PRODUCT ON ANTI FUZZY GRAPHS

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Abstract

The anti fuzzy graph is introduced by M.S.Sunitha, A.Vijayakumar, R.Muthuraj and A.Sasirekha. A new anti fuzzy graph can be obtained from two given anti fuzzy graphs using the operations, cartesian product, composition, tensor product and normal product. In this paper, we study about the degree of a vertex in anti fuzzy graphs which are obtained from two given anti fuzzy graph using the operations cartesian product, composition, tensor product and normal product of two anti fuzzy graph

Keywords

Anti fuzzy graph, Cartesian product, Composition, Tensor product, Normal product.

Introduction

The concept of fuzzy graph was first introduced by Kaufmann from the fuzzy relation introduced by Zadeh. Although Rosenfield introduced another elaborated definition including fuzzy vertex and fuzzy edge and also introduced the notion of fuzzy graph. The operation of union, join, Cartesian product and composition on two fuzzy graphs were defined by Moderson J.N and Peng C.S. In this paper we study about the degree of a vertex in anti fuzzy graphs which are obtained from two given anti fuzzy graphs using the operations Cartesian product , Composition, Tensor product and Normalproduct

Definition 1.1

An **anti fuzzy graph**$\mathcal{A} = (\sigma, \mu)$ is a pair of functions σ:V→[0,1] and $\mu: V \times V$ →[0, 1] with μ(u,v) ≥ σ(u) ∨σ(v) for all u,v in V whereV is a finite non empty set and ∨ denote maximum.

Definition 1.2

The graph $\mathcal{A}^*$= (V,E) is called the **underlying crisp graph** of an anti fuzzy graph $\mathcal{A}$ where V = {u / σ(u) $\neq 0$} and E = {(u,v) ∈ V× V / μ(u,v) ≠ 0}.

Definition 1.3

Let $\mathcal{A}$ = (σ,μ) be an anti fuzzy graph. The **degree of a vertex σ(u) of an anti fuzzy graph** is sum of degree of membership of all those edges which are incident on vertex σ(u) and is denoted by $d_{\mathcal{A}}(\sigma(u))$ = d(σ(u)) = $\sum_{u\neq v}$ μ(u, v) = $\sum_{uv\in E}$ μ(u, v).

Main Results

Definition 2.1

The Cartesian Product of two anti fuzzy graph $\mathcal{A}_1$ and $\mathcal{A}_2$ is defined as a anti fuzzy graph $\mathcal{A}$=$\mathcal{A}_1 \times \mathcal{A}_2$:($\sigma_1 \times \sigma_2$, $\mu_1 \times \mu_2$) on $\mathcal{A}^*$: (V,E) where V= $V_1 \times V_2$ and

E={$((u_1,u_2)(v_1,v_2))/\ u_1 = v_1,\ u_2v_2 \in E_2 (or) u_2 = v_2, u_1v_1 \in E_1$} with

$(\sigma_1 \times \sigma_2)(u_1,u_2)$=$\sigma_1(u_1) \vee \sigma_2(u_2)$

$\forall\ (u_1,u_2) \in V_1 \times V_2$ and

$(\mu_1 \times \mu_2)((u_1,u_2)(v_1,v_2))$=

$$\begin{cases} \sigma_1(u_1) \vee \mu_2(u_2v_2) & if\ \ u_1 = v_1,\ u_2v_2 \in E_2 \\ \sigma_2(u_2) \vee \mu_1(u_1v_1) & if\ \ u_2 = v_2,\ u_1v_1 \in E_1 \end{cases}$$

2.2 Degree of vertex in Cartesian product

By the definition for any vertex

$(u_1,u_2)\ in\ V_1 \times V_2$

$d_{\mathcal{A}_1 \times \mathcal{A}_2}(u_1,u_2) = \sum_{(u_1,u_2)(v_1,v_2)\in E} (\mu_1 \times \mu_2)((u_1,u_2)(v_1,v_2))$

$= \sum_{u_1 = v_1, u_2v_2 \in E_2} \sigma_1(u_1) \vee \mu_2(u_2v_2) + \sum_{u_2 = v_2, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1)$

In the following theorem ,we find the degree of (u_1,u_2) in $\mathcal{A}_1 \times \mathcal{A}_2$ in terms of those in $\mathcal{A}_1 and \mathcal{A}_2$ in some particular cases.

Theorem 2.3

Let $\mathcal{A}_1$:(σ_1,μ_1) and $\mathcal{A}_2$:(σ_2,μ_2) be two anti fuzzy graphs. If $\sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$ then $d_{\mathcal{A}_1 \times \mathcal{A}_2}(u_1,u_2) = d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$

Proof

From the definition of a degree of a vertex in Cartesian product

$d_{\mathcal{A}_1 \times \mathcal{A}_2}(u_1,u_2)$=

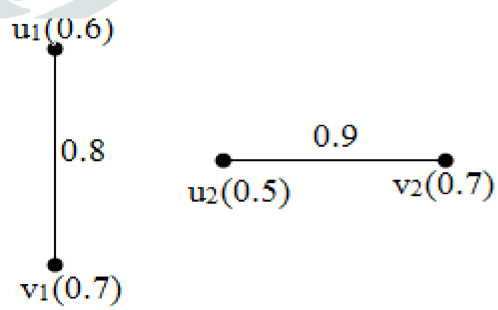
$\sum_{u_1 = v_1, u_2v_2 \in E_2} \sigma_1(u_1) \vee \mu_2(u_2v_2)$ + $\sum_{u_2 = v_2, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1)$

$=\sum_{u_2v_2 \in E_2} \mu_2(u_2v_2) + \sum_{u_1v_1 \in E_1} \mu_1(u_1v_1)$

Since $\sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$

$= d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$

Example 2.4



$\mathcal{A}_1 \mathcal{A}_2$

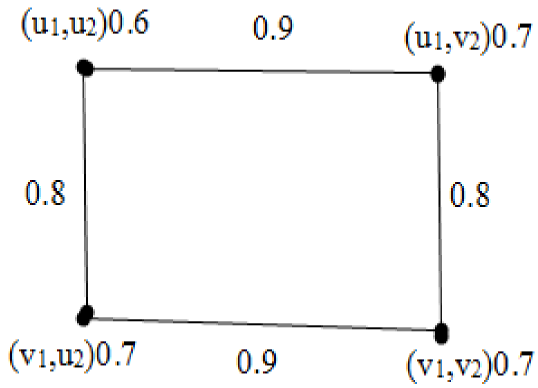


Fig 1

Cartesian product of $\mathcal{A}_1$&$\mathcal{A}_2$($\mathcal{A}_1 \times \mathcal{A}_2$)

Here $\mathcal{A}_1$ and $\mathcal{A}_2$ are two anti fuzzy graph, $\sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$.

$$d_{\mathcal{A}_1 \times \mathcal{A}_2}(u_1, u_2) = d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$$
$$= 0.8 + 0.9 = 1.7$$
$$d_{\mathcal{A}_1 \times \mathcal{A}_2}(u_1, v_2) = d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(v_2)$$
$$= 0.8 + 0.9 = 1.7$$

Similarly we can find all the vertices of $\mathcal{A}_1$ and $\mathcal{A}_2$

Definition 2.5

The composition of two anti fuzzy graph $\mathcal{A}_1$ and $\mathcal{A}_2$ is defined as a anti fuzzy graph $\mathcal{A}$=$\mathcal{A}_1$ [$\mathcal{A}_2$]:($\sigma_1 o \sigma_2$,$\mu_1 o \mu_2$) on $\mathcal{A}^*$= (V,E) whereV=$V_1 \times V_2$and E={$((u_1, u_2)(v_1, v_2))/$ $u_1 = v_1,\ u_2v_2 \in E_2 (or) u_2 \neq v_2, u_1v_1 \in E_1\ (or) u_2 = v_2, u_1v_1 E_2$}with

$(\sigma_1 o \sigma_2)(u_1,u_2)= \sigma_1(u_1) \vee \sigma_2(u_2)$

for all (u_1,u_2) $in\ V_1 \times V_2$.

$(\mu_1 o \mu_2)((u_1, u_2)(v_1, v_2))=$

$$\begin{cases} \sigma_1(u_1) \vee \mu_2(u_2v_2)\ \ if\ \ u_1 = v_1,\ u_2v_2 \in E_2 \\ \sigma_2(u_2) \vee \mu_1(u_1v_1)\ \ if\ u_2 = v_2, u_1v_1 \in E_1 \\ \sigma_2(u_2) \vee \mu_1(u_1v_1)\ \ if\ u_2 \neq v_2, u_1v_1 \in E_1 \end{cases}$$

2.6 Degree of a vertex in Composition

By the definition, for any vertex $(u_1, u_2)\ in\ V_1 \times V_2$

$$d_{\mathcal{A}_1[\mathcal{A}_2]}(u_1, u_2) = \sum_{(u_1,u_2)(v_1,v_2) \in E} (\mu_1\, o \mu_2\,)(\,(u_1, u_2)(v_1, v_2)$$
$$= \sum_{u_1 = v_1, u_2v_2 \in E_2} \sigma_1(u_1) \vee \mu_2(u_2v_2) + \sum_{u_2 = v_2, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1) + \sum_{u_2 \neq v_2, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1)$$

Theorem 2.7

Let $\mathcal{A}_1$:(σ_1, μ_1) and $\mathcal{A}_2$:(σ_2, μ_2) be two anti fuzzy graphs. If $\sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$ then $d_{\mathcal{A}_1[\mathcal{A}_2]}(u_1, u_2)$= $P_2 d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$.

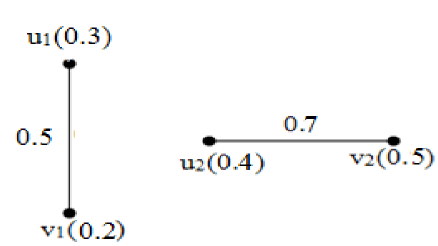
Proof

$$d_{\mathcal{A}_1[\mathcal{A}_2]}(u_1, u_2) = \sum_{u_1 = v_1, u_2v_2 \in E_2} \sigma_1(u_1) \vee \mu_2(u_2v_2) + \sum_{u_2 = v_2, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1) + \sum_{u_2 \neq v_2, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1)$$
$$= \sum_{u_1 = v_1, u_2v_2 \in E_2} \mu_2(u_2v_2) + \sum_{u_2 = v_2, u_1v_1 \in E_1} \mu_1(u_1v_1) + \sum_{u_2 \neq v_2, u_1v_1 \in E_1} \mu_1(u_1v_1)$$

since $\sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$

$$= d_{\mathcal{A}_2}(u_2) + |V_2| \sum_{u_1v_1 \in E_1} \mu_1(u_1v_1)$$
$$= d_{\mathcal{A}_2}(u_2) + P_2 d_{\mathcal{A}_1}(u_1)$$

Example 2.8



$\mathcal{A}_1\mathcal{A}_2$

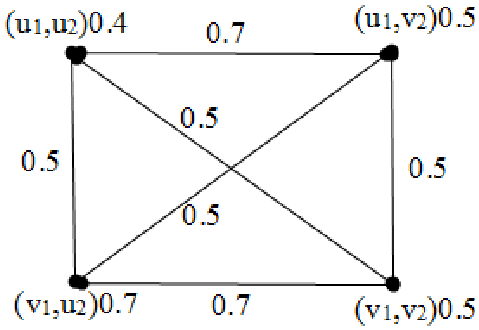


Fig 2

Composition of $\mathcal{A}_1$&$\mathcal{A}_2$ *is* $\mathcal{A}_1$ [$\mathcal{A}_2$]

Consider the anti fuzzy graph $\mathcal{A}_1$ and $\mathcal{A}_2$ with $\sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$.

$d_{\mathcal{A}_1[\mathcal{A}_2]}(u_1, u_2)$= P2$d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$.

= 2(0.5)+0.7=1.7

$d_{\mathcal{A}_1[\mathcal{A}_2]}(u_1, v_2)$= P2$d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(v_2)$.

= 2(0.5)+0.7=1.7

Similarly we can find degree of all the vertices in $\mathcal{A}_1$ [$\mathcal{A}_2$]

Definition 2.9

The **tensor product** of two anti fuzzy graphs (σ_i, μ_i) on G_i=(V_i, X_i),i=1,2 is defined as a anti fuzzy graph ($\sigma_1 \otimes \sigma_2$,$\mu_1 \otimes \mu_2$) on $\mathcal{A}$=(V,X) where V= $V_1 \times V_2$ and X={(u1,u2),(v1,v2)/(u1,v1)∈ X_1, (u2,v2)∈ X_2}.Anti fuzzy sets $\sigma_1 \otimes \sigma_2$ and $\mu_1 \otimes \mu_2$ are defined as

($\sigma_1 \otimes \sigma_2$)(u_1,u_2)= $\sigma_1(u_1) \vee \sigma_2(u_2)$for all (u1,u2) $in\ V_1 \times V_2$.

($\mu_1 \otimes \mu_2$){(u1,u2),(v1,v2)}= { $\mu_1(u_1v_1) \vee \mu_2(u_2v_2)$}∀ (u1,v1)∈ X_1, (u2,v2)∈ X_2

2.10 Degree of a vertex in Tensor product

By the definition, for any vertex $(u_1, u_2)\ in\ V_1 \times V_2$

$d_{\mathcal{A}_1 \otimes \mathcal{A}_2}(u_1, u_2)$=∑$(\mu_1 \otimes \mu_2)${(u1,u2),(v1,v2)}

=$\sum_{(u1,v1)\in E_1} \mu_1(u_1v_1) \vee \mu_2(u_2v_2)$

Theorem 2.11

Let $\mathcal{A}_1$:(σ_1, μ_1) and $\mathcal{A}_2$:(σ_2, μ_2) be two anti fuzzy graphs. If $\mu_2 \leq \mu_1$ then (u_1, u_2)= $d_{\mathcal{A}_1 \otimes \mathcal{A}_2}(u_1, u_2) = d_{\mathcal{A}_1}(u_1)$ and if $\mu_1 \leq \mu_2$ then (u_1, u_2)= $d_{\mathcal{A}_1 \otimes \mathcal{A}_2}(u_1, u_2) = d_{\mathcal{A}_2}(u_2)$

Proof

$d_{\mathcal{A}_1 \otimes \mathcal{A}_2}(u_1, u_2)$=

$\sum_{(u1,v1)\in E_1} \mu_1(u_1v_2) \vee \mu_2(u_2v_2)$

$= \sum \mu_1(u_1v_2) \quad (\mu_2 \leq \mu_1)$

$= d_{\mathcal{A}_1}(u_1)$

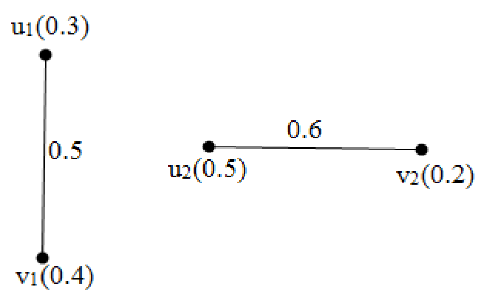
$d_{\mathcal{A}_1 \otimes \mathcal{A}_2}(u_1, u_2)$=

$\sum_{(u1,v1)\in E_1} \mu_1(u_1v_2) \vee \mu_2(u_2v_2)$

$= \sum \mu_2(u_1v_2) \quad (\mu_1 \leq \mu_2)$

$= d_{\mathcal{A}_2}(u_2)$

Example 2.12



{(u1,w)(v1,w)/ (u1,v1)∈ X1 , w ∈ X2 } ∪{(u1,u2)(v1,v2)/ (u1,v1)∈ X1 ,(u2,v2) ∈ X2 }

Anti Fuzzy sets $\sigma_1\ o\ \sigma_2$ and $\mu_1\ o\ \mu_2$ *are defined as* $(\sigma_1 o \sigma_2)(u_1,u_2)$= $\sigma_1(u_1) \vee \sigma_2(u_2)$for all (u1,u2) $in\ V_1 \times V_2$.

$(\mu_1 o \mu_2)((u,u_2)(u,v_2)) = \{\sigma_1(u) \vee \mu_2(u_2v_2)\}$ $\forall$ u∈V_1,(u_2v_2) ∈ X_2

$(\mu_1 o \mu_2)((u_1,w)(v_1,w))=$ $\{\mu_1(u_1v_1) \vee \sigma_2(w)\}$ $\forall$ (u_1,v_1) ∈ X_1,w∈V_2

$\mathcal{A}_1\mathcal{A}_2$

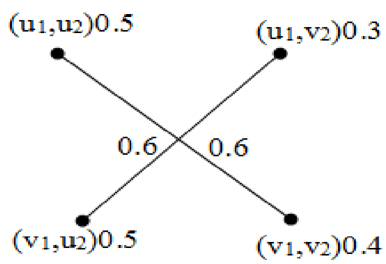


Fig 3

Tensor product of $\mathcal{A}_1$&$\mathcal{A}_2$ *is* $\mathcal{A}_1 \otimes \mathcal{A}_2$

Consider the anti fuzzy graphs $\mathcal{A}_1$ and $\mathcal{A}_2$ with $\mu_1 \leq \mu_2$

$d_{\mathcal{A}_1 \otimes \mathcal{A}_2}(u_1,u_2) = d_{\mathcal{A}_2}(u_2)$ =0.6

$d_{\mathcal{A}_1 \otimes \mathcal{A}_2}(v_1,v_2) = d_{\mathcal{A}_2}(v_2)$ =0.6

Definition2.13

The normal product of two anti fuzzy graph on $\mathcal{A}$=(V,X)where V= $V_1 \times V_2$ and X={(u,u_2)(u,v_2)/u∈V_1,(u_2,v_2) ∈X_2}∪

2.14 Degree of a vertex in normal product

By the definition, for any vertex (u_1,u_2) *in* $V_1 \times V_2$

$d_{\mathcal{A}_1 o \mathcal{A}_2}(u_1,u_2) =$

$\sum_{(u_1,u_2)(v_1,v_2)\in E}(\mu_1 \, o \mu_2)((u_1,u_2)(v_1,v_2))$

$=\sum_{u_1= v_1, u_2v_2 \in E_2} \sigma_1(u_1) \vee \mu_2(u_2v_2) +$

$\sum_{u_2=v_2, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1) +$

$$\sum_{u_2v_2 \in E_2 u_1v_1 \in E_1} \mu_2(u_2v_2) \vee \mu_1(u_1v_1)$$

$(\mu_1 o \mu_2)((u_1,v_2)(u_1,v_1)\}=$ $\{\mu_1(u_1v_1) \vee \mu_2(u_2v_2)\}\forall$ (u_1,v_1) ∈ X_1, (u_2,v_2) ∈X_2

Theorem 2.15

Let $\mathcal{A}_1$:(σ_1,μ_1) and $\mathcal{A}_2$:(σ_2,μ_2) be two anti fuzzy graphs. If $\sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$ and $\mu_2 \leq \mu_1$ then $d_{\mathcal{A}_1 o \mathcal{A}_2}(u_1,u_2)$= $P_2 d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$.

Proof

$d_{\mathcal{A}_1 o \mathcal{A}_2}(u_1,u_2) =$

$\sum_{(u_1,u_2)(v_1,v_2)\in E}(\mu_1 \, o \mu_2)((u_1,u_2)(v_1,v_2))$=

$\sum_{u_1= v_1, u_2v_2 \in E_2} \sigma_1(u_1) \vee \mu_2(u_2v_2) +$

$\sum_{u_2=v_2, u_1v_1 \in E_1} \sigma_2(u_2) \vee \mu_1(u_1v_1) +$

$$\sum_{,u_2v_2 \in E_2 u_1v_1 \in E_1} \mu_2(u_2v_2) \vee \mu_1(u_1v_1)$$

$=\sum_{u_1= v_1, u_2v_2 \in E_2} \mu_2(u_2v_2) +$

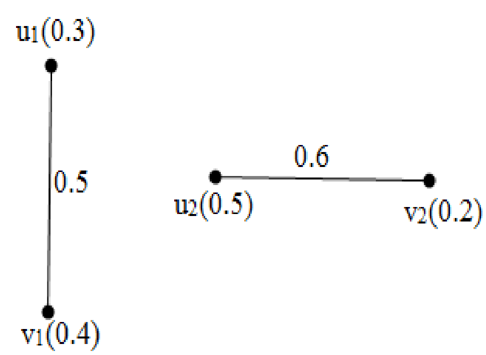
$\sum_{u_2=v_2, u_1v_1 \in E_1} \mu_1(u_1v_1)$+$\sum_{,u_1v_1 \in E_1} \mu_1(u_1v_1)$

since $\sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$ and $\mu_1 \leq \mu_2$

$= |V_2| d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$

$= P_2 d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$

Example 2.16



$\mathcal{A}_1\mathcal{A}_2$

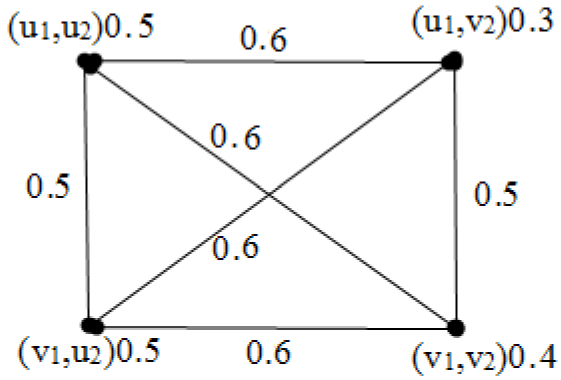


Fig 4

Normal product of $\mathcal{A}_1$&$\mathcal{A}_2$ is $\mathcal{A}_1 o \mathcal{A}_2$

Consider the anti fuzzy graph $\mathcal{A}_1$ and $\mathcal{A}_2, \sigma_1 \leq \mu_2$ and $\sigma_2 \leq \mu_1$ and $\mu_2 \leq \mu_1$

$d_{\mathcal{A}_1 o \mathcal{A}_2}(u_1, u_2)$= P$_2 d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$

= 2(0.5)+0.6 = 1.6

$d_{\mathcal{A}_1 o \mathcal{A}_2}(v_1, v_2)$= P$_2 d_{\mathcal{A}_1}(u_1) + d_{\mathcal{A}_2}(u_2)$.

= 2(0.5) + 0.6 = 1.6

Similarly we can find the degree of all the vertices in$\mathcal{A}_1$ o$\mathcal{A}_2$.

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