A_2 e^{-ScV_0 y} + A_3 e^{-\lambda_1 y} \\ + e^{-\lambda_1 y} [A_{41} \cos(nt - \lambda_2 y) - A_{42} \sin(nt - \lambda_2 y)] \\ + Rc [A_5 (e^{-PrV_0 y} - e^{-\lambda_1 y}) + A_6 (e^{-ScV_0 y} - e^{-\lambda_1 y})] A_7 e^{-\lambda_1 y} + V_0 y e^{-\lambda_1 y} [M_1 \cos(nt - \lambda_2 y) \\ - M_2 \sin(nt - \lambda_2 y)] + n y e^{-\lambda_1 y} [M_3 \cos(nt - \lambda_2 y) + M_4 \sin(nt - \lambda_2 y)]$$

$$\theta = e^{-PrV_0 y}$$

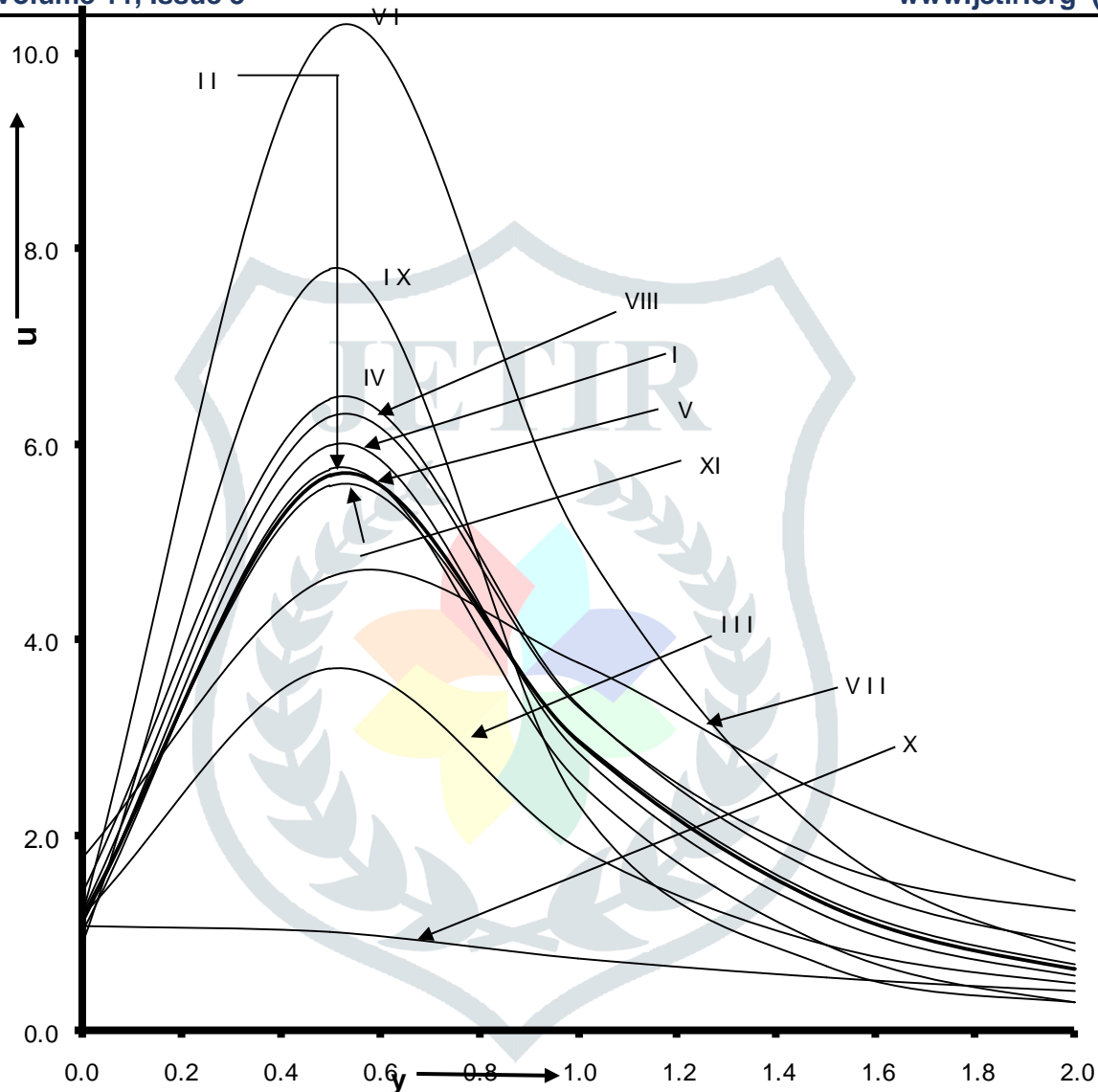
$$\theta = e^{-ScV_0 y}$$

## CONCLUSION

Heavier species with higher Prandtl number fluid contributes to decrease the velocity. Lorentz force contributes to enhance the velocity at all points except a few layer away from the plate.

2. The effect of the permeability parameter is just opposite to that of magnetic parameter.
3. Rarefaction parameter contributes to decelerate the fluid particles in the flow domain. Elastic elements contributes to sudden rise of the velocity near the plate.
4. The buoyancy effect due to mass transfer enhances the velocity.
5. Magnetic force enhances the skin friction as the time elapses.
6. The skin friction decreases as Rarefaction parameter increases, but skin friction increases as elasticity parameter increases.
7. Harmonic oscillation of the plate contributes to the sinusoidal variation of skin friction





**Fig - 1 Velocity profiles when  $Pr = 0.71$ ,  $n = 2$ ,  $Gr = 5$ ,  $t = 0.2$**

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