Structure Theorem for B(1,2) $\overline{s}$ - Near Subtraction Semigroups

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Abstract: In this paper, we introduce the concept of B (1,2) $\overline{s}$- Near Subtraction Semigroups and give the structure theorem for the same. By x we mean a zero-symmetric near subtraction semigroups. Define x to be a p_k (p_k') near subtraction semigroups if $a^k X = aXa(Xa^k = aXa)$ for all a∈X and a near subtraction semigroups x is said to be a p_k $(m,n)(p_k'(m,n))$ near substraction semigroups if $a^k X = a^m X a^n$ $(Xa^k = a^m X a^n)$ for all a∈ X. Motivated by these concept we introduce B(1,2) near subtraction semigroups and their generalization and similarities. A near subtraction semigroups X is said to be a B(1,2) near subtraction semigroups is the right (left) X-subalgebra of near subtraction semigroups X generated by 'a'.

Keywords: Left permutable, $\overline{s}$ -near subtraction semigroups, $B_k(B_k')$ near subtraction semigroups, B(1,2) near subtraction semigroups.

1.Introduction

B.M. Schein [10] considered system of the form (X;**0**;\) , where X is set of functions closed under the composition "0" of functions (and hence (X;0) is a function semigroups) and the set theoretic subtraction "\" (and hence (X;\) is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroups is isomorphic to a difference semigroups of invertible function B.Zelinka [11] discussed a problem proposed by B.M.Schein concerning the structure of multiplication in a subtraction semigroups. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. For basic definition one may refer to Pilz[8].Motivated by the study of B(1,2) near subtraction semigroups in "A Study on Regularities in Near - ring" by S.Jayalakshmi .We introduced new concepts "B(1,2) near subtraction semigroups" .

2. Preliminary

Definition :2.1

A non empty set X together with two binary operation '–' and '•' is said to be $\overline{s}$ **− near subtraction semigroups** if it satisfies the following

i) (X;–) is a subtraction algebra.

ii) (X;•) is a semigroups

iii) $x(y-z) = xy - xz$ and $(x-y)z = xz - yz$ for every $x, y, z \in X$

Definition:2.2

A near subtraction semigroups X is said to have **property(α)** if aX is a subalgebra of (X,-) for every a∈X.

Definition:2.3

A near subtraction semigroups X is called a **generalized near-field** if for each a $\in X$ there exists unique b$\in X$ such that a=aba and b =bab.

Theorem :2.4

Let X be a near subtraction semigroups . Then the following are equivalent.

i) X is a GNF

ii) X is a regular and each idempotent is central

iii) X is regular and sub commutative

Lemma:2.5

If X is a K(1,2) near subtraction semigroups, E ⊆ C(X)

Remark :2.6

If X is a s-near subtraction semigroups with property(α) , then $< a >_r = aX$ and $< a >_l = Xa$, for all a∈ X.

Lemma:2.7

Let X be a zero –symmetric near subtraction semigroups without non-zero nilpotent elements. Then ab=0 implies ba=0.

Remark:2.8

Whenever a zero-symmetric near subtraction semigroups contains no non-zero nilpotent elements in view of above lemma2.7,X has IFP.

Theorem:2.9

Let X be a $\bar{s}$- near substraction semigroup with property (α). If X is a B(1,2) near subtraction semigroups , then $M_1 \cap M_2 = M_1M_2$ for any two left x-subalgebra M_1 *and* M_2 of X.

Corollary:2.10

Let X be a B(1,2) $\bar{s}$ -near subtraction semigroups with property(α) .Then X is strongly regular.

Corollary:2.11

Let X be a B(1,2) $\bar{s}$ - near subtraction semigroups with property(α). Then X is regular.

3. Structure Theorem for B(1,2) $\bar{s}$-Near Subtraction Semigroups

In this section first we study certain properties involving structure theorem for B(1,2) in the class of near subtraction semigroups.

Definition:3.1

We say that a near subtraction semigroups X has the property B(m,n) , if there exist positive integers m,n such that $< a >_r^m X = X < a >_l^n$, for all a in X.

Example:3.2

Let X= {0,a,b,1} in which "–'' and "•"are defined by,

$-$	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

One can check the $< 0 >_r =< 0 >_l = \{0\}$ $< a >_r =< a >_l = \{0, a\}$, $< a >_r = < b >_l = \{0, b\}$, $< 1 >_r =< 1 >_l = \{0, a, b, 1\}$ and so this X is B(m,n) near subtraction semigroups, for all positive integer m and n.

Proposition:3.3

Let X be a $\bar{s}$ -near subtraction semigroups .Then X is a B (1,2) near subtraction semigroups with property(α) if and only if X is a K(1,2) near subtraction semigroups.

Proof:

Assume that X is a B(1,2) near subtraction semigroups with property(α) .By corollary2.11,X is regular and so, for a ∈ X, a=axa , for some x ∈ X.Now aX= axaX ⊆ aXXX ⊆ aXX = $< a >_r$ X =X$< a >_l^2$ =XXaXa ⊆ Xa ie, aX ⊆ Xa. Similarly Xa ⊆ aX and so aX = Xa ie, X is a sub commutative. Since X is regular and sub commutative ,by Theorem 2.4 , X is regular and E⊆C(X).Let $X_1 \in aX$. Then for y∈X, X_1= ay =axay =a(xay)=ay(xaxa) =ayxxaa=(ay x^2a^2) ∈ Xa^2 .ie, aX ⊆ Xa^2.Trivially Xa^2 ⊆ Xa =aX . Thus aX = Xa^2 ,for all a in X .ie., X is a K(1,2) near subtraction semigroups .conversely, Let X be a K (1,2) near subtraction semigroups.

Since X is a $\bar{s}$ - near subtraction semigroups a∈ aX =Xa^2.ie, X is strongly regular and so X is regular. Since X is a K(1,2) near subtraction semigroups, by Lemma2.5, E⊆C(X).Then , by Theorem2.4 ,X is regular and sub commutative .In the review of Remark2.6 $< a >_r$X =aXX =XaX =XXa =XXaxa ∈ XXaXa =X $< a >_l^2$ .ie, $< a >_r X \subseteq X < a >_l^2$. Also $X < a >_l^2 = XXaXa \subseteq XXa = XaX = aXX =< a >_r X$. ie, X$< a >_l^2 \subseteq < a >_r X$. These two imply that $< a >_r X = X < a >_l^2$ .ie, X is a B(1,2) near subtraction semigroups.

Proposition:3.4

Let X be a $\bar{s}$ -near subtraction semigroups with property (α). Then X is a B(1,2) near subtraction semigroups if and only if X is a GNF.

Proof:

Assume that X is a GNF. Now for a∈ X, $< a >_r X = aXX = axaXX \in aXaXX = XXaXa = X < a >_l^2$.(ie) $< a >_r X \subseteq X < a >_l^2$ .Similarly X $< a >_l^2 = XXaXa \subseteq XXa = XaX = aXX = < a >_r X$.(ie) $X < a >_l^2 \subseteq < a >_r X$ .From these, we get that $< a >_r X = X < a >_l^2$.(ie) X is a B(1,2) near subtraction semigroups. Conversely, assume that X is a B(1,2) near subtraction semigroups .Since X is a $\bar{s}$ -near subtraction semigroups with property (α) and by Corollary2.11, X is regular. By Theorem2.4, X is a K(1,2) near subtraction semigroups. Again by Lemma2.5 ,E ⊆ $C(X)$. X is GNF.

Proposition:3.5

Let X be a B(1,2) $\bar{s}$ - near subtraction semigroups with property(α) and let A and B be any two left X -subalgebra of X. Then we have the following :

i. $\sqrt{A}$=A
ii. $A \cap B = AB$
iii. $A^2 = A$
iv. If A ⊂ B then AB=A
v. $A \cap XB = AB$
vi. If A is proper, then each element of A is a zero divisor

vii. A is a completely semiprime ideal of X.

Proof:

i) For x∈ $\sqrt{A}$, there exists some positive integer k such that $x^k \in A$. Since x is a B(1,2) $\bar{s}$ -near subtraction semigroups with property(α) . By Corollary2.10, X is strongly regular. If x∈ X, then x=ax^2,for some a∈ X.This implies x=a$x^2 = (ax)x = a(ax^2)x = a^2x^3 = \cdots = a^{k-1}x^k \in XA \subseteq A$.(ie)x ∈ A .Thus $\sqrt{A} \subseteq A$ obviously A⊂$\sqrt{A}$ and so A =$\sqrt{A}$

ii) Since X is a $\bar{s}$ -near subtraction semigroups with property(α) , by the Theorem2.9,AB=A∩ B

iii) Taking B=A in (ii) we get A=A^2

iv) Suppose that A ⊂ B. Then $A \cap B = A$ and (ii) gives A=AB

v) $A \cap XB \subset A \cap B$ and so $A \cap XB \subset AB$ (by(ii)) .Also $AB = A \cap B \subset A$ and $AB \subset XB$. Therefore $AB \subset A \cap XB$. Hence $AB = A \cap XB$

vi) By the Remark2.8, X has the IFP. Then the concept of left zero –divisors, right zero-divisors and zero–divisors are equivalent in X. Thus we need only to prove that A^* consists of only zero-divisors. Let a∈ $A^*$by (iii) for the principal left x-subalgebra Xa, Xa= $(Xa)^2$=XaXa Consequently, for any x∈ X, there exists y,z∈ X such that xa=yaza . (ie)(x-yaz)a=0. Similarly (yaz-x)a=0 .If a is not a zero-divisor , then x-yaz=0 and yaz-x=0. This implies x=yaz ∈ $XAX \subset A$ (ie) $X \subset A$. Hence X=A which is a contradiction to the hypothesis that A is proper.Thus a∈ A^*. Hence 'a' is a zero-divisor.

vii) Let $a^2 \in A$.X has strong IFP. So axa ∈ A By Corollary2.11 X is regular. Then a∈ A . Hence A is completely semi prime.

Proposition:3.6

Let X be a $\bar{s}$ - near subtraction semigroups with property(α) .Then X is a B(m,n) near subtraction semigroups, for all positive integer m,n if and only if X is a B(1,2) near subtraction semigroups.

Proof:

Assume that X is a B(1,2) near subtraction semigroups. By Proposition3.3,X is a GNF. Therefore by Theorem2.4. X is regular and E ⊆ C(X) .Let a ∈ X, here $< a >_r^m X =< a >_r < a >_r^{m-1} \subseteq < a >_r X = X < a >_l^2 \subseteq X < a >_l Xa = X < a >_l Xaxa \subseteq Xxa \subseteq X(xa)^n \in X(Xa)^n = X < a >_l^n$. Similary $X < a >_l^n = X < a >_l^{n-2} < a >_l^2 \in X < a >_l^2 = < a >_r X \subseteq aXX = axaXX \in axX = (ax)^m X \in (aX)^m X =< a >_r^m X$ $ie., X < a >_l^n \subseteq < a >_r^m$. So $< a >_r^m X = X < a >_l^n$ and hence X is B(m,n) near subtraction semigroups, for all positive integer m,n. Converse part is trivial.

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