On Generalized (m,n) Bi-ideals in Near Subtraction Semigroups

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Abstract:

In this paper we introduced the concept of generalized (m,n) bi-ideal in a class of near subtraction semigroups (i.e.) A near subtraction semigroups X satisfying the condition: A subalgebra B of $(X,-)$ is said to be a generalized (m,n) bi-ideal of X if $B^m X B^n \subseteq B$, where m and n are positive integers. Also we study certain properties of bi-ideals in a left permutable s-near subtraction semigroups. Also we generalize the notion of bi-ideals and obtain the properties of generalized bi-ideals in certain classes of near subtraction semigroups. Further it is shown that the concept of generalized bi-ideals and bi-ideals are equivalent in a left permutable s- near subtraction semigroups.

Key words: Bi-ideal, s-near subtraction semigroup, property(α).

1. Introduction

B.M.Schein [12] considered systems of the form (X;**o;**\),where X is a set of functions closed under the composition "**o**" of functions (and hence (X;**o**) is a function semigroup) and the set theoretic subtraction "\" (and hence (X**;**\) is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B.Zelinka[16] discussed a problem proposed by B.M.Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. For basic definition one may refer to Pilz[11]. The notion of bi-ideals was introduced by Tamizh Chelvam and Ganesan[14]. Further Tamizh Chelvam [15] obtained certain properties of bi-ideals in a class of near-rings and also obtained equivalent conditions for a generalized near-field in terms of bi-ideal in a sub commutative s-near ring. With this in mind, initially we study certain properties of bi-ideals in left permutable near subtraction semigroup. Also we generalize the notion of bi-ideals and obtain the properties of generalized bi-ideals in certain classes of near subtraction semigroups. Further it is shown that the concept of generalized bi-ideals and bi-ideals are equivalent in a left permutable s- near subtraction semigroups.

2.Preliminaries on Near Subtraction Algebra

Definition 2.1:

A nonempty set X together with binary operations "-" and is said to be a **subtraction algebra** if it satisfies the following

(i) $x-(y-x)=x$

(ii)$x-(x-y)=y-(y-x)$

(iii)$(x-y)-z=(x-z)-y$, for every $x,y,z \in X$.

Definition 2.2:

A nonempty set X together with two binary operations "-'' and "·" is said to be **subtraction semigroup** if it satisfies the following:

(i) (X,-) is a subtraction algebra.

(ii) (X,·) is a semigroup.

(iii$x(y-z)=xy-x$ and $(x-y)z=xz-yz$, for every $x,y,z \in X$.

Definition 2.3:

A nonempty set X together with two binary operations "-'' and "·" is said to be a **near subtraction semigroup** (right) if it satisfies the following:

(i) (X,-) is a subtraction algebra.

(ii) (X,·) is semigroup.

(iii) $(x-y)z=xz-yz$ for every $x,y,z \in X$.

Definition 2.4:

A nonempty subset S of a subtraction semigroup X is said to be a **subalgebra** of X if $x-x' \in X$ whenever $x,x' \in S$.

Note 2.5:

Let X be a near subtraction semigroup. Given two subsets A and B of X, $AB=\{ab/a \in A, b \in B\}$. Also we define another operation "*"
A*B=$\{ab-a(a'-b)\ /a,a' \in A, b \in B\}$.

Definition 2.6:

We say that X is an **s(s')-near subtraction semigroup** if $a \in Xa(aX)$, for all $a \in X$.

Definition 2.7:

A s-near subtraction semigroup X is said to be a **$\bar{s}$-near subtraction semigroup** if $x=xX$, for all $a \in X$.

Definition 2.8:

An element $a \in X$ is said to be **regular** if for each $a \in X$, $a=aba$ for some $b \in X$.

Definition 2.9:

A near subtraction semigroup X is said to have **property(α)** if xX is a subalgebra of (X,-), for every $x \in X$.

Definition 2.10:

A near subtraction semigroup X is said to be **left permutable** if $abc=bac$, for all $a,b,c \in X$.

Definition 2.11:

A near subtraction semigroup X is called **generalized near-field (GNF)** if for each $a \in X$, there exists a unique $b \in X$ such that $a=aba$ and $b=bab$.

Definition 2.12:

A subalgebra B of (X,-) is said to be **bi-ideal** of X if $BXB \cap (BX)*B \subseteq B$. In the case of a zero symmetric near subtraction semigroup, a subalgebra B of X is a quasi-ideal if $BXB \subseteq B$.

Definition 2.13:

A family $\mathcal{J}$ of subsets of a set A is called a **Moore system** if (i) A$\in\mathcal{J}$ and (ii) is closed under arbitrary intersections.

Lemma 2.14:

If X has the condition, $eX = eXe = Xe$, for all $e \in E$, then $E \subseteq C(X)$.

Lemma 2.15:

Let X be a zero-symmetric near subtraction semigroup. If L={0}, then $en = ene$, for $0 \neq e \in E$ and $n \in X$.

3. On Generalized (m,n) Bi-ideals

In this section first we study certain properties involving generalized (m,n) bi-ideals in a class of near subtraction semigroups. Also we see certain properties of bi-ideals in a left permutable s-near subtraction semigroup, which is different from the class so far we have discussed.

Definition 3.1:

A subalgebra B of (X,-) is said to be a **generalized (m,n) bi-ideal** of X if $B^m X B^n \subseteq B$, where m and n are positive integers.

Remark 3.2:

Note that every bi-ideal is a generalized (m,n) bi-ideal, for all positive integers m and n. However the converse is not true. This can be seen from the example given below.

Example 3.2.1:

Let X={0,1,2,3,4,5,6,7} in which "-" and "·" are defined by,

-	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	1	0	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	0	3	3	3	3
4	4	4	4	4	0	4	4	4
5	5	5	5	5	5	0	5	5
6	6	6	6	6	6	6	0	6
7	7	7	7	7	7	7	7	0

.	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	2	4	2	0	2	4	6
2	0	4	0	4	0	4	0	4
3	0	6	4	6	0	6	4	2
4	0	0	0	0	0	0	0	0
5	0	2	4	2	0	2	4	6
6	0	4	0	4	0	4	0	4
7	0	6	4	6	0	6	4	2

One can check that B={0,7} is a subalgebra of (X,-) and $B^3XB^2 \subseteq B$, Whereas $BXB \not\subset B$. Hence B is a generalized (3,2) bi-ideal. But not a bi-ideal.

Proposition 3.3:

The set of all generalized (m,n) bi-ideals of X form a Moore system on X.

Proof:

Let $\{B_i\}_{i\in I}$ be a set of generalized (m,n) bi-ideals of X. Let $B = \cap B_i$. Then $B^m X B^n \subseteq B_i^m X B_i^n \subseteq B_i \subseteq B$. Hence B is a generalized (m,n) bi-ideal of X, for all positive integers m and n.

Proposition 3.4:

Let B a generalized (m,n) bi-ideal B of X. Then Bb and $b'B$ are generalized (m,n) bi-ideals of X, Where, $b, b' \in X$ and b' is a distributive element in X.

Proof:

Clearly Bb is a subalgebra of (X,-) and $(Bb)^m X (Bb)^n \subseteq B^m X B^n b \subseteq Bb$.Since b' is distributive,$b'B$ is a subalgebra of (X,-) and $(b'B)^m X (b'B)^n = b'B^m X B^n \subseteq b'B$. Hence Bb and $b'B$ are the generalized (m,n) bi-ideals of X.

Corollary 3.5:

Let B be a generalized (m,n) bi-ideal of X. If b is a distributive element of X and

$c \in X$, then bBc is a generalized (m,n) bi-ideal of X, for all positive integers m and n.

Proof:

Follows from the proposition 3.4.

Theorem 3.6:

Let X be a left permutable s-near subtraction semigroup. Then $B = B^m X B^n$, for every generalized (m,n) bi-ideal B of X, for all positive integers m and n if and only if X is regular.

Proof:

Assume that $= B^m X B^n$, for every generalized (m,n) bi-ideal B of X. For every $a \in X$, Xa is a generalized (m,n) bi-ideal of X. Since X is a s-near subtraction semigroup, $a \in Xa = (Xa)^m X (Xa)^n \subseteq XaXa$. (i.e.) $a = c_1 a c_2 a$, for some $c_1, c_2 \in X$. Since X is a left permutable near subtraction semigroup $a = a c_1 c_2 a \in aXa$. (i.e.,) a is regular andso X is regular.

Conversely, let $b \in B$. Since X is regular, $b = bab$, for some $a \in X$. Therefore $b = bababab = b^2 a^3 b^2 \in b^m X b^n \in B^m X B^n$ andso $B \subseteq B^m X B^n$. By the definition $B^m X B^n \subseteq B$.Hence $B = B^m X B^n$, for every generalized (m,n) bi-ideal B of X.

Theorem 3.7:

Let X be a left permutable s-near subtraction semigroup. Then $B = B^m X B^n$, for every generalized (m,n) bi-ideal B of X if and only X is a GNF, for all $a \in X$.

Proof:

Assume that X is a GNF. Then X is regula andso from the above Theorem 3.6, $B = B^m X B^n$, for every generalized (m,n) bi-ideal B of X.

Conversely, assume that $B = B^m X B^n$ for every generalized (m,n) bi-ideal B of X. From the above Theorem 3.6, X is a regular and to prove X is a GNF, it is enough to prove that $E \subseteq C(X)$. First let us show that $aXa = Xa^2 = Xa$, forall $a \in X$. Let $x \in aXa$. Since X is a left permutable near subtraction semigroup, $x = aca = ca^2 \in Xa^2$. Therefore $aXa \subseteq Xa^2$.Similarly one can prove that $Xa^2 \subseteq aXa$. Thu $aXa = Xa^2$. Since X is regular, for $a \in X$, $a = aca$, for some $c \in X$. From this, we get that $Xa = Xaca \subseteq XaXa \subseteq aXa \subseteq Xa$.Hence $Xa = Xa^2 = aXa$. Since X is a s-near subtraction semigroup, the above implies that X is strongly regular and $Xe = eXe$, for $e \in E$. Now by the Lemma 2.15, $eXe = eX$,for $e \in E$.Thus,$eXe = Xe = eX$ for every $e \in E$. Now by the Lemma 2.14, $E \subseteq C(X)$.

Theorem 3.8:

Let X be a left permutable s-near subtraction semigroup. Then $B = BXB$, for every bi-ideal B of X if and only if $B = B^m X B^n$, for every generalized (m,n) bi-ideal B of X

Proof:

Assme that $B = B^m X B^n$, for every generalized (m,n) bi-ideal B of X. By the Theorem 3.6, X is regular and so $B = BXB$, for every bi-ideal B of X.

Conversly assume that $B = BXB$, for every bi-ideal B of X. Hence by the Theorem 3.6, X is a GNF and hence the result follows from the Theorem 3.7.

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