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Effects of a Magnetic Field and Rotation on the Onset of Double-Diffusive Micropolar Fluids Convection with Couple-Stress in a Saturated Porous Medium

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Abstract: The linear stability analysis was performed to study the effects of magnetic field and rotation on the onset of double-diffusive micro-polar fluid convection in a horizontal porous layer. The perturbation method is used to analyze the combined effect of the magnetic field and temperature variation on fluid stability through a porous medium. It found that the convective behavior is significantly reduced or decreased by the impact of the critical thermal Rayleigh number. It likewise found that the rotation, couple stress parameter, Chandrasekhar number, Lewis number, and the number of solutes Rayleigh have stabilizing effects**.**

***IndexTerms* – rotation, micro-polar fluid, couple-stress fluid, Double-diffusive convection, Galerkin methods, magnetic field, porous medium.**

I. INTRODUCTION

Convective porous medium instability, under the influence of an imposed magnet field, has only recently gained traction in its various engineering and technological applications, particularly in the production of magnetic fluids. Additional significant applications for this research are geophysics and earth core analysis, where the molting fluid performed, that, due to diffusion and the analysis of petroleum reservoir quality, can become convectively unstable [1]. A good review was conducted by [2], of most findings concerning the above subject. While the subject of study is quite old, there is little literature on this topic. In a porous medium, the magneto-convective was investigated by [3], [4], [5], [6], [7], [8], [9], [10], and [11]. Centrifugal convection was recently investigated in a magnetically fluid-saturated porous medium of zero gravity [12].

Nevertheless, the standard temperature gradient (z vertical coordinate) is taken into consideration in all the above research but might not be so in several specific relevant situations. For instance, a time-based temperature gradient (temperature modulation) may contribute to the performance and structure of the produced material, thus impacting the transmission phase in the centrifugal molding of metals. Therefore, the temperature gradient can be taken for space and time and is used to stabilize the convective flow as an effective mechanism. [13], [14], [15], [16] and [17] are some of the studies available concerning the effects of temperature control on the convective fluid surface. Temperature modulation has been investigated for different physical models for the effect on the porous medium convective flow by [18], [19], [20] and [21].

Research into such fluids is important with the increasing relevance of non-Newtonian fluids containing suspension particles in technological advances and industries. The research of such liquids is conducted using several processes in the industry; exotic stimulation, colloidal suspension solutions, extrusion of polymer fluids, liquid crystal solidification, and cooling metal plate in a bath. Such fluids become deformed and induce a rotation field because the suspension particle micro-polar fluid formed by [22], is micro-rotation. Micropolar fluids take care of the effects of microstructures and microfluidic internal movements. The spin effect, because of the micro-rotation of freely rotating particles, produces anti-symmetric stress, called couple stress and hence forms a couple of stress fluids. According to [22], a special case of micropolar fluid whenever the normal vorticity of the fluid balances with the microrotation is a couple of stress fluids. In the non-Newtonian category, a couple of stress fluids have different properties like polar effects. Models of fluids that microstructure is mechanically important are polar fluids theory and associated theories. [23] developed constitutive formulas for a couple of stress fluids. Stokes' theory of micro-fluids is simplistic which makes polar effects like couple stress, body couples, and non-symmetric tensors. On the Rayleigh – Bénard problem, there are several studies on the subject of couple stress fluids, such as on stability/onset.

The external rotation has attracted important exploratory as well as theoretical interest in thermal convection. As geophysical and oceanic flow occurrences in general, the effects of the Coriolis force on the thermal convection mechanism and the flow property are necessary to understand. A system obtained by rotating thermal convection to investigate hydrodynamic stability, pattern development and space-time chaos in nonlinear dynamic systems. In theory as in practice, the analysis of thermal convection in porous rotating mediums is based on engineering applications in certain important areas, such as the processing of foodstuffs, chemical processes, centrifugal coating of metals, solidification and rotating machines. Several scientists have worked hard over the last two decades to examine the external rotation effect on the convection of Rayleigh-Bénard. Most literature studies are concerned with explaining how Coriolis's strength affects the beginnings of thermal convection. The aim of this paper is to study the effects of an electrically conducting fluid and rotation on the onset of micro-polar fluid convection in a horizontal porous layer.

II. MATHEMATICAL FORMULATION

We regard a porous micropolar, electrically conducting liquid, among two horizontal parallel walls with z = 0 and z = d. The walls in x and y are free and infinitely stretched. The rotates of the porous layer around the z-axis in uniform together with a steady angular velocity. The temperatures Tl and Tu with Tl > Tu and solute concentrations Sl and Su with Sl > Su are imposed at the bottom and top boundaries, respectively. The boundaries are impermeable, and we assume that the fluid and solid phases are in local thermal equilibrium. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The interaction between heat and mass transfer, known as Soret and Dufour effects, is supposed to not influence the convective flow, so they are ignored. The velocities are assumed to be small so that the advective and Forchheimer inertia effects are ignored. The flow in the porous medium is governed by the modified Darcy's law, which includes the time derivative and the Coriolis terms employed as a momentum equation. The basic state is assumed to be quiescent, and we superpose infinitesimal perturbations on this basic state. The equations for the perturbation quantities under the Boussinesq approximation are as follows:

$$\nabla \cdot \boldsymbol{q} = 0, \tag{1}$$

$$\nabla \cdot \boldsymbol{H} = 0, \tag{2}$$

$$\frac{\rho_0}{\psi}\frac{\partial \boldsymbol{q}}{\partial t} = -\nabla P + \rho_0(\beta_T T - \beta_T S)\boldsymbol{g} - \frac{2\rho_0}{\psi}\boldsymbol{\Omega} \times \boldsymbol{q} + (2\zeta + \eta)\nabla^2 \boldsymbol{q} + (\zeta \nabla \times \omega) + \mu_m (\boldsymbol{H} \cdot \nabla)\boldsymbol{H}, \tag{3}$$

$$\rho_0 I \left[\frac{\partial \omega}{\partial t}\right] = (\lambda' + \eta')\nabla(\nabla \cdot \omega) + (\eta' \nabla^2 \omega) + \zeta(\nabla \times \boldsymbol{q} - 2\omega), \tag{4}$$

$$\gamma \frac{\partial T}{\partial t} + (\boldsymbol{q} \cdot \nabla)T = k_T \nabla^2 T + \frac{\beta}{\rho_0 C_v}(\nabla \times \omega).\nabla T, \tag{5}$$

$$\psi \frac{\partial S}{\partial t} + (\boldsymbol{q} \cdot \nabla)S = k_S \nabla^2 S, \tag{6}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} + \boldsymbol{q} \cdot \nabla \boldsymbol{H} = \eta_m \nabla^2 \boldsymbol{H} + \boldsymbol{H} \cdot \nabla \boldsymbol{q}, \tag{7}$$

Where $\boldsymbol{H}$ is the magnetic field, $\boldsymbol{q}$ is the velocity, T and S are the temperature and concentration, respectively, ρ is the density, ρ_0 is the density of the fluid at reference $T = T_0$, P is the hydromagnetic pressure, β_T and β_S are the coefficients of thermal and solute expansion, respectively, ζ is the coupling viscosity coefficient or vortex viscosity, ω is the angular velocity, I is the moment of inertia, η denotes shear kinematic viscosity coefficient, λ' and η' are bulk and shear spin viscosity coefficients, ψ and k are the porosity and permeability of the porous medium, g is the acceleration due to gravity, $\boldsymbol{\Omega}$ is the angular velocity of rotation, k_T is the thermal diffusivity, k_S is the solute diffusivity. Further, $\gamma = \frac{(\rho c)_m}{(\rho c_p)_f}$, where $(\rho c)_m = (1-\psi)(\rho c)_s + \psi(\rho c_p)_f$ is the volumetric heat capacity of the saturated medium as a whole and $(\rho c_p)_f$ volumetric heat capacity of the fluid, with the subscripts, are the properties of the f fluid, s solid and m porous matrix, $\eta_m = \frac{1}{\mu_m \sigma_m}$ is the magnetic viscosity : (σ_m: electrical conductivity and μ_m: magnetic permeability), α is the coefficient of thermal expansion. Dimensionless using the following transformations: d for length, ΔT for temperature, ΔS for concentration, k_T/d for velocity, H_0 for the magnetic field, γd^2/k_T for time and k_T/d^2 for modulation frequency and χ/d^3 for the angular velocity. After some mathematical calculations, we then get the below non-dimensional, linear equations for disturbed variables; namely, W is the velocity vertical component, ω_y is the angular velocity, T is the temperature, Sis the concentration, and H_zis the magnet field vertical component:

$$(1 + N_1)\nabla^4 W - Ta\frac{\partial^2 W}{\partial z^2} + N_1 \nabla^2 \omega_y + \nabla_1^2(Ra_T - Ra_S) + QP_m \nabla^2 \frac{\partial H_z}{\partial z} = 0, \tag{8}$$

$$N_3 \nabla^2 \omega_y + N_1 \nabla^2 W - 2N_1 \omega_y = 0, \tag{9}$$

$$\nabla^2 T - W + N_5 \omega_y = 0, \tag{10}$$

$$\frac{1}{Le}\nabla^2 S - W = 0, \tag{11}$$

$$\frac{\partial W}{\partial z} + P_m \nabla^2 H_z = 0, \tag{12}$$

Where $Ta = \left(\frac{2\boldsymbol{\Omega} k}{(\zeta+\eta)\psi}\right)^2$ is the Taylor number, $N_1 = \frac{\zeta}{(\zeta+\eta)}$ represent the coupling parameter, $N_3 = \frac{\eta'}{(\zeta+\eta)d^2}$ denotes the couple stress parameter, $N_5 = \frac{\beta}{\rho_0 C_v d^2}$ resembles the micropolar heat conduction parameter, $Ra_T = \frac{\beta_T g \Delta T d k}{(\zeta+\eta)k_T}$ thermal Rayleigh number, $Ra_S = \frac{\beta_S g \Delta S d k}{(\zeta+\eta)k_T}$ solute Rayleigh number, $Q = \frac{\mu_m \vec{H}_0 d^2}{\lambda_m(\zeta+\eta)}$ denotes the Chandrasekhar number $P_m = \frac{(\zeta+\eta)}{\rho_0 \gamma_m}$ resembles the magnetic Prandtl number, $Le = \frac{k_T}{k_S}$ is Lewis's number. The infinitesimal disturbance W, ω_y, T, S and H_z are supposed to be periodic waves and therefore allow for a normal solution as a mode

$$(W, \omega_y, T, S, H_z) = [W(z), \Gamma(z), \Theta(z), \Phi(z), H_z(z)]exp[i(lx + my)], \tag{13}$$

(l, m: horizontal wavenumbers) . Substitute Eq.(13) with the linear form of Eqs.(8)–(12) we have

$$(1+N_1)(D^2-a^2)^2W+TaD^2W+N_1(D^2-a^2)\Gamma-a^2Ra_T\Theta+a^2Ra_S\Phi - QP_m(D^2-a^2)(DH_z)=0, \quad (14)$$

$$(N_3(D^2-a^2)-2N_1)\Gamma+N_1(D^2-a^2)W=0, \quad (15)$$

$$(D^2-a^2)\Theta-W+N_5\Gamma=0, \quad (16)$$

$$\frac{1}{Le}(D^2-a^2)\Phi-W=0, \quad (17)$$

$$DW+P_m(D^2-a^2)H_z=0, \quad (18)$$

here, $D=\frac{d}{dz}$, $a^2=l^2+m^2$. Equations (14)-(18) are solved for free-free, isothermal and permeable boundaries and hence we have

$$W=\frac{\partial^2 W}{\partial z^2}=\Phi=\Gamma=\Theta=H_z=0 \text{ at } z=0,1. \quad (19)$$

We assume the solutions fulfil the boundary conditions (19) by the form

$$(W(z),\Gamma(z),\Theta(z),\Phi(z),H_z(z))=(W_0,\Gamma_0,\Theta_0,\Phi_0,,H_{z0})sin\,\pi z \quad (20)$$

Therefore, substituting Eq. (20) into Eqs. (14)– (18), we obtain a matrix equation

$$Ra_T=\frac{\upsilon^3N_1(N_3-N_1)+(N_3\upsilon-2N_1)(\upsilon^3+\pi^2Ta+a^2LeRa_S+\pi^2\upsilon Q)}{a^2((N_1N_5+N_3)\upsilon-2N_1)}, \quad (21)$$

where $\upsilon=\pi^2+a^2$

The aforementioned findings may be viewed as akin to that of Bhadauria and Sherani[24] for convection in an electrically conducting fluid-saturated porous media, also Plam and Tyvand [25] for convection in a rotating porous medium when the absence of solute and the couple stress parameter. When $(Q=0)$ which is the situation in non-magnetoconvection, the critical value $Ra=4\pi^2$for $a_c=\pi^2$ was determined by Horton and Rogers[26], and Lapwood[27] . In the limiting situation when $(Q\to\infty)$, the critical value achieved in Nakagawa's [28] experiment for magneto-convection in an ordinary fluid layer.

III. RESULT AND DISCUSSION

Magneto-convection linear stability is studied in a porous rotating micropolar fluid-saturated medium. The terms for the stationary and oscillatory Rayleigh numbers for parameters of different values, including such Taylor number, solute Rayleigh number, couple stress parameter, the coupling parameter, Lewis number, Chandrasekhar number and, the results are determined and the micropolar heat conduction parameter shown in figures. For different parameter values, neutral stability curves are shown in Figures (1-9) in the (Ra_S, a) plane. We have set the parameter values as Ta = 50, Ra_S= 100, Le = 20, N3=1, Q = 50, N1=0.5, N5=1, except the varying parameter. The neutral curves are linked in a topological sense in all of these results. Taylor effect Ta is indicated in Figure 1. (a) and (b) on neutral curves for all other parameters with fixed values, the minimum value Rayleigh number increases to increase Ta, demonstrating that the stabilization of the system is the effect of Taylor's number. Thus, with increasing Ta, the critical number of waves increases (i.e., the convection cell size decreases).

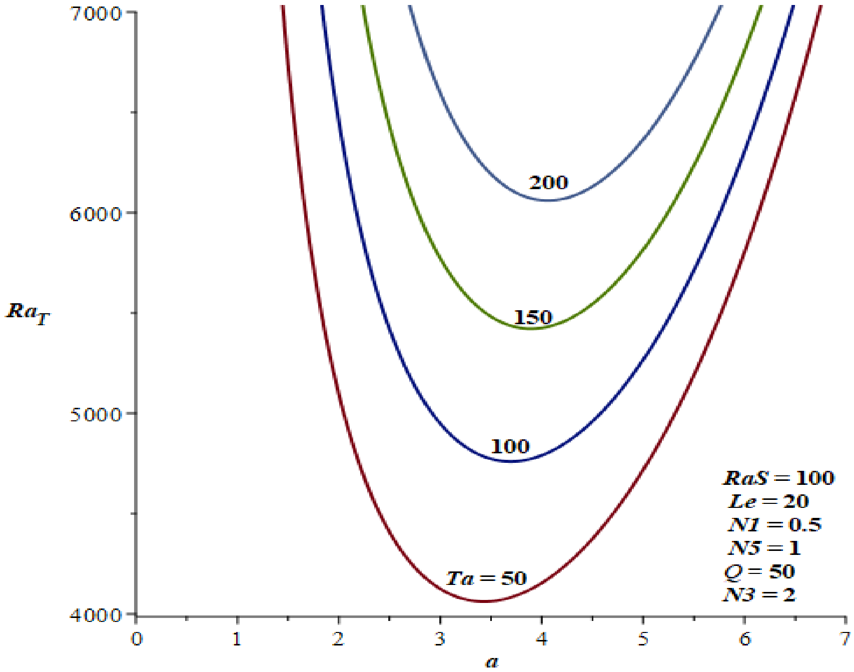

Figure 1: Plot of Ra_T versus a for different values of Taylor number Ta.

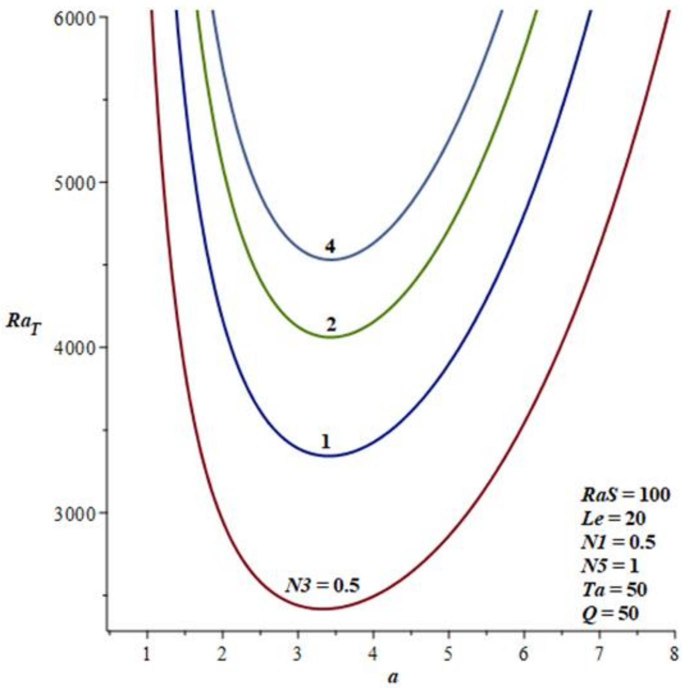

Figure 2: Plot of Ra_T verses a for different values of couple stress parameter N3

Figure 2 shows the effect on neutral stability curves of a couple of stress parameters N3. Due to an increase in the value of the couple stress parameters, the minimum Rayleigh value increases which implies a delay in magneto-convection.

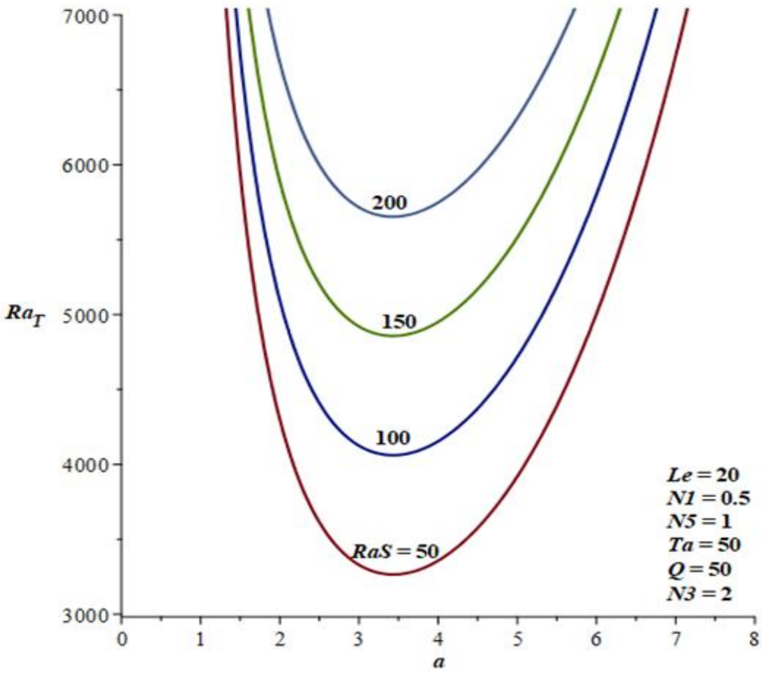

Figure 3: Plot of Ra_T verses a for different values of solute Rayleigh number Ra_S.

The impact of the solute Rayleigh number on the beginning criteria is shown in Figure 3. We notice that by increasing the value of the solute Rayleigh number, the minimum Rayleigh number increases, which means an improvement in system stability. The Lewis number Le shown in Figure 4 effect of fixed values for other parameters on neutral stability curves. With the increase of Le, the critical values of Rayleigh numbers and the corresponding wave numbers increase. Furthermore, the Lewis number contrasts with the stability of the system.

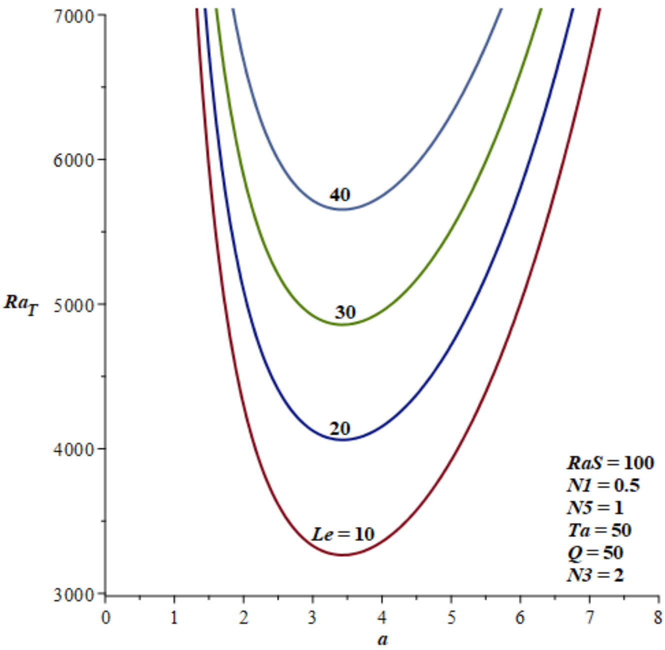

Figure 4: Plot of Ra_T verses a for different values of Lewis number Le.

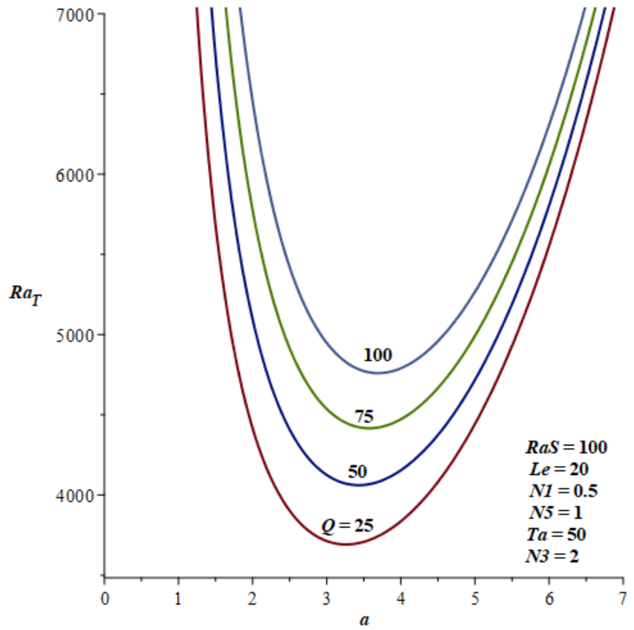

Figure 5: Plot of Ra_T verses for a different values of Chandrasekhar number Q.

Figure 5 shows the effect on the neutral stability curves of Darcy Chandrasekhar. The Rayleigh number's minimum value increases showing a delay in the onset of magnetic convection as the Chandrasekhar number increases. The onset of the convection would be delayed

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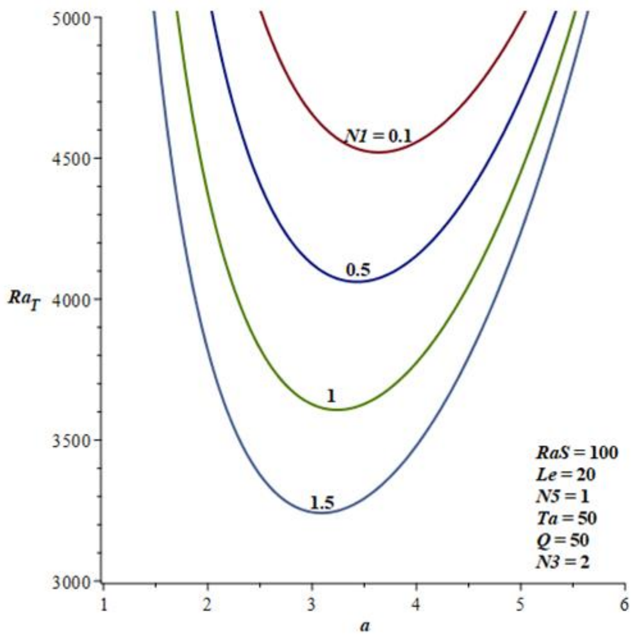

Figure 6: Plot of Ra_T verses a for different values of the coupling parameter N1.

The coupling parameter N1, was decreased in the process as in Figure 6, The existence of the buoyancy force has been observed, meanwhile, promoting convection beginning. So it is concluded that the system is destabilized by the magnetic mechanism. The figure also indicates that the increase in N1 value destabilized the system.

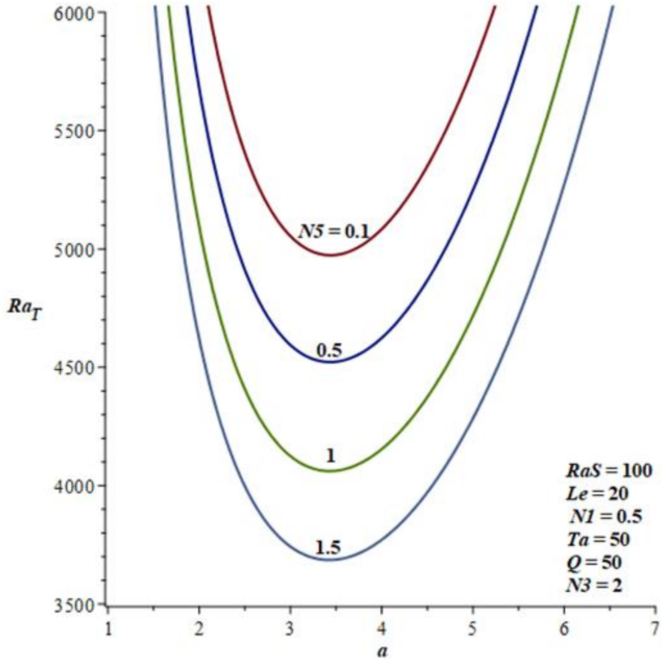

Figure 7: Plot of Ra_T verses a for different values of the micropolar heat conduction parameter number $N5$

In Figure 7 the marginal stability curves are drawn for the different micropolar heat conduction parameter number N5 values. We consider that an increase in the micropolar heat conduction parameter number N5 decreases Rayleigh's number, implying that the number of micropolar heat conduction parameters advances convection.

IV. CONCLUSION

The results in this work show that the onset of the convection with a rotating porous layer in a micropolar fluid was investigated using stability analysis. Depending on the parameters operated, Rayleigh statements are produced. Taylor number, solute Rayleigh number, couple stress parameter, the coupling parameter, Lewis number, Chandrasekhar number and the micropolar heat conduction parameter, the results are determined and graphically show the effect on convection. The critical number of Rayleigh increases the functions of Taylor. The number of Taylor, the couple stress parameter, the Lewis number, the Chandrasekhar number and the solute number of Rayleigh have a stable impact on the convection. The effect of increasing numbers of the coupling parameter and the micropolar heat conduction parameter have destabilizing effects on the onset of convection.

REFERENCES

[1] W. E. Wallace, C. I. Pierce, and W. K. Sawyer, *Experiments on the flow of mercury in porous media in a transverse magnetic field*, vol. 7259. US Department of the Interior, Bureau of Mines, 1969.

[2] D. A. Nield and A. Bejan, *Convection in porous media*. Springer International Publishing, 2017.

[3] P. R. Patil and N. Rudraiah, “Stability of hydromagnetic thermo-convective flow through a porous medium,” *J. Appl. Mech. Trans. ASME*, vol. 40, no. 4, pp. 879–884, 1973, doi: 10.1115/1.3423181.

[4] N. Rudraiah and D. Vortmeyer, “Stability of finite-amplitude and overstable convection of a conducting fluid through fixed porous bed,” *Wärme - und Stoffübertragung*, vol. 11, no. 4, pp. 241–254, Dec. 1978, doi: 10.1007/BF02587788.

[5] N. Rudraiah, “Linear and non-linear magnetoconvection in a porous medium,” *Proc. Indian Acad. Sci. - Math. Sci.*, vol. 93, no. 2–3, pp. 117–135, Dec. 1984, doi: 10.1007/BF02840655.

[6] S. Alchaar, P. Vasseur, and E. Bilgen, “Effects of a magnetic field on the onset of convection in a porous medium,” *Heat Mass Transf.*, vol. 30, no. 4, pp. 259–267, Apr. 1995, doi: 10.1007/BF01602772.

[7] W. Bian, P. Vasseur, and E. Bilgen, “Effect of an external magnetic field on buoyancy-driven flow in a shallow porous cavity,” *Numer. Heat Transf. Part A Appl.*, vol. 29, no. 6, pp. 625–638, 1996, doi: 10.1080/10407789608913810.

[8] S. E. Borglin, G. J. Moridis, and C. M. Oldenburg, “Experimental studies of the flow of ferrofluid in porous media,” *Transp. Porous Media*, vol. 41, no. 1, pp. 61–80, Oct. 2000, doi: 10.1023/A:1006676931721.

[9] C. M. Oldenburg, S. E. Borglin, and G. J. Moridis, “Numerical simulation of ferrofluid flow for subsurface environmental engineering applications,” *Transp. Porous Media*, vol. 38, no. 3, pp. 319–344, 2000, doi: 10.1023/A:1006611702281.

[10] T. Desaive, M. Hennenberg, and P. C. Dauby, “Stabilité thermomagnétoconvective d’un ferrofluide dans une couche poreuse en rotation,” *Mec. Ind.*, vol. 5, no. 5, pp. 621–625, Sep. 2004, doi: 10.1051/meca:2004065.

[11] Sunil, Divya, and R. C. Sharma, “Effect of rotation on ferromagnetic fluid heated and soluted from below saturating a porous medium,” *J. Geophys. Eng.*, vol. 1, no. 2, pp. 116–127, Jun. 2004, doi: 10.1088/1742-2132/1/2/003.

[12] S. Saravanan and H. Yamaguchi, "Onset of centrifugal convection in a magnetic-fluid-saturated porous medium," *Phys. Fluids*, vol. 17, no. 8, pp. 1–9, Aug. 2005, doi: 10.1063/1.1999547.

[13] G. Veneziant, "Effect of modulation on the onset of thermal convection," 1969.

[14] S. Rosenblat and G. A. Tanaka, "Modulation of thermal convection instability," *Phys. Fluids*, vol. 14, no. 7, pp. 1319–1322, Aug. 1971, doi: 10.1063/1.1693608.

[15] M. N. Roppo, S. H. Davis, and S. Rosenblat, "Bénard convection with time-periodic heating," *Phys. Fluids*, vol. 27, no. 4, pp. 796–803, Jun. 1984, doi: 10.1063/1.864707.

[16] B. S. Bhadauria and P. K. Bhatia, "Time-periodic Heating of Rayleigh-Benard Convection," *Phys. Scr.*, vol. 66, no. 1, pp. 59–65, Jan. 2002, doi: 10.1238/physica.regular.066a00059.

[17] B. S. Bhadauria, "Time-periodic heating of Rayleigh-Benard convection in a vertical magnetic field," *Phys. Scr.*, vol. 73, no. 3, pp. 296–302, Mar. 2006, doi: 10.1088/0031-8949/73/3/010.

[18] B. Chhuon and J. P. Caltagirone, "Stability of a horizontal porous layer with timewise periodic boundary conditions," *J. Heat Transfer*, vol. 101, no. 2, pp. 244–248, May 1979, doi: 10.1115/1.3450954.

[19] N. Rudraiah and M. S. Malashetty, "Effect of modulation on the onset of convection in a sparsely packed porous layer," *J. Heat Transfer*, vol. 112, no. 3, pp. 685–689, Aug. 1990, doi: 10.1115/1.2910441.

[20] M. S. Malashetty and D. Basavaraja, "Rayleigh-Benard convection subject to time-dependent wall temperature/gravity in a fluid-saturated anisotropic porous medium," *Heat Mass Transf. und Stoffuebertragung*, vol. 38, no. 7–8, pp. 551–563, Aug. 2002, doi: 10.1007/s002310100245.

[21] B. S. Bhadauria, "Fluid convection in a rotating porous layer under modulated temperature on the boundaries," *Transp. Porous Media*, vol. 67, no. 2, pp. 297–315, Mar. 2007, doi: 10.1007/s11242-006-9027-x.

[22] A. C. Eringen, "Theory of Micropolar Fluids," *J. Math. Mech.*, vol. 16, no. 1, pp. 1–18, 1966, [Online]. Available: http://www.jstor.org/stable/24901466.

[23] V. K. Stokes, "Couple stresses in fluids," *Phys. Fluids*, vol. 9, no. 9, pp. 1709–1715, Dec. 1966, doi: 10.1063/1.1761925.

[24] B. S. Bhadauria and A. Sherani, "Onset of darcy-convection in a magnetic-fluid-saturated porous medium subject to temperature modulation of the boundaries," *Transp. Porous Media*, vol. 73, no. 3, pp. 349–368, Jul. 2008, doi: 10.1007/s11242-007-9176-6.

[25] E. Palm and P. A. Tyvand, "Thermal convection in a rotating porous layer," *ZAMP Zeitschrift für Angew. Math. und Phys.*, vol. 35, no. 1, pp. 122–123, Jan. 1984, doi: 10.1007/BF00945182.

[26] C. W. Horton and F. T. Rogers, "Convection currents in a porous medium," *J. Appl. Phys.*, vol. 16, no. 6, pp. 367–370, 1945, doi: 10.1063/1.1707601.

[27] E. R. Lapwood, "Convection of a fluid in a porous medium," *Math. Proc. Cambridge Philos. Soc.*, vol. 44, no. 4, pp. 508–521, 1948, doi: 10.1017/S030500410002452X.

[28] Y. Nakagawa, "Experiments on the inhibition of thermal convection by a magnetic field," *Proc. R. Soc. London. Ser. A. Math. Phys. Sci.*, vol. 240, no. 1220, pp. 108–113, Apr. 1957, doi: 10.1098/rspa.1957.0070.