ON THE ENERGY-DEPENDENT ALPHA-ALPHA POTENTIAL

M. Majumder, +U. Laha and A. K. Behera
Department of Physics, National Institute of Technology, Jamshedpur, Jharkhand
J. Bhoi
Department of Physics, Veer Surendra Sai University of Technology, Burla, Odisha

Abstract: An equivalent energy-dependent local potential for a two-term nonlocal potential is constructed to study the alpha-alpha system. The s-wave phase shifts are computed by exploiting an efficient approach, the phase function method, to judge the merit of our constructed potential. Reasonable agreement in phase shifts is achieved with experimental results.

I INTRUDUCTION

Investigation of the scattering theory is important because many important discoveries in the nuclear and the atomic physics have been made by bombarding a nucleus or an atom by particles and measuring the number of particles scattered in various directions. The subject matter to α-α interaction dates back to the discovery of radioactivity by Becquerel and Curies in 1896-1898 when the α particle was discovered. However, the first α-α scattering was performed by Rutherford and Chadwick (1927) [1] when they investigated the scattering of α particle by He-nuclei. Oppenheimer (1927) [2] and Gordon (1928) [3] put the classical theory on the table to solve the problem however Mott (1930) [4] called it a partial wave analysis problem. Years later, Mott's formulae was verified by Heydenberg and Temmer (1956) [5], who covered the energy range from 150keV to 3MeV (lab frame) between lab angles of 10˚ and 80˚. They also concluded that the nuclear interactions come into play at energies greater than 400 keV as phase shift decreases from 180˚ at 400 keV to about 120˚ at 3 MeV. They were also able to explain and explore the ground state energy region of Be^8.

As the technology advanced, higher energy scattering experiments were performed by several workers [6-12]. Summarising, the energy phase shift relations so far, we have: Below 400 keV there is no evidence of nuclear interactions. The S wave phase shift starts at this energy point with a value of 180˚ and decreases monotonically with energy while passing through zero at 22 MeV and then becomes negative. The D wave starts at 2.5 MeV, attains a maximum of 120˚ at about 8 MeV and then starts decreasing. The G wave starts at about 4 MeV and then increases with energy. The I wave is first observed at 20 MeV and is positive. The three energy levels established so far are: the 0^+ level at ~92 keV, the 2^+ level at ~3 MeV and 4^+ level at ~11 MeV. From the aforesaid experimental results, many attempts were made to construct an α-α potential which could reproduce the experimental phase shifts. There have been two famous approaches to the problem: The two nucleon interaction and the Resonating Group Formalism.

Earliest attempt in constructing phenomenological α-α potential came from Haefner (1951) [13]. Latter on Humphrey (1957) [14] reproduced the phase shifts for 0-22 MeV with modified Haefner potential with his best fit which required an *l*- dependent well depth. Verification of the above idea came when Van der Spuy and Pienaar (1958) [15] indicated that for square well analysis even at low energies (E<6MeV), one needs a velocity dependent interaction with a core of radius of about 1.8 fm. This schematised the idea of *l* dependence of the core. Igo (1960) [16] made an optical model analysis with the use of complex potential. However the introduction of a non zero imaginary part of the potential which was necessary to reproduce the reaction cross section, had negligible effect on the real part of the phase shifts. An attempt at effective range theory by Russell, Phillips and Reich (1956) [7] could only schematise that if the velocity dependence is to be attributed to *l*-dependence then the α-α interaction should be characterised by an *l* dependent inner core rather than an *l* -independent one.

For the part of the "Resonating Group Formalism" of Wheeler (1937b) [17], the wave functions of the composite nucleus are written as a totally anti-symmetrised combination of the wave functions for the various possible groups in the nucleons. However as pointed out later by Herzenberg (1955, 1957) [18], the Be^8 state to be virtual which reduced the eight body problem to a two body interaction. Herzenberg also revived the α Particle Model of nuclei and also explored several interesting features of α-α interaction. In 1977 Buck, Friedrich and Wheatley [19] proposed a two-parameter angular momentum and energy-independent local Gaussian potential. Marquez [20] successfully described the $\alpha-\alpha$ system by considering a Woods-Saxon type potential for the nuclear part of the interaction. Subsequently, in 1984 it was proved [21] that the Potential of Buck et al. [19] and the one proposed by Marquez [20] are identical. In the meantime, several sophisticated potential models for the light nuclei systems have also been proposed [22-26]. The present text addresses itself to the study of alpha-alpha scattering by an energy-dependent potential. This energy-momentum dependent interaction will be constructed from a two-term nonlocal potential. For simplicity of calculation we treat only the s-wave scattering. In section II we present the prescription for constructing energy-dependent potential equivalent to a nonlocal potential. Section III is devoted to calculation of phase shifts, related results and discussions. Finally, we conclude in section IV.

III ENERGY-MOMENTUM DEPENDENT POTENTIAL

Here we shall deal with a two-term separable potential for the nuclear part of the ($\alpha-\alpha$) interaction and examine the effect of electromagnetic distortion on the nuclear scattering phases. The potential for $\ell=0$ has an attractive and a repulsive component. The strong repulsion due to Pauli Exclusion Principle obeyed by the nucleons is thus accounted for in a phenomenological way. The two-term separable potential is written as

$$V(r,r') = \lambda_1 g_1(\alpha,r) g_1(\alpha,r') + \lambda_2 g_2(\beta,r) g_2(\beta,r') \tag{1}$$

with the form factors

$$g_1(\alpha,r) = e^{-\alpha r};\; g_2(\beta,r) = e^{-\beta r}. \tag{2}$$

For electromagnetic part, that takes care of the charges, we consider a screened Coulomb potential, the Hulthén [27] one expressed as

$$V_H(r) = V_0 \exp(-r/a)/(1-\exp(-r/a)),\; a>0. \tag{3}$$

The strength parameter V_0 for the Hulthén potential is real and positive. The quantity a is the screening radius. The S-wave Schrödinger's equation for rank-2 separable potential is written as

$$\left[\frac{d^2}{dr^2} + k^2\right]\varphi_N(k,r) = \sum_{i=1}^{2} \lambda_i\, g_i(\beta_i,r) \int_0^\infty ds\, g_s(\beta_s,s)\, \varphi_N(k,s) \tag{4}$$

For the regular solution $\varphi_N(k,r)$ the integral equation corresponding to Eq. (4) is written in the form

$$\varphi_N(k,r) = \frac{\sin kr}{k} + \lambda_1\, d_1(k)\, I_1^{(R)}(\beta_1,k,r) + \lambda_2\, d_2(k)\, I_2^{(R)}(\beta_2,k,r) \tag{5}$$

with

$$I_1^{(R)}(\beta_1,k,r) = \frac{1}{\beta_1^2 + k^2}\left[e^{-\beta_1 r} + \frac{\beta_1}{k}\sin kr - \cos kr\right], \tag{6}$$

$$I_2^{(R)}(\beta_2,k,r) = \frac{1}{\beta_2^2 + k^2}\left[e^{-\beta_2 r} + \frac{\beta_2}{k}\sin kr - \cos kr\right], \tag{7}$$

$$d_1(k) = \int_0^\infty ds\, g_1(\beta_1,s)\, \varphi_N(k,s) \tag{8}$$

and

$$d_2(k) = \int_0^\infty ds\, g_2(\beta_2,s)\, \varphi_N(k,s). \tag{9}$$

The regular solution corresponding to local and non-local potentials [28] is related by

$$\varphi_N(k,r) = A(k,r)\, \varphi_L(k,r) \tag{10}$$

where $A(k,r)$ is the damping function. The damping factor $A(k,r)$ is related to the on shell irregular solution through relation

$$A(k,r) = \left(J(k,r)\right)^{1/2} = \left[\frac{1}{k}\operatorname{Im} g\left[f^*(k,r)\, f'(k,r)\right]\right]^{1/2}. \tag{11}$$

Here $f^*(k,r)$ and $f'(k,r)$ represents the complex conjugate of the on shell irregular solution for rank-2 potential and its derivatives with respect to r. The on shell irregular solution for rank -2 potential reads as

$$f(k,r) = e^{ikr} + \lambda_1 d_1(k) I_1^{(I)}(\beta_1,r) + \lambda_2 d_2(k) I_2^{(I)}(\beta_2,r). \tag{12}$$

The quantities used in Eq. (12) are as follows:

$$I_1^{(I)}(\beta_1,r) = \frac{e^{-\beta_1 r}}{(\beta_1^2 + k^2)} \quad \text{and} \quad I_2^{(I)}(\beta_2,r) = \frac{e^{-\beta_2 r}}{(\beta_2^2 + k^2)}, \tag{13}$$

$$d_1(k) = \frac{A_{22}R_{Y1} + A_{12}R_{Y2}}{A_{11}A_{22} - A_{12}A_{21}} + i\,\frac{A_{22}M_{Y1} + A_{12}M_{Y2}}{A_{11}A_{22} - A_{12}A_{21}}, \tag{14}$$

$$d_2(k) = \frac{A_{11}R_{Y2} + A_{21}M_{Y1}}{A_{11}A_{22} - A_{12}A_{21}} + i\,\frac{A_{11}M_{Y2} + A_{21}M_{Y1}}{A_{11}A_{22} - A_{12}A_{21}}, \tag{15}$$

$$A_{11} = 1 - \frac{\lambda_1}{2\beta_1(\beta_1^2 + k^2)},\quad A_{12} = \frac{\lambda_2}{(\beta_1 + \beta_2)(\beta_2^2 + k^2)},\quad A_{21} = \frac{\lambda_1}{(\beta_1 + \beta_2)(\beta_1^2 + k^2)}$$
$$A_{22} = 1 - \frac{\lambda_2}{2\beta_2(\beta_2^2 + k^2)} \tag{16}$$

and

$$R_{Y1} = \frac{\beta_1}{(\beta_1^2 + k^2)},\ M_{Y1} = \frac{k}{(\beta_1^2 + k^2)},\ R_{Y2} = \frac{\beta_2}{(\beta_2^2 + k^2)},\ M_{Y2} = \frac{k}{(\beta_2^2 + k^2)}. \tag{17}$$

Using Eqs. (12-16), (20) and (21) in Eq.(8) one can easily obtain the irregular solution for the corresponding local potential. The equivalent local potential for rank-2 separable potential is obtained as [28]

$$V(k,r) = -\frac{1}{2}\frac{J''(k,r)}{J(k,r)} + \frac{3}{4}\left(\frac{J'(k,r)}{J(k,r)}\right)^2 - \frac{1}{J(k,r)}\int_0^\infty V(r,s)\frac{d}{dr}Q(k,r,s)\,ds. \tag{18}$$

Here $J'(k,r)$ and $J''(k,r)$ are the first and second order derivatives of $J(k,r)$ with respect to r and the term $Q(k,r,s)$ is related to the on -shell irregular solution as

$$Q(k,r,s) = k^{-1}\,\mathrm{Im}\,g\left(f^*(k,r)\,f(k,s)\right). \tag{19}$$

Again

$$\int_0^\infty V(r,s)\frac{d}{dr}Q(k,r,s)\,ds = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7\,. \tag{20}$$

The terms $S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7$ are given by

$$S_1 = -\frac{\lambda_1}{\beta_1^2 + k^2}\left[\beta_1 \cos kr + 2k \sin kr\right]e^{-\beta_1 r} - \frac{\lambda_2}{\beta_2^2 + k^2}\left[\left(\beta_2 \cos kr + 2k \sin kr\right)\right]e^{-\beta_2 r}\ , \tag{21}$$

$$\begin{aligned} S_2 = &-\frac{\lambda_1\, d_{1M} \sin kr}{2\beta_1\left(\beta_1^2 + k^2\right)(\beta_1 + \beta_2)^2}\left[\lambda_1(\beta_1 + \beta_2)^2 e^{-\beta_1 r} + \lambda_2\,\beta_1(2\beta_1 + 3\beta_2)e^{-\beta_2 r}\right] \\ &- \frac{\lambda_1\lambda_2\, d_{2M} \sin kr}{2\left(\beta_2^2 + k^2\right)^2(\beta_1 + \beta_2)^2}\left[2k^2(\beta_1 + \beta_2)\right] - \frac{\lambda_2^2\, d_{2M}\, e^{-\beta_2 r} \sin kr}{4\beta_2\left(\beta_2^2 + k^2\right)^2}\left(2k^2 - \beta_2^2\right), \end{aligned} \tag{22}$$

$$\begin{aligned} S_3 = &-\frac{\lambda_1^2\, d_{1R} \cos kr}{2\beta_1\left(\beta_1^2 + k^2\right)}e^{-\beta_1 r} - \frac{\lambda_1\,\lambda_2\, d_{1R} \cos kr}{2\beta_2\left(\beta_1^2 + k^2\right)(\beta_1 + \beta_2)^2}e^{-\beta_2 r}\left(2\beta_1(\beta_1 + \beta_2) + 1\right) \\ &- \frac{\lambda_1\,\lambda_2\, d_{2R} \cos kr}{2\left(\beta_2^2 + k^2\right)^2(\beta_1 + \beta_2)^2}\left(k^2(\beta_2 + 2\beta_1) - \beta_2^3\right)e^{-\beta_1 r} - \frac{\lambda_2^2\left(7k^2 - \beta_2^2\right)d_{2R}}{8\beta_2\left(\beta_2^2 + k^2\right)^2}e^{-\beta_2 r} \cos kr\ , \end{aligned} \tag{23}$$

$$S_4 = -\frac{\left[\lambda_1 d_{1M}\,\chi_1 + \lambda_2 d_{2M}\,\chi_2\right]}{2k}\left[2\lambda_1\frac{\beta_1 e^{-\beta_1 r}}{\left(\beta_1^2 + k^2\right)} + \lambda_2\frac{\beta_2}{\left(\beta_2^2 + k^2\right)^2}\left(2 - k^2 + \beta_2^2\right)e^{-\beta_2 r}\right], \tag{24}$$

$$\chi_1 = -\frac{\beta_1 e^{-\beta_1 r}}{\beta_1^2 + k^2}\,, \tag{25}$$

$$\chi_2 = -\frac{\beta_2 e^{-\beta_2 r}}{2\left(\beta_2^2 + k^2\right)}\left[1 + \frac{2k^2}{\beta_2^2 + k^2} - \beta_2 r\right], \tag{26}$$

$$S_5 = \left[\lambda_1 d_{1R}\,\chi_1 + \lambda_2 d_{2M}\,\chi_2\right]\left[\left[\frac{\lambda_1 e^{-\beta_1 r}}{\beta_1^2 + k^2}\right] + \lambda_2\frac{e^{-\beta_2 r}}{\left(\beta_2^2 + k^2\right)^2}\left(2\beta_2^2 + k^2\right)\right], \tag{27}$$

$$S_6 = -\frac{\lambda_1\,\lambda_2\,\chi_2\left(d_{1M}d_{2R} + d_{1R}d_{2M}\right)}{2k\beta_1\left(\beta_1^2 + k^2\right)(\beta_1 + \beta_2)^2}\left[\lambda_1(\beta_1 + \beta_2)^2 e^{-\beta_1 r} - \lambda_2\beta_1 e^{-\beta_2 r}(2\beta_1 + 3\beta_2)\right], \tag{28}$$

$$S_7 = -\frac{\lambda_1\,\lambda_2\,\chi_1\left(d_{1R}d_{2M} - d_{1M}d_{2R}\right)}{4\beta_2\left(\beta_2^2 + k^2\right)^2(\beta_1 + \beta_2)^2}\left[2\lambda_1\left(k^2\left(2\beta_1^2 + \beta_2^2\right) - \beta_2^3\right)\right]e^{-\beta_1 r} + \lambda_2 k^2(\beta_1 + \beta_2)^2\ e^{-\beta_2 r}, \tag{29}$$

Combining Eqs. (11)- (29) one is now in a position to write an expression for the equivalent energy-dependent local potential for the two-term nonlocal potential. Using this potential we shall compute the s-wave alpha-alpha scattering phase shifts by applying the phase function method (PFM) [29].

III PHASE SHIFTS, RESULTS AND DISCUSSIONS

The phase function method is an efficient approach to compute the scattering phase shifts for quantum mechanical problems involving local [29] and nonlocal interactions [30-32] and is based on the separation of radial wave function of the Schrödinger equation into an amplitude part $\alpha_\ell(k,r)$ and an oscillating part with variable phase $\delta_\ell(k,r)$. The function $\delta_\ell(k,r)$ called the phase function, describes the meaning of phase shift, at each point of the wave function for scattering by the potential truncated at a distance r. For a local potential $\delta_\ell(k,r)$ satisfy a first order non-linear differential equation given by

$$\delta'_\ell(k,r) = -k^{-1}V(r)\left[\hat{j}_\ell(kr)\cos\delta_\ell(k,r) - \hat{\eta}_\ell(kr)\sin\delta_\ell(k,r)\right]^2 \qquad (30)$$

with $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ the Riccati Bessel functions. Here $\delta'_\ell(k,r)$ indicates the derivative of $\delta_\ell(k,r)$ with respect to r. We shall follow the phase convention of Calogaro [29] with Riccati Hankel function of first kind written as $\hat{h}^1_\ell(x) = -\hat{\eta}_\ell(x) + i\,\hat{j}_\ell(x)$. The scattering phase shift $\delta_\ell(k)$ is achieved by solving the equation from origin to asymptotic region with the initial condition $\delta_\ell(k,0)=0$. Finally, one gets the phase shift $\delta_\ell(k) = \underset{r\to\infty}{Lim}\,\delta_\ell(k,r)$.

Now one will be able to compute scattering phase shifts by utilizing our constructed local potential in conjunction with Eq. (30). We have chosen to work with $V_0 a = 0.2758\ fm^{-1}, a = 20\ fm$ and $\hbar^2/2m = 10.3675\,MeV\ fm^2$. The parameters for our nonlocal potential are $\lambda_1 = -0.97\ fm^{-3}$, $\lambda_2 = 9.0159\ fm^{-3}$, $\alpha = 0.37\ fm^{-1}$ and $\beta = 0.90\ fm^{-1}$. As the screening radius is considered to be $a = 20\ fm$, much more than nuclear range, the atomic Hulthén potential reproduces the effect of the Coulomb potential. The phase shifts are plotted in Fig. 1 along with the experimental data [33].

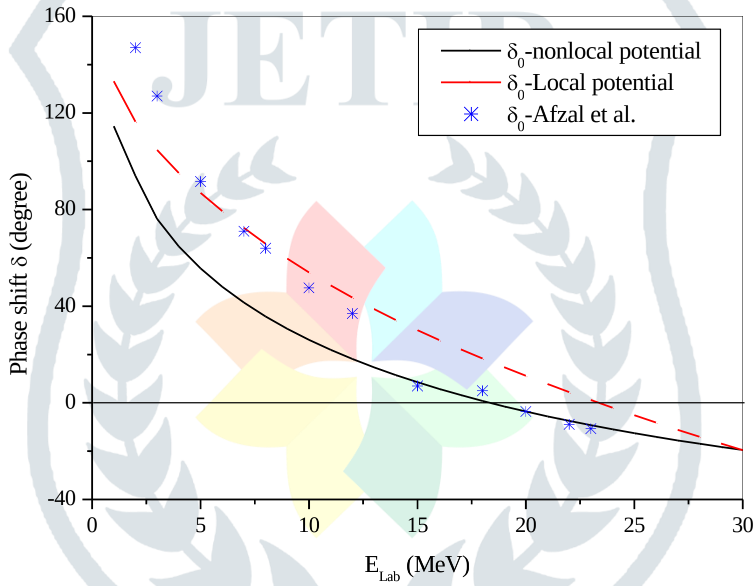

Fig. 1: Alpha-alpha phase shifts as a function of laboratory energy.

Looking closely into fig. 1 it is observed that the s-wave phase shift is positive at lower energies and becomes negative as energy increases. Our results for the nonlocal potential discern from standard data [33] in the low energy range i.e. up to 12.5 MeV and beyond that match well with those of ref. [33]. The phase shift changes sign at about 19.5 MeV which is in good agreement with experimental result [33]. In contrast to this our equivalent local potential produces better agreement with ref. [33]. They slightly differ from experimental result [33] beyond 15 MeV and changes sign at 22.5 MeV. In the low energy range the agreement in phase shifts is quite good. Our phase shift values for nonlocal potential indicate pure nuclear phase shift while the same for the local one is electromagnetic plus nuclear. Thus, the equivalent local potential is superior to its nonlocal counterpart.

IV CONCLUSION

The use of separable nonlocal interactions to fit two-nucleon and nucleus-nucleus phase shifts in various angular momentum states is well established. An equivalent local potential analysis to a nonlocal one is quite common in optical potential model. These methods include a comparison between the characteristics of nonlocal potentials and the phenomenological local potentials. The present technique includes a method for generating smooth potential and computation of phase shifts by exploiting a very accurate method like the PFM. It is worthwhile to mention that the nonlocal separable or energy-dependent local interactions of various shapes are generally used in the folding models for alpha-nucleus scattering. The present method can easily be extended for nonlocal potentials of higher rank. Therefore, it is our belief that this straightforward approach to the problem deserves some attention for treating complex nucleus-nucleus systems.

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