THE EFFECT OF DUST CHARGE FLUCTUATION ON ELECTRON BEAM DRIVEN GOULD-TRIVELPIECE (TG) MODE IN MAGNETIZED DUSTY PLASMA CYLINDER

Daljeet Kaur^{*} Amity Institute of Applied Sciences, Amity University. Suresh C. Sharma Department of Applied Physics, Delhi Technological University. R.S. Pandey Amity Institute of Applied Sciences, Amity University

ABSTRACT- Excitation of Gould-Trivelpiece(TG) mode by an electron beam via Cerenkov interaction in a magnetized dusty plasma cylinder is studied considering the effect of variation in the size and number density of dust grains. The unstable wave's frequency increases with wave number and $\delta = n_{i0} / n_{e0}$ (relative density of negatively charged dust grains). However, the growth rate decreases with size and density of dust grains due to dust charge fluctuations. Our work may advantageous in plasma processing of material experiments because dust appeared to be key source of contamination during material manufacturing.

KEYWORDS-Electron beam, Cerenkov interaction, Dust grains, dust charge fluctuations, Growth rate.

I.INTRODUCTION

Gould-Trivelpiece (TG) waves are electrostatic waves possess unique property of interaction with ions moving in the transverse direction and electrons moving in the magnetic field direction. Due to this property it has been investigated for many decadesboth theoretically and experimentally[1-5] hence are of immense significance in space and laboratory plasmas.

Stenzeland Urrutia [6]have studied the TG mode in uniform unbounded plasma. Through their work they concluded that these modes occur without radial standing waves. In another investigation, it was reported that low-energy beam of electron excite the TG wave of higher harmonics by Praburam and Sharma [7].

Recently, a great deal of interest has been shown by the researchers in dusty plasma[8-11]. In laboratory plasma the dust waves have been taken innon-magnetized [8] and in weekly magnetized [9] surroundings. Sharma et al. [10] have developed a model in which ion-acoustic wave (IAWs) are excited by an ion beam in a magnetized dusty plasma cylinder. Excitation of dust-acoustic waves (DAWs) by an ion beam with negatively charged dust grainsforfinite geometry has been reported by Sharma et al. [11]

In the present work, we have developed a model on excitation of Gould-Trivelpiece (TG) mode by an electron beamin magnetized dusty plasma cylinder. The instability analysis will be carried out in section II. Using fluid treatment, we have obtained the response of beam and plasmas. Further, by using first order perturbation theory we have derived the expression for the instability's growth. Summary of the work is given in Section III. The conclusion of the work is given in section IV.

II.INSTABILITY ANALYSIS

We consider acylinder of radius 'a₁' containing plasma with negatively charged dust grains, electrons and ionshaving equilibrium electron density n_{e0} , ion density n_{i0} , and dust grains density n_{d0} . The column is engrossed in the static magnetic field B_s in the z-direction. The electrons are defined by (-e, m_e, T_e), ions by (e, m_i, T_i) and dust particles by (-Q_{d0}, m_d, T_d). Consider an electrostatic wave, say, Gould-Trivelpiece (TG) wave, propagating perpendicular to the external magnetic field, being propagation vector **k** in the x-z plane. The electron beam is propagating along the z-axis in the direction of the magnetic field with density n_{b0} and equilibrium velocity $v_{b0}\hat{z}$. The beam and plasma system prior to perturbation is quasineutral, such that

 $(-n_{e0} + n_{i0} - n_{b0} - n_{d0} \square 0)$ since we have taken the plasma density much greater than the beam density. This equilibrium is perturbed due to electrostatic perturbation and potential associated with it is given by

$$\phi = \phi_0 \exp[-i(\omega t - k_x x - k_z z)]. (1)$$

The components are taken as fluids and refer by the equation of motion and continuity as given

$$m_{e} = \left\lfloor \frac{\partial \vec{\upsilon}}{\partial t} + (\vec{\upsilon}.\nabla)\vec{\upsilon} \right\rfloor = -e\vec{E} - \frac{e}{c}\vec{\upsilon} \times \vec{B}_{S(2)}$$
$$\frac{\partial n}{\partial t} + \nabla .(n\vec{\upsilon}) = 0.$$
(3)

Further, on linearization the equations of motion and continuity [cf. Eqs. (1) & (2)] leads to the electron and ion density perturbations, dust density perturbation and electron beam density perturbation as:

$$\begin{split} n_{1e} &= -\frac{n_{e0}e\phi}{m_e} \left[\frac{-\nabla_{\perp}^2 \phi}{\omega^2 - \omega_{ce}^2} + \frac{k_z^2 \phi}{\omega^2} \right], \\ n_{1i} &= \frac{n_{i0}e\phi}{m_i} \left[\frac{-\nabla_{\perp}^2 \phi}{\omega^2 - \omega_{ci}^2} + \frac{k_z^2 \phi}{\omega^2} \right], \\ n_{1d} &= \frac{-n_{d0}Q_{d0}k^2 \phi}{m_d \omega^2}, \\ \end{split}$$

and

$$n_{1b} = \frac{-n_{b0}ek_z^2\phi}{m_e(\omega - k_z v_{b0})^2} .$$
(7)

In this case, dust is taken as unmagnetized since $\omega \gg \omega_{cd}$ with $\omega_{cd} \left(= \frac{Q_{d0}B_s}{m_d c} \right)$ is the dust cyclotron frequency. Further

applying the probe theory to a dust grain, Q_d (dust grain's charge) is said to be well-adjusted with the plasma currents present on the grain surface as

$$-\frac{dQ_d}{dt} = I_e + I_i.$$
(8)

Following Refs. [12, 13], the dust grain surface will have electron and ion currents and can be expressed as

$$I_{e} = -\pi a^{2} e^{\binom{8T_{e}}{\pi m_{e}}^{1/2}} n_{e} \exp^{\binom{e(\phi_{g0} - V)}{T_{e}}}$$
$$I_{i} = \pi a^{2} e^{\binom{8T_{i}}{\pi m_{i}}^{1/2}} n_{i} \left(1 - \frac{e(\phi_{g0} - V)}{T_{i}}\right).$$

Here, *a* is the radius of the dust grain sphere, $(\phi_{g0} - V)$ is the difference between the surface potential of the dust grain and plasma potential. In the absence of dust grains the electron density is n_e and ion density is n_i. In equilibrium, $|I_{e0}| = |I_{i0}|$, where $|I_{e0}|$ and $|I_{i0}|$ are the equilibrium currents due to electron and ion on the grain surface. For instance, $|I_{e0}|$ can be written as $|I_{e0}| = \pi a^2 e \left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_{e0} \exp\left(\frac{e\phi_{g0}}{T_e}\right)$ because there is no plasma potential in equilibrium. Then, the equation

written below is expressed the term 'charge fluctuation'

$$\frac{dQ_{1d}}{dt} + \eta Q_{1d} = -\left|I_{eo}\right| \left(\frac{n_{1i}}{n_{i0}} - \frac{n_{1e}}{n_{e0}}\right),\tag{10}$$

where
$$\eta = \left(\frac{|I_{e0}|e}{C_g}\right) \left(\frac{1}{T_e} + \frac{1}{T_i - e\phi_{g0}}\right)$$
 is the dust charging rate and Q_{1d} is the perturbed dust grain charge and is given

as $Q_{1d}=Q_d+Q_{d0}$. The capacitance of dust grain is denoted by $C_g = \left(a + a^2 \lambda_{De}^{-1}\right)$ and λ_{De} is the electron Debye length.

Replacing $\frac{d}{dt}$ by $-i\omega$ in Eq. (10), we deduce the dust grain charge fluctuation

$$Q_{1d} = \frac{\left| I_{e0} \right|}{i(\omega + i\eta)} \left(\frac{n_{1i}}{n_{i0}} - \frac{n_{1e}}{n_{e0}} \right). \tag{11}$$

Substituting the values of n_{1e} and n_{1i} from Eqs. (4) and (5) in Eq. (11), we obtain

$$Q_{1d} = \frac{\left|I_{e0}\right|}{i(\omega + i\eta)} \left[\frac{k_x^2 \phi}{(\omega^2 - \omega_{ci}^2)m_i} + \frac{k_z^2 \phi}{m_i \omega^2} + \frac{k_x^2 \phi}{m_e (\omega^2 - \omega_{ce}^2)} + \frac{k_z^2 \phi}{m_e \omega^2} \right].$$

 $\begin{array}{l} m & i \left(\omega + i\eta\right) \left[\left(\omega^2 - \omega_{ci}^2\right) m_i & m_i \omega^2 & m_e \left(\omega^2 - \omega_{ce}^2\right) & m_e \omega^2 \right] \\ \text{Under the view of overall charge neutrality in equilibrium, we can write, } -en_{i0} + en_{e0} + Q_{d0}n_{d0} = 0 \quad \text{or} \\ \frac{1}{2} \left[n_{e0} - 1 \right] \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} - \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2} \left[\frac{1}{2} - \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) \right] \\ \frac{1}{2}$

$$n_{d0}/n_{e0} = \{\delta - 1\} \{e/Q_{d0}\}, \text{ where } \delta = n_{i0}/n_{e0}.$$
Substituting the values from Eqs. (4)-(7) and (12) inPoisson's equation
$$\nabla^{2} \phi = 4\pi (n_{1e}e - n_{1i}e + n_{1b}e + n_{d0}Q_{1d} + Q_{d0}n_{1d}), \text{ and taking } \omega << \omega_{ce}, \text{ we obtain}$$

$$\nabla^{2}_{\perp} \left(-\frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} + \frac{\omega_{pi}^{2}}{\omega^{2} - \omega_{ci}^{2}} + \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pe}^{2}m_{e}}{(\omega^{2} - \omega_{ci}^{2})m_{i}} - \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} + \frac{\omega_{pd}^{2}}{\omega^{2}} \right) \phi$$

$$+ \left(-1 + \frac{\omega_{pe}^{2}}{\omega^{2}} + \frac{\omega_{pi}^{2}}{\omega^{2}} - \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pe}^{2}m_{e}}{\omega^{2}m_{i}} + \frac{i\beta\omega_{pe}^{2}}{(\omega + i\eta)\omega^{2}} + \frac{\omega_{pd}^{2}}{\omega^{2}} \right) k_{z}^{2} \phi$$

$$= \frac{\omega_{pb}^{2}k_{z}^{2} \phi}{(\omega - k_{z}\upsilon_{b0})^{2}}.$$
(13)

where
$$\omega_{pe}^2 = \frac{4\pi n_{e0}e^2}{m_e}$$
, $\omega_{pi}^2 = \frac{4\pi n_{i0}e^2}{m_i}$, $\omega_{pd}^2 = \frac{4\pi n_{d0}e^2}{m_d}$, $\omega_{pb}^2 = \frac{4\pi n_{b0}e^2}{m_b}$ and

 $\beta \left(= \frac{\left| \mathbf{I}_{e0} \right| n_{d0}}{e n_{e0}} \right)$ is the dust plasma coupling parameter.

Using charge neutrality condition mentioned by Prakash and Sharma [14], we can also write dust plasma coupling parameter as $\beta = 0.1\pi a^2 n_{d0} v_{te}$, where $v_{te} \left(=\sqrt{T_e/m_e}\right)$ is the electron thermal velocity. The dust charging rate can be given as

$$\eta = 0.01 \omega_{pe} \frac{n_{e0}}{n_{i0}} \frac{a}{\lambda_D}$$
. Rewriting Eq. (13) for axially symmetric case, we obtain
$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + p^2 \phi = -\frac{\omega_{pb}^2 k_z^2 \phi}{(\omega - k_z v_{b0})^2 l_2^2}$$
(14)

where
$$p^{2} = \frac{l_{1}^{2}}{l_{2}^{2}}$$
,
 $l_{1}^{2} = \left(-k_{z}^{2} + \frac{\omega_{pe}^{2}k_{z}^{2}}{\omega^{2}} + \frac{\omega_{pi}^{2}k_{z}^{2}}{\omega^{2}} - \frac{i\beta}{(\omega + i\eta)}\frac{\omega_{pe}^{2}m_{e}k_{z}^{2}}{\omega^{2}m_{i}} + \frac{i\beta\omega_{pe}^{2}k_{z}^{2}}{(\omega + i\eta)\omega^{2}} + \frac{\omega_{pd}^{2}k_{z}^{2}}{\omega^{2}}\right)$
 $l_{2}^{2} = \left(-\frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} + \frac{\omega_{pi}^{2}}{\omega^{2} - \omega_{ci}^{2}} + \frac{i\beta}{(\omega + i\eta)}\frac{\omega_{pe}^{2}m_{e}}{(\omega^{2} - \omega_{ci}^{2})m_{i}} - \frac{i\beta}{(\omega + i\eta)}\frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} + \frac{\omega_{pd}^{2}}{\omega^{2}}\right)$.

If we neglect the terms containing dust, we can rewrite Eq. (14) as

$$p^{2} = \frac{\left(-1 + \frac{\omega_{pe}^{2}}{\omega^{2}} + \frac{\omega_{pi}^{2}}{\omega^{2}}\right)k_{z}^{2}}{\left(-\frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} + \frac{\omega_{pi}^{2}}{\omega^{2} - \omega_{ci}^{2}}\right)}$$
(15)

In the absence of the electron beam, Eq. (14) can be rewritten as

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + p^2 \phi = 0.(16)$$

Equation (16) is a Bessel equation and will give a familiar solution as $\phi = AJ_0(p_{n1}r) + BY_0(p_{n1}r)$, where A and B are constants, J_0 and Y_0 are the zeroth order Bessel function of 1^{st} and 2^{nd} kind, respectively. At r = 0, $Y_0 \rightarrow \infty$ and hence B=0,

$$\phi = AJ_0(p_{n1}r)$$
, $p = p_{n1}$. At $r = a_1$, ϕ must vanish, hence, $J_0(p_{n1}a_1) = 0$, $p_{n1} = \frac{x_n}{a_1}$, $(n = 1, 2, 3...)$, where

 X_n are the zeros of the Bessel function $J_0(X)$. The electron beam being present, the solution of wave-function ϕ can be stated as orthogonal sets of wave-function

$$\phi = \sum_{m} A_m J_0(p_m r). \tag{17}$$

Further, using the value of ϕ from Equation (17) in Equation (13) and multiplying both the sides of Eq.(13) by $rJ_0(p_{n1}r)$ and integrating over r from 0 to a_1 , here a_1 is the plasma radius, retaining only the dominant mode m=n, we obtain

$$p^{2} - p_{n1}^{2} = -\frac{\omega_{pb}^{2} k_{z}^{2} \beta''}{(\omega - k_{z} \upsilon_{b0})^{2} l_{2}^{2}}, (18)$$

where $\beta'' = \frac{\int_{0}^{r_{b}} r J_{0}(p_{m}r) J_{0}(p_{n1}r) dr}{\int_{0}^{r} r J_{0}(p_{m}r) J_{0}(p_{n1}r) dr}$ if $r_{b} \neq a_{1}$.

Putting the value of p^2 in Eq. (18) from Eq. (14), we obtain

$$(\omega^{2} - \alpha_{2}^{2}) \left(\omega - k_{z} \upsilon_{b0}\right)^{2} = \frac{\omega_{pb}^{2} k_{z}^{2} \omega^{2} \beta''}{(p_{n1}^{2} + k^{2})}$$
(19)

where

$$\begin{split} &\alpha_{2}^{2} = \frac{h_{1}^{2}}{h_{2}^{2}}, \\ &h_{1}^{2} = m_{1} + im_{2} \ , \\ &m_{1} = \left(\frac{\omega_{pe}^{2} k_{z}^{2}}{p_{n1}^{2} + k_{z}^{2}} \right) + \omega_{pd}^{2} - \frac{\beta\eta}{(\omega^{2} + \eta^{2})} \left(\frac{k_{z}^{2}}{p_{n1}^{2} + k_{z}^{2}} \right) \right) \omega_{pe}^{2} \left(\frac{m_{e}}{m_{i}} - 1 \right), \\ &m_{2} = -\frac{\beta\omega}{(\omega^{2} + \eta^{2})} \left(\frac{k_{z}^{2}}{p_{n1}^{2} + k_{z}^{2}} \right) \right) \omega_{pe}^{2} \left(\frac{m_{e}}{m_{i}} - 1 \right), \\ &h_{2}^{2} = m_{3} + im_{4} \ , \\ &m_{3} = 1 + \frac{p_{n1}^{2}}{(p_{n1}^{2} + k_{z}^{2})} \frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} + \frac{p_{n1}^{2}}{(p_{n1}^{2} + k_{z}^{2})} \frac{\omega_{pi}^{2}}{\omega^{2} - \omega_{ci}^{2}} - \frac{\beta\eta}{(\omega^{2} + \eta^{2})} \frac{p_{n1}^{2}}{(p_{n1}^{2} + k_{z}^{2})} \left(\frac{\omega_{pe}^{2}}{\omega^{2} - \omega_{ci}^{2}} + \frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} \right). \end{split}$$

Therefore,

$$\left|\alpha_{2}\right| = \left[\left(\frac{m_{1}m_{3} + m_{2}m_{4}}{m_{3}^{2} + m_{4}^{2}}\right)^{2} + \left(\frac{m_{2}m_{3} + m_{1}m_{4}}{m_{3}^{2} + m_{4}^{2}}\right)^{2}\right]^{1/4}$$

Considering Eq. (20) and applying condition essential for TG wave $(k_z \ll k_\perp, \omega_{pi} \ll \omega \ll \omega_{ce})$ we will get $\omega = (\omega_{pe}k_z)/k_\perp$ as $(\omega_{pe}/\omega_{ce}) \ll 1$ [15, 16], where $k_\perp = \frac{X_n}{a_1}$. In the presence of beam, we expand the

(21)

frequency $\omega as \omega = \chi_1 + \delta_1 = k_z \upsilon_{b0} + \delta_1$, where δ_1 is the small frequency discrepancy due to the finite value on RHS of Eq. (19).

Following to Mikhailovski [16], the growth rate of the unstable mode is

$$\gamma = \operatorname{Im} \delta_{1} = \frac{\sqrt{3}}{2} \left[\frac{\omega_{pb}^{2} k_{z}^{2} \alpha_{2} \beta''}{2[(X_{n}/a_{1})^{2} + k_{z}^{2}]h_{2}^{2}} \right]^{1/3} . (22)$$

The real part of unstable mode's frequency is

$$\omega_r = k_z \left(\frac{2eV_b}{m_b}\right)^{1/2} - \frac{1}{2} \left[\frac{\omega_{pb}^2 k_z^2 \alpha_2 \beta''}{2[(X_n/a_1)^2 + k_z^2]h_2^2}\right]^{1/3}.$$
(23)

In this also the real part of the frequency of unstable mode increases with beam voltage similar to the experimental observation of Chang [17].

The phase velocity of the unstable mode is

$$v_{ph} = \frac{\omega_r}{k_z} = \left(\frac{2eV_b}{m_b}\right)^{1/2} - \frac{1}{2k_z} \left[\frac{\omega_{pb}^2 k_z^2 |\alpha_2|\beta''}{2[(X_n/a_1)^2 + k_z^2]h_2^2}\right]^{1/3} .(24)$$

From Eq. (18), we can say that the phase velocity of the unstable mode increases with the beam voltage V_b.

III. NUMERICAL RESULT AND DISCUSSIONS

The common dusty plasma parameters are taken for calculations, experiment. Using Eq. (21)we have plotted the dispersion curves of Gould-Trivelpiece(TG) mode (cf.Fig.1) for the different values of $\delta = 1, 2, 3, 5$ ($\delta = 1$ specifies for absence of dust). The parameters used for calculations are: ion plasma density $n_{i0}=10^9$ cm⁻³, electron plasma density $n_{e0}=0.1\times10^9-1\times10^9$ cm⁻³, $m_d=10^{12}m_p$ (for the size of 1µm grain , a mass of density of ~1g cm⁻³), temperatures of electron and ion is taken equal to 0.2eV, plasma radius $a_1=2$ cm, beam radius $r_b=1.5$ cm, guide magnetic field $B_s=4\times10^3$ G, dust grain density $n_{d0}=1\times10^4$ cm⁻³, $m_i/m_e=7.16\times10^4$ for potassium plasma and the average dust grain size $a=5\mu$ m. The 1st zero of the Bessel function (i.e., mode number=1) is taken.

The effect of dust charge fluctuation has direct relation with β and η . β is same as an effective collision frequency therefore it is called as coupling parameter. β comes into picture due to the coupling of the TG mode with the dust chargefluctuations. Also, the dust charge fluctuation has a decay rate η . The electron and ion currents into the dust particles opposed any change in the grain potential. Smaller values of a and n_{d0} lower the dust charge fluctuations.

Considering dust charge fluctuation negligible, we have plotted the electron beam modewith beam velocity= 1.2×10^8 cm/sec (beam energy=10.4 eV). We have choose the velocity of beam such that it intersects the TG mode, here the condition which must be considered for the existence of TG mode is $k_{\perp} = k$, $k_Z \ll k_{\perp}$. Further, the frequencies and the corresponding wave numbers of the unstable wave are acquired by the point of intersection between beam mode and the TG mode (cf. Fig.1) which are given in Table I.At these points of intersection the TG wave in plasma interact with beam mode. Through the interaction, the beam transfers its energy to the wave and makes it unstable and hence wave grows.

Taking the values from Table I and using Eq. (22) we have plotted the growth rate of unstable mode $\gamma (\text{sec}^{-1})$ against δ (cf. Fig.2). As the value of δ increases, the number density of electron n_{e0} decreases, this lead to the disturbance in the proportion of negative charge per unit volume inherent by the dust particles which becomes more, as a result, the unstable mode frequency enhances, which in turn decreases the wave damping. Our theoretical results are in line with the experimental results of Barkan et al.[18] and theoretical findings of Chow and Rosenberg [19].

Using Eq. (22) we have plotted the growth rate $\gamma(\sec^{-1})$ of unstable mode as a function of size of dust particles a (cm) for different values of δ (cf. Fig.3). It is found that the growth rate of unstable mode decreases with size of the dust particles. It is because of the appetite of dust grains for electrons. As the size of dust particles increase more and more electrons will stick to the dust particles which reduce their surface potential and hence average dust grain charge Q_{d0} also drops. To compensate this decline in the surface potential, the particles of dust grains capture more electrons from the ambient plasma thus the growth rate associated with TG mode decreases.

Again, using Eq. (22) we have plotted the growth rate $\gamma(\sec^{-1})$ of unstable mode as a function of number density of dust grain $n_{d0}(\text{cm}^{-3})$ for different values of δ (cf. Fig.4). It is found that the growth rate of unstable mode falls with increase in the density of dust particles. As the number density of dust grain increases their need for electrons also increases, hence growth rate of TG wave decreases to compensate the fall in average dust grain charge. Our result resembles with theoretical outcomes of Tribecheet al.[21].

Moreover, we can see that the unstable mode's real frequency varies as square root of the beam voltage (cf. Eq.23). Our results are similar with the experimental findings of Chang [17] without dust grains case. Also, Eq. (24) shows that the phase velocity of the TG mode increases with the beam energy.

TABLE I. From Fig.2 (beam of electron + plasma containing negatively charged dust grains) we acquire the values of frequencies of unstable modeand axial wave number k_z(cm⁻¹)diverse values of δ.











FIG.4 Growth rateas a function of dust density for different values of δ .

IV CONCLUSION

The excitation of Gould-Trivelpiece mode by an electron beam drives to instability in a dusty plasma cylinder. The instability occurs due to interaction of electron beam with dusty plasma species via Cerenkov interaction. The effect of dust charge fluctuations on electron driven TG mode is studied. The frequency and the growth rate of unstable TG mode up-rises with the negatively charged dust grains and decreases with size and number density of dust particles. It is also found that the growth rate scales as the (1/3)rdpower of the beam density. This result is similar to the experimental observations of Chang [17]. The growth rate results with negatively charged dust grains in our work qualitatively similar to the experimental observations of Barkan*et al.*[18] and theoretical prediction of Chow and Rosenberg [19] and Tribeche*et al.* [20]. This model of charging dust particles is applicable only for conducting dust particles which are in interstellar clouds and in laboratory plasma/tokomaks. Our work may be advantageous in plasma processing in laboratory[21] and in Space Shuttle exhaust [22].

REFERENCES

- [1] A. W. Trivelpieceand R. W. Gould, *Space Charge Waves in Cylindrical Plasma Columns*, Jour. Appl. Phys. **30**, pp. (1959), 1784.
- [2]J. H. Malmberg and C. B. Wharton, Dispersion of Electron Plasma Waves, Phys. Rev. Lett. 17, (1966), 175.
- [3]W. M. Mannheimer, Nonlinear Development of an Electron Plasma Wave in a Cylindrical Waveguide, Phys. Fluids 12, (1969), 2426.
- [4] J. P.Lynov, P.Michelsen, H. L. Pecseli, J. Juul Rasmussen, K.Saeki, and V. A. Turikov, Observations of Solitary Structures in a Magnetized, Plasma Loaded Waveguide, Phys. Scr. 20, (1979), 328.
- [5]H.Schamel, Theory of Electron Holes, Phys. Scr. 20, (1979), 336.
- [6] R. L Stenzel, and J. M. Urrutia, Trivelpiece-Gould modes in a uniform unbounded plasma, Phys. Plasmas 23, (2016), 092103.
- [7] G. Praburamand A. K. Sharma, Second-harmonic excitation of a Gould- Trivelpiece mode in a beam-plasma system, J. Plasma Phys. **48**, (1992), 3.
- [8] J. B. Pieper and J.Goree, Dispersion of Plasma Dust Acoustic Waves in the Strong Coupling Regime, Phys. Rev. Lett., 77,(1996), 3137.
- [9]C. Thompson, A. Barkan, N. DAngelo, R. L. Merlino, *Dust acoustic waves in a direct current glow discharge*, Phys.Plasmas 4, (1997), 2331.
- [10]Suresh. C. Sharma, K.Sharmaand R.Walia, *Ion beam driven ion- acousticwaves in a plasma cylinder with negatively charged dust grains,* Phys. Plasmas **19**, (2012), 073706.
- [11]Suresh C. Sharma, DaljeetKaur, Ajay Gahlot, and Jyotsna Sharma, *Excitation of dust acoustic waves by an ion beam in a plasma cylinder with negatively charged dust grains*, Phys. Plasmas **21**, (2014), 103702.
- [12] E. C. Whipple, T. G. Northdrop and D. A. Mendis, The electrostatics of dusty plasma, J. Geophys. Res. 90, (1985),7405.
- [13]M. R. Jana, A. Sen, and P. K. Kaw, Collective effects due to charge fluctuation dynamics in a dusty plasma, Phys. Rev.E 48, (1993), 3930.
- [14] VedPrakash and Suresh C. Sharma, Excitation of surface plasma waves by an electron beam in a magnetized dusty plasma, Phys. Plasmas 16, (2009), 093703.
- [15]V.K. Jain and P.Khristiansen, *Excitation of Electron Cyclotron Harmonic Instabilities in a thin Beam-Plasma System*, J. Plasma Phys. and contr. Fusion **26**,(1984), 613.
- [16]V.K. Jain, and P. Khristiansen, *Recurrence of Electron Cyclotron Harmonic Wave Instability in a Beam Plasma System*, J. Plasma Phys. and contr. Fusion **25**, (1983), 1169.
- [17] R.P.H.Chang, Lower-Hybrid Beam-Plasma Instability, Phys. Rev. Lett. 35, (1975), 28.
- [18]A. B. Mikhailovski, *Theory of Plasma instabilities vol.1Instabilities of a Homogeneous Plasma*, New York ConsultantBurea, (1974).
- [19]A.Barkan, N. D' Angelo, and R.L.Merlino, Laboratory experiments on electrostatic ion cyclotron waves in a dusty plasma, Planet. Space Sci. 43, (1995), 905.
- [20]V.W. Chow and M. Rosenberg, Electrostatic ion cyclotron instability in dusty plasmas, Planet. Space Sci. 43, (1995), 613.
- [21] M.Tribeche and T.H. Zerguini, Current-driven dust ion-acoustic instability in a collisional dusty plasma with charge fluctuations, Phys. Plasmas, 41, (2001) 394.
- [22]K. Ostrikov, U. Cvelbar, and A. B. Murphy, *Plasma nanoscience: setting directions, tackling grand challenge*, J.Phys.Appl.D:Phys.44, (2011), 174001.
- [23] P.A., Bernhardt, G. Ganguli, M.C. Kelley, and W.E. Swartz, *Enhanced radar backscatter from space shuttle exhaust in the ionosphere*, J. Geopys. Res. **100**, (1995), 23811.