GENERALIZATION OF PAIRWISE FUZZY CONNECTEDNESS

T.Thangam[1] , K.Bageerathi[2]

[1]Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur-628215, Tamilnadu. e-mail:thangammathavan@yahoo.com

[2]Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur-628216, Tamilnadu. e-mail:sivarathi_2006@yahoo.co.in

Manonmanium Sundaranar University, Abhishekapatti, Tirunelveli – 627012.

Abstract

Focus of this paper is to introduce the concept of Pairwise fuzzy $\mathfrak{C}$ -connected, Pairwise fuzzy $\mathfrak{C}$ - super connected Pairwise fuzzy $\mathfrak{C}$ - strongly connected and Pairwise fuzzy $\mathfrak{C}$ - extremally disconnected in fuzzy bitopological space where $\mathfrak{C} : [0,1] \rightarrow [0,1]$ is arbitrary complement function. Several examples are given to illustrate the concepts introduced in this paper.

Keywords : Fuzzy complement function $\mathfrak{C}$, Pairwise fuzzy $\mathfrak{C}$ -connected, Pairwise fuzzy $\mathfrak{C}$ - super connected Pairwise fuzzy $\mathfrak{C}$ - strongly connected and Pairwise fuzzy $\mathfrak{C}$ - extremally disconnected and fuzzy bitopological spaces.

1.Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh [8] in the year 1965. The theory of fuzzy topological space was introduced and developed by C. L. Chang [5]. A. Kandil [6] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological space. The concept of Pairwise fuzzy connected spaces, Pairwise fuzzy super connected spaces Pairwise fuzzy strongly connected and Pairwise fuzzy extremally disconnected were introduced and studied by V.Chandrasekar and G.Balasubramanian in[3] In this paper the concept of Pairwise fuzzy $\mathfrak{C}$ -connected, Pairwise fuzzy $\mathfrak{C}$ - super connected in fuzzy bitopological spaces introduced and several examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries

In this section we list some definitions and results that are needed. Any function $\mathfrak{C} : [0, 1]\rightarrow[0, 1]$ defined from the interval [0, 1] to itself is called a complement function. Throughout the paper $\mathfrak{C}$ denotes an arbitrary complement function and (X, τ_1, τ_2) is a fuzzy bitopological space in the sense of A.Kandil[6].Throughout this paper,for fuzzy set λ of a fuzzy bitopological space (X, τ_1, τ_2), τ_i - intλ and τ_j - cl$_{\mathfrak{C}}\lambda$ means, respectively, the interior and closure of λ with respect to fuzzy topologies τ_i and τ_j.

Definition 2.1[2]: If λ is a fuzzy subset of X then the complement $\mathfrak{C}\lambda$ of a fuzzy set λ is a fuzzy subset with membership function defined by $\mu_{\mathfrak{C}}\lambda$ (x) = $\mathfrak{C}(\mu\lambda$ (x)) for all $x \in X$.

A subset λ of a fuzzy topological space is fuzzy closed if its standard complement λ', where λ' (x) = 1–λ(x) is fuzzy open. Several fuzzy topologists used this type of complement while extending the concepts in general topological spaces to fuzzy topological spaces. But there are other complements available in the fuzzy literature.

The properties of fuzzy complement function $\mathfrak{C}$ and $\mathfrak{C}\lambda$ are given in George Klir **[5]** and Bageerathi et al **[2]**. Also some complement function and results refered from [1, 4, 7].

3. Pairwise fuzzy $\mathfrak{C}$ - connected space

In this section we define the notion of pairwise fuzzy $\mathfrak{C}$ - connected and pairwise fuzzy $\mathfrak{C}$ - disconnected and discuss some of their properties.

Definition 3.1: Let (X, τ_1, τ_2) be a fuzzy bitopological space. Then X is said to be pairwise fuzzy $\mathfrak{C}$ - connected space if X has no proper fuzzy set λ_1 and λ_2 such that $\lambda_1 \in \tau_1$, $\lambda_2 \in \tau_2$ and $\lambda_1 = \mathfrak{C}\lambda_2$ or $\lambda_2 = \mathfrak{C}\lambda_1$, where $\mathfrak{C}$ is any arbitrary complement function. A fuzzy bitopological space (X, τ_1, τ_2) is called pairwise fuzzy $\mathfrak{C}$ - disconnected if it is not pairwise fuzzy $\mathfrak{C}$ - connected.

Example 3.2: Let X = {a, b, c}, τ_1 = {0, {a.$_1$, c.$_2$}, {a.$_3$, b.$_2$, c.$_5$}, 1} and τ_2 = {0, {a.$_2$, c.$_3$}, {a.$_8$, b.$_7$, c.$_8$}, 1}. Let $\mathfrak{C}(x) = \frac{1-x^2}{1+x^2}$, $0 \le x \le 1$ be a complement function that does not satisfy monotonicity and involutive properties. Then the family of all τ_i - fuzzy $\mathfrak{C}$ - closed sets are $\mathfrak{C}(\tau_1)$ = {1, {a.$_9$, b$_1$, c.$_8$}, {a.$_7$, b.$_8$, c.$_6$}, 0} and $\mathfrak{C}(\tau_2)$ = {1, {a.$_8$, b$_1$, c.$_7$}, {a.$_3$, b.$_4$, c.$_3$}, 0}. Then X has no proper fuzzy set λ_1 and λ_2 such that $\lambda_1 \in \tau_1$, $\lambda_2 \in \tau_2$ and $\lambda_1 = \mathfrak{C}\lambda_2$ or $\lambda_2 = \mathfrak{C}\lambda_1$. This implies that by Definition 3.1, (X, τ_1, τ_2) is pairwise fuzzy $\mathfrak{C}$ - connected.

Proposition 3.3: Let (X, τ_1, τ_2) be a fuzzy bitopological space. Let $\mathfrak{C}$ be any arbitrary complement function that satisfies the involutive property. Then the following statements are equivalent.

(i) (X, τ_1, τ_2) is pairwise fuzzy $\mathfrak{C}$ - connected.

(ii) There exist no τ_1 - fuzzy open set $\lambda_1 \neq 0$ and τ_2 - fuzzy open set $\lambda_2 \neq 0$ such that $\lambda_1 = \mathfrak{C}\lambda_2$ or $\lambda_2 = \mathfrak{C}\lambda_1$.

(iii) There exist no τ_1 - fuzzy $\mathfrak{C}$ - closed $\lambda_1 \neq 1$ and τ_2 - fuzzy $\mathfrak{C}$ - closed $\lambda_2 \neq 1$ such that $\lambda_1 = \mathfrak{C}\lambda_2$ or $\lambda_2 = \mathfrak{C}\lambda_1$.

(iv) X contains no fuzzy set $\lambda \neq 0$, 1 and it is both τ_1 - fuzzy open and τ_2 - fuzzy $\mathfrak{C}$ - closed (or) both τ_2 - fuzzy open and τ_1 - fuzzy $\mathfrak{C}$ - closed.

4. Pairwise fuzzy $\mathfrak{C}$ - super connected spaces via fuzzy $\mathfrak{C}$ - regular open set

In this section we define the notion of pairwise fuzzy $\mathfrak{C}$ - regular open, pairwise fuzzy $\mathfrak{C}$ - super connected and discuss some of their properties.

Definition 4.1: Let (X, τ_1, τ_2) be any fuzzy bitopological space and $\mathfrak{C}$ be any arbitrary complement function and λ be any fuzzy set in X. Then

(i) λ is said to be (τ_1 , τ_2) - fuzzy $\mathfrak{C}$ - regular open if τ_1 - Int (τ_2 - $cl_{\mathfrak{C}}$ (λ)) = λ.

(ii) λ is said to be (τ_2 , τ_1) - fuzzy $\mathfrak{C}$ - regular open if τ_2 - Int (τ_1 - $cl_{\mathfrak{C}}$ (λ)) = λ and

(iii) λ is said to be pairwise fuzzy $\mathfrak{C}$ - regular open if λ is both (τ_1 , τ_2) - fuzzy $\mathfrak{C}$ - regular open and (τ_2 , τ_1) - fuzzy $\mathfrak{C}$ - regular open.

Definition 4.2: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be any arbitrary complement function. Then

(i) a fuzzy set λ of X is said to be (τ_1 , τ_2) - fuzzy $\mathfrak{C}$ - regular closed if τ_1 - $cl_{\mathfrak{C}}$ (τ_2 - Int (λ)) = λ.

(ii) a fuzzy set λ of X is said to be (τ_2 , τ_1) - fuzzy $\mathfrak{C}$ - regular closed if τ_2 - $cl_{\mathfrak{C}}$ (τ_1 - Int (λ)) = λ

(iii) λ is said to be pairwise fuzzy $\mathfrak{C}$ - regular closed if λ is both (τ_1 , τ_2) - fuzzy $\mathfrak{C}$ - regular closed and (τ_2 , τ_1) - fuzzy $\mathfrak{C}$ - regular closed.

Theorem 4.3: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be any arbitrary complement function that satisfies monotonicity and involutive properties. Then a fuzzy subset λ is pairwise fuzzy $\mathfrak{C}$ - regular open set if and only if $\mathfrak{C}\lambda$ is pairwise fuzzy $\mathfrak{C}$ - regular closed.

The following example shows that if the complement function $\mathfrak{C}$ does not satisfy the monotonicity and involutive properties then the conclusion of the above theorem is not true.

Example 4.4: Let X = {a, b}, τ_1 = {0, {$a_{.1}$, $b_{.2}$ }, {$a_{.2}$, $b_{.1}$ }, {$a_{.1}$, $b_{.1}$ }, {$a_{.2}$, $b_{.2}$}, 1} and τ_2 = {0, {$a_{.1}$, $b_{.1}$ }, {$a_{.2}$, $b_{.2}$}, 1}. Let $\mathfrak{C}(x) = \frac{x}{3-2x}$, $0 \le x \le 1$ be a complement function that does not satisfy the involutive and monotonicity properties. Then the family of all τ_i - fuzzy $\mathfrak{C}$ - closed sets are $\mathfrak{C}(\tau_1)$ = {0, {$a_{.3}$, $b_{.4}$}, {$a_{.4}$, $b_{.3}$}, {$a_{.3}$, $b_{.3}$}, {$a_{.4}$, $b_{.4}$}, 1} and $\mathfrak{C}(\tau_2)$ = {0, {$a_{.3}$, $b_{.3}$}, {$a_{.4}$, $b_{.4}$}, 1}. Let λ = {$a_{.2}$, $b_{.2}$}. Then it can be calculated that τ_1 - Int (τ_2 - $cl_{\mathfrak{C}}$ (λ)) = {$a_{.2}$, $b_{.2}$} = λ and τ_2 - Int (τ_1 - $cl_{\mathfrak{C}}$ (λ)) = {$a_{.2}$, $b_{.2}$} = λ. We see that τ_1 - Int (τ_2 - $cl_{\mathfrak{C}}$ (λ)) = λ and τ_2 - Int (τ_1 - $cl_{\mathfrak{C}}$ (λ)) = λ. By Definition 4.1 shows that λ is pairwise fuzzy $\mathfrak{C}$ - regular open. Also
$\mathfrak{C}\lambda$ = {$a_{.1}$, $b_{.1}$ }. Then it can be calculated that τ_1 - $cl_{\mathfrak{C}}$ (τ_2 - Int ($\mathfrak{C}\lambda$)) = {$a_{.3}$, $b_{.3}$ } $\neq \mathfrak{C}\lambda$ and τ_2 - $cl_{\mathfrak{C}}$ (τ_1 - Int ($\mathfrak{C}\lambda$)) = {$a_{.3}$, $b_{.3}$ } $\neq \mathfrak{C}\lambda$.We note that τ_1 - $cl_{\mathfrak{C}}$ (τ_2 - Int ($\mathfrak{C}\lambda$)) $\neq \mathfrak{C}\lambda$ and τ_2 - $cl_{\mathfrak{C}}$ (τ_1 - Int ($\mathfrak{C}\lambda$)) $\neq \mathfrak{C}\lambda$. By Definition 4.2, shows that $\mathfrak{C}\lambda$ is not pairwise fuzzy $\mathfrak{C}$ - regular closed.

Definition 4.5: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be any arbitrary complement function. Then (X, τ_1, τ_2) is said to be pairwise fuzzy $\mathfrak{C}$ - super connected if it has no proper pairwise fuzzy $\mathfrak{C}$ - regular open.

Here the following example shows that pairwise fuzzy $\mathfrak{C}$ - super connected need not be pairwise fuzzy $\mathfrak{C}$ - connected.

Example 4.6: Let X = {a, b}, τ_1 = {0, λ_1= {a_1, b_0}, 1} and τ_2 = {0, λ_2= {a_0, b_1}, 1}. Let $\mathfrak{C}(x) = \frac{1-x}{1+x}$, $0 \leq x \leq 1$ be a complement function that satisfy monotonicity and involutive properties. Then the family of all τ_i - fuzzy $\mathfrak{C}$ - closed sets are $\mathfrak{C}(\tau_1)$ = {1, {a_0, b_1, }, 0} and $\mathfrak{C}(\tau_2)$ = {1, {a_1, b_0}, 0}. Since X has proper fuzzy sets λ_1 and λ_2 such that $\lambda_1 \in \tau_1$, $\lambda_2 \in \tau_2$ and $\lambda_1 = \mathfrak{C}\lambda_2$ or $\lambda_2 = \mathfrak{C}\lambda_1$, by Definition 3.1, (X, τ_1, τ_2) is not pairwise fuzzy $\mathfrak{C}$ - connected. Also it has no proper pairwise fuzzy $\mathfrak{C}$ - regular open, by Definition 4.5, (X, τ_1, τ_2) is a pairwise fuzzy $\mathfrak{C}$ - super connected.

Proposition 4.7: Let (X, τ_1, τ_2) be any fuzzy bitopological space and $\mathfrak{C}$ be a complement function that satisfies monotonicity and involutive properties. Then (i) $\Rightarrow$ (ii) and (ii) $\Rightarrow$ (iii).

(i) (X, τ_1, τ_2) is pairwise fuzzy $\mathfrak{C}$ - super connected space.

(ii) The τ_2 - fuzzy $\mathfrak{C}$ - closure (τ_1 - fuzzy $\mathfrak{C}$ - closure) of a pairwise fuzzy $\mathfrak{C}$ - regular open set which is different from 0 is 1.

(iii) The τ_2 - Interior (τ_1 - Interior) of a pairwise fuzzy $\mathfrak{C}$ - regular closed set which is different from 1 is 0.

5. Pairwise fuzzy $\mathfrak{C}$ - strongly connected spaces

In this section we define the notion of pairwise fuzzy $\mathfrak{C}$ - strongly connected and study some of their properties.

Definition 5.1: Let (X, τ_1, τ_2) be any fuzzy bitopological space. Then X is said to be pairwise fuzzy $\mathfrak{C}$ - strongly connected if it has no proper fuzzy sets λ_1, λ_2 such that λ_1 is τ_1 - fuzzy $\mathfrak{C}$ - closed (or) τ_2 - fuzzy $\mathfrak{C}$ - closed and λ_2 is τ_1 - fuzzy $\mathfrak{C}$ - closed (or) τ_2 - fuzzy $\mathfrak{C}$ - closed with $\lambda_1 \leq \mathfrak{C}\lambda_2$ (or) $\lambda_2 \leq \mathfrak{C}\lambda_1$ where $\mathfrak{C}$ is any arbitrary complement function.

If (X, τ_1, τ_2) is not pairwise fuzzy $\mathfrak{C}$ - strongly connected then it will be called pairwise fuzzy $\mathfrak{C}$ - weakly connected.

Proposition 5.2: Let (X, τ_1, τ_2) be any fuzzy bitopological space and $\mathfrak{C}$ be any arbitrary complement function that satisfies monotonicity and involutive properties. Then X is pairwise fuzzy $\mathfrak{C}$ - weakly connected iff it has proper fuzzy sets λ, μ such that $\lambda \in \tau_1$ (or) τ_2 and $\mu \in \tau_1$ (or) τ_2 such that $\lambda \geq \mathfrak{C}\mu$ (or) $\mu \geq \mathfrak{C}\lambda$.

Remark 5.3: Every pairwise fuzzy $\mathfrak{C}$ - strongly connected is pairwise fuzzy $\mathfrak{C}$ - connected but the converse is not true as shown by following example.

Example 5.4: Let X = {a, b, c}. Let τ_1 = {0, {$a_{.7}$, $b_{.7}$, $c_{.8}$}, 1} and τ_2 = {0, {$a_{.2}$, $b_{.2}$, $c_{.3}$}, 1}. Let $\mathfrak{C}(x) = \frac{1-x}{1+x}$, $0 \leq x \leq 1$ be a complement function that satisfies monotonicity and involutive properties . Then the family of all τ_i - fuzzy $\mathfrak{C}$ - closed sets are $\mathfrak{C}(\tau_1)$ ={1,{$a_{.2}$, $b_{.2}$, $c_{.1}$}, 0} and

$\mathfrak{C}(\tau_2) = \{1, \{a._7, b._7, c._5\}, 0\}$. Then $\{a._7, b._7, c._8\} \neq \mathfrak{C}(\{a._2, b._2, c._3\})$ and $\mathfrak{C}(\{a._7, b._7, c._8\}) \neq \mathfrak{C}(\{a._2, b._2, c._3\})$. This implies that X is pairwise fuzzy $\mathfrak{C}$ - connected. Let $\lambda_1 = \{a._2, b._2, c._1\}$ $\lambda_2 = \{a._7, b._7, c._5\}$. Then $\lambda_1 = \{a._2, b._2, c._1\} \leq \mathfrak{C}\lambda_2 = \{a._2, b._2, c._{33}\}$ and $\lambda_2 = \{a._7, b._7, c._5\} \leq \mathfrak{C}\lambda_1 = \{a._7, b._7, c._8\}$. This implies by Definition 5.1, X is not pairwise fuzzy $\mathfrak{C}$ - strongly connected.

6. Pairwise fuzzy $\mathfrak{C}$ - extremally disconnected space

In this section we define the notion of pairwise fuzzy $\mathfrak{C}$ - extremally disconnected and investigate some of their properties.

Definition 6.1: Let (X, τ_1, τ_2) be a fuzzy bitopological space and $\mathfrak{C}$ be any arbitrary complement function. Then X is said to be pairwise fuzzy $\mathfrak{C}$ - extremally disconnected space if τ_1 - fuzzy $\mathfrak{C}$ - closure of each τ_2 - fuzzy open set is τ_2 - fuzzy open and τ_2 - fuzzy $\mathfrak{C}$ - closure of each τ_1 - fuzzy open set is τ_1 - fuzzy open.

Proposition 6.2: Let (X, τ_1, τ_2) be any fuzzy bitopological space and $\mathfrak{C}$ be any complement function that satisfies monotonicity and involutive properties.Then the following are equivalent:

(i) (X, τ_1, τ_2) is pairwise fuzzy $\mathfrak{C}$ - extremally disconnected.

(ii) Whenever λ is τ_1 - fuzzy $\mathfrak{C}$ - closed set, τ_2 - Int λ is a τ_1 - fuzzy $\mathfrak{C}$ - closed set. whenever μ is τ_2 - fuzzy $\mathfrak{C}$ - closed set, τ_1 - Int μ is a τ_2 - fuzzy $\mathfrak{C}$ - closed set.

(iii) When λ is τ_1 - fuzzy open set (τ_2 - fuzzy open set) $\tau_1 - cl_{\mathfrak{C}} (\mathfrak{C}(\tau_2 - cl_{\mathfrak{C}} (\lambda))) = \mathfrak{C}(\tau_2 - cl_{\mathfrak{C}} (\lambda))$ and $\tau_2 - cl_{\mathfrak{C}} (\mathfrak{C}(\tau_1 - cl_{\mathfrak{C}} (\lambda))) = \mathfrak{C}(\tau_1 - cl_{\mathfrak{C}} (\lambda))$.

References

[1] C. Alegre, J. Ferrer and V. Gregori, On pairwise Baire bitopological spaces, publ. Math.s Debrecen, 55(1999), 3 - 15.

[2] K. Bageerathi, G. Sutha and P. Thangavelu, A generalization of fuzzy closed sets. International Journal of fuzzy systems and rough systems, 4(1), 1 - 5(2011).

[3] V. Chandrasekar and G. Balasubramanian, Weaker forms of connectedness and stronger forms of disconnectedness in fuzzy bitopological spaces,Indian J. pure appl math, 33(7) : 955-965. July 2002.

[4] C. L. Chang, Fuzzy topological spaces, J.Math. Anal. Apspl., 24(1968), 182 - 190.

[5] George J.Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic Theory and Applications, *Prentice – Hall, Inc*., 2005.

[6] A. Kandil, Biproximaties and fuzzy bitopological spaces, Simon steven 63(1989)

[7] T.Thangam , K.Bageerathi Generalization of Fuzzy Bi-closed sets and Pair wise Baire Spaces Proceedings of the national Seminar On Advances in Fuzzy Algebra,Fuzzy Topology and Fuzzy Graphs(AFATG2015) A.P.C.Mahalaxmi College, Thoothukudi, Jan 22-23, (2015).

[8] L. A. Zadeh, Fuzzy sets, Information and control, 8(1965), 338 - 353.