SOME NEW SEPARATION AXIOMS IN TOPOLOGICAL SPACES

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Abstract:

A subset A of a space (X,τ) is said to be sbĝ closed if scl(A) ⊆ U whenever A ⊆ U and U is bĝ-open in X . The complement of sbĝ closed set is called sbĝ-open set. A function f: (X,τ) → (Y,σ) is said to be sbĝ- continuous if $f^{-1}(v)$ is sbĝ closed in (X,τ) for every closed set V in (Y,σ) . A function f: (X,τ) → (Y,σ) is said to be sbĝ- irresolute if $f^{-1}(V)$ is Sbĝ closed in (X, τ) . In this paper we introduce three new topological spaces namely sbĝ-T_0 , sbĝ-T_1 and sbĝ-T_2 spaces via sbĝ- open sets. Also we characterize their properties.

Keywords:

sbĝ closed set, sbĝ- open set, sbĝ-T_0 spaces, sbĝ-T_1 spaces, sbĝ-T_2 spaces, sbĝ-kernel.

1.Inroduction:

Caldas and Jafari[7] introduced and studied b-T_0,b-T_1 and b-T_2 via b-open sets. K.Bala Deepa Arasi and S.Navaneeatha Krishnan[2] introduced the concept of sbĝ-closed set in Topological spaces. And they introduced and studied sbĝ-continuous functions[3], contra sbĝ-continuous functions [4], sbĝ-quotientmap[5], sbĝ-connected spaces and sbĝ-compact spaces[6].

In this paper we introduce three new spaces namely,sbĝ-T_0 ,sbĝ-T_1 and ,sbĝ -T_2 spaces via sbĝ-open set .We investigate relation between these spaces with other existing spaces . Also we study some properties of these spaces.

2.Preliminaries:

Throughout this paper (X , τ) & (Y ,σ) respectively and on which no separation axioms are assumed unless otherwise explicity stated . Let A be a subset of the space X. The interior and closure of a set A in X are denoted by int(A) and cl(A). The complement of A is denoted by (X-A) or A^c.

Definition 2.1: A subset A of a topological space (X,τ) is said to be an sbĝ-closed set if scl(A)⊆ U whenever A ⊆ U and U is bĝ-open in X. The family of all sbĝ-closed sets of X is denoted by sbĝ-C(X). The complement of sbĝ-closed set is called sbĝ-open set. The family of all sbĝ-open sets of X is denoted by sbĝ-O(X).

Definition 2.2:[7] A topological space (X,τ) is said to be

i) b-T_0 if for each pair of distinct points in X, there is a b-open set containing one of the points but not the other.

ii)b-T_1 if for each pair of distinct points x and y of X, there exist b-open sets U and V containing x and y respectively such that y ∉ U and x ∉ V.

iii)b-T_2 if for each pair of distinct points x and y of X, there exist disjoint b-open sets U and V containing x and y respectively.

Definition 2.3:[8] A topological space (X,τ) is said to be

i)α-T_0 if for each pair of distinct points in X, there exists an α-open set containing one of the points but not the other.

ii) α-T_1 if for each pair of distinct points x and y of X, there exist α-open sets U and V containing x and y respectively such that y ∉ U and x ∉ V.

iii) α-T_2 if for each pair of distinct points x and y of X, there exist disjoint α-open sets U and V containing x and y respectively.

Definition 2.4:[9] A topological space (X,τ) is said to be

i)gb-T_0 if for each pair of distinct points in X, there is a gb-open set containing one of the points but not the other.

ii)gb-T_1 if for each pair of distinct points x and y of X, there exist gb-open sets U and V containing x and y respectively such that y ∉ U and x ∉ V.

iii)gb-T_2 if for each pair of distinct points x and y of X, there exist disjoint gb-open sets U and V containing x and y respectively.

Definition 2.5: A topological space (X,τ) is said to be

i) g*b-T_0 if for each pair of distinct points in X, there is a g*b-open set containing one of the points but not the other.

ii) g*b-T_1 if for each pair of distinct points x and y of X, there exist g*b-open sets U and V containing x and y respectively such that y ∉ U and x∉V

iii) g*b-T_2 if for each pair of distinct points x and y of X, there exist disjoint g*b-open sets U and V containing x and y respectively.

Definition 2.6: A topological space (X,τ) is said to be

i)bĝ-T_0 if for each pair of distinct points in X, there is a bĝ-open set containing one of the points but not the other.

ii)bĝ-T_1 if for each pair of distinct points x and y of X, there exist bĝ-open sets U and V containing x and y respectively such that y ∉ U and x ∉ V.

iii)bĝ-T_2 if for each pair of distinct points x and y of X, there exist disjoint bĝ-open sets U and V containing x and y respectively.

Proposition 2.7:[2] Every α-closed set is sbĝ-closed set

Proposition 2.8:[2]

i) Every sbĝ-closed set is b-closed set.
ii) Every sbĝ-closed set is gb-closed set.
iii)Every sbĝ-closed set is g*b-closed set.
iv)Every sbĝ-closed set is bĝ-closed set.

Proposition 2.9: Let A and B be subsets of a topological space (X,τ). Then

1.sbĝ-cl(ϕ) = ϕ and sbĝ-cl(X) = X.
2.If A ⊆ B, then sbĝ-cl(A) ⊆ sbĝ-cl(B).
3.sbĝ-cl(A∩B) ⊆ sbĝ-cl(A) ∩ sbĝ-cl(B).
4.sbĝ-cl(A∪B) ⊇ sbĝ-cl(A) ∪ sbĝ-cl(B).
5.A is an sbĝ-closed set in (X,τ) if and only if A = sbĝ-cl(A).
6.sbĝ-cl(sbĝ-cl(A)) = sbĝ-cl(A).

Proposition 2.10:

Let (X,τ) be a topological space. Then x ∈ sbĝ-cl(A) if and only if U∩A ≠ ϕ for every sbĝ-open set U containing x.

3. sbĝ-T_i SPACES, i = 0, 1, 2.

Definition3.1: A topological space (X,τ) is said to be sbĝ-T_0 if for each pair of distinct point x,y in X, there exists an sbĝ-open set U of X such that either x∈U and y∉U or x∉U and y∈U.

Example 3.2: Let X={a,b,c}with a topology τ={X,ϕ,{a}}. Here, sbĝ-O(X)={X,ϕ,{a},{a,b},{a,c}}. Clearly, X is an sbĝ-T_0 space.

Definition 3.3: A topological space (X,τ) is said to be sbĝ-T_1 if for each pair of distinct points x,y in X, there exist two sbĝ-open sets U and V such that x ∈ U but y ∉ U and x ∉ V but y ∈ V.

Example 3.4: Let X ={a,b,c}with a topology τ={X,ϕ,{a},{b},{a,b}}. sbĝO(X)={X, ϕ, {a}, {b},{a,b},{a,c}, {b,c}}. Clearly, X is an sbĝ-T_1 space.

Definition 3.5: A topological space (X,τ) is said to be sbĝ-T_2 (or sbĝ-Hausdorff) if for each distinct points x,y in X, there exist two disjoint sbĝ-open sets U and V of X containing x and y respectively.

Example 3.6: Let X = {a,b,c} with a topology τ ={X,ϕ, {b},{c},{b,c}}.Here sbĝ-

O(X)={X,ϕ,{b},{c},{a,b},{a,c},{b,c}}. Clearly, X is an sbĝ-T_2 space.

Proposition 3.7: For a topological space (X,τ), the following properties hold:

i) Every α-T_i space is sbĝ-T_i space, i = 0, 1, 2.

ii) Every sbĝ-T_i space is b-T_i space, i = 0, 1, 2.

iii)Every sbĝ-T_i space is gb-T_i space,i = 0,1, 2.

iv) Every sbĝ-T_i space is g^*b-T_i space i= 0,1, 2.

v) Every sbĝ-T_i space is bĝ-T_i space, i = 0, 1, 2.

Proof: The proofs follow from Proposition 2.7, 2.8(i), 2.8(ii), 2.8(iii) and 2.8(iv).

The following examples show that the converse of the above propositions need not be true.

Example 3.8:

(i) Let X = {a,b,c} with a topology τ = {X,ϕ,{a},{b},{a,b}}. Here α-O(X)={X,ϕ,{a},{b},{a,b}} and sbĝ-O(X)={X,ϕ,{a},{b},{a,b},{a,c},{b,c}}.Clearly (X,τ) is sbĝ-T_1 and sbĝ-T_2 space but not α-T_1 and α-T_2 space.

(ii) Let X = {a,b,c} with a topology τ = {X,ϕ,{a},{b,c}}. Here b-O(X) ={X,ϕ, {a},{b},{c},{a,b},{a,c},{b,c}}and sbĝ-O(X)={X, ϕ, {a, b},{c}} Clearly (X,τ) is b-T_i space but not sbĝ-T_i space, i=0,1,2.

(iii) Let X = {a,b,c} with a topology τ ={X,ϕ,{a,b},{c}}. Here gb-O(X) = **{**X,ϕ, {a},{b},{c},{a,b},{a,c},{b,c}}and sbĝ-O(X)= {X,ϕ,{a,b},{c}}. Clearly (X,τ) is gb-T_i space but not sbĝ-T_i space, i=0,1,2.

(iv) Let X = {a,b,c} with a topology τ ={X,ϕ,{a,c}}. Here g*b-O(X) = **{**X,ϕ, {a},{c},{a,b},{a,c},{b,c}} and sbĝ-O(X)={X,ϕ,{a,c}}. Clearly (X,τ) is g*b-T_i space but not sbĝ-T_i space, i=0,1,2.

v) Let X = {a,b,c,d} with a topology τ = {X,ϕ,{a},{b,c,d}}. Here bĝ O(X)**={**X,ϕ, {a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b, d},{c,d},{a,b,c},{a,b,d}{a,c,d}{b,c,d}}and sbĝ -O(X)={X,ϕ, {a},{b,c,d}}. Clearly (X,τ) is bĝ- -T_i space but not sbĝ-T_i space, i=0,1,2.

Proposition 3.9:

A topological space (X,τ) is sbĝ-T_0 if and only if for each pair of distinct points x,y of X, sbĝ-cl({x}) ≠ sbĝ-cl({y}).

Proof: Let (X,τ) be an sbĝ-T_0 space and x, y be any two distinct points of X. There exists an sbĝ-open set U containing x or y, say x but not y. Then X\U is an sbĝ-closed set which does not contain x but contains y. Since sbĝ-cl({y}) is the smallest sbĝ-closed set containing y, sbĝ-cl({y}) ⊆ X\U. Therefore, x ∉ sbĝ-cl({y}). Consequently, sbĝ-cl({x}) ≠ sbĝ-cl({y}). Conversely, suppose that x, y ∈ X, x ≠ y and sbĝ-cl({x}) ≠ sbĝ-cl({y}). Let z ∈ X such that z ∈ sbĝ-cl({x}) but z ∉ sbĝ-cl({y}). We claim that x ∉ sbĝ-cl({y}). Suppose x ∈ sbĝ-cl({y}). Then by proposition 2.9(2) and (6), sbĝ-cl({x}) ≠ sbĝ-cl(sbĝ-cl({y})) = sbĝ-cl({y}). Therefore, z ∈ sbĝ-cl({y}) which is a contradiction. Thus, x ∉ sbĝ-cl({y}). Now, X\sbĝ-cl({y}) is an sbĝ-open set in X such that x ∈ X\sbĝ-cl({y}) and y ∉ X\sbĝ-cl({y}). Hence, (X,τ) is an sbĝ-T_0 space.

Proposition 3.10: A topological space (X,τ) is sbĝ-T_1 if and only if the singletons are sbĝ-closed sets.

Proof: Let (X,τ) be an sbĝ-T_1 space and x be any point of X. Suppose y ∈ X\{x}. Then x ≠ y. So, there exists an sbĝ-open set U such that y ∈ U but x ∉ U. Consequently, y ∈ U □□X\{x}, that is X\{x} = ⋃{U: y ∈ X\{x}} which is sbĝ-open. So, singletons in (X,τ) are sbĝ-closed sets. Conversely, suppose {p} is sbĝ-closed for every p ∈ X. Let x, y ∈ X with x ≠ y. Since x ≠ y, y ∈ X\{x}. So, X\{x} is an sbĝ-open set contains y but not x. Similarly, X\{y} is an sbĝ-open set contains x but not y. Hence, X is an sbĝ-T_1 space.

Proposition 3.11: Let (X,τ) be a topological space. Then the following statements are equivalent:

1) X is an sbĝ-T_2 space.

2) Let x ∈ X. For each y ≠ x, there exists an sbĝ-open set U containing x such that y ∉ sbĝ-cl(U).

3) For each x ∈ X, ∩{sbĝ-cl(U) : U ∈ sbĝ-O(X) and x ∈ U} = {x}.

Proof:

(1)⇒ (2): Since X is an sbĝ-T_2 space, for each y ≠ x, there exist two disjoint sbĝ-open sets U and V such that x ∈ U and y ∈ V. If F = X\V, then F is an sbĝ-closed set such that U ⊆ F. By proposition 2.9(2), sbĝ-cl(U) ⊆ sbĝ-cl(F) = F. Since y ∉ F y ∉ sbĝ-cl(U).

(2) ⇒ (3): Suppose for any x ∈ X and for each y ≠ x, there exists an sbĝ-open set U such that x ∈ U and y ∉ sbĝ-cl(U). Therefore, y ∉ ∩ {sbĝ-cl(U) :U ∈ sbĝ-O(X) and x ∈ U}. Hence, ∩{sbĝ-cl(U) : U ∈ sbĝ-O(X) and x ∈ U} = {x}.

(3)⇒ (1): Let x,y ∈ X and x ≠ y. Then y ∉ {x} = ∩ {sbĝ-cl(U) : U ∈ sbĝ-O(X) and x ∈ U}. This implies that, there exists an sbĝ-open set U containing x such that y ∉ sbĝ-cl(U). Let V = X \ sbĝ-cl(U). Then V is sbĝ-open and y ∈ V. Now, U∩V = U ∩ (X\sbĝ-cl(U)) ⊆ U ∩ (X\U) = ϕ. Therefore, X is an sbĝ-T_2 space.

Proposition 3.12: Every sbĝ-T_1 space is sbĝ-T_0 space.

Proof: The proof follows from definition 3.1 and 3.3.

The following example shows that the converse of the above proposition need not be true.

Example:

Let X = {a,b,c} with a topology τ = {X,ϕ,{a},{b},{a,b},{a,c}}.Here sbĝ-O(X)={X, ϕ,{a},{b},{a,b},{a,c}}. Clearly, (X,τ) is an sbĝ-T0 space but not sbĝ-T1 space.

Proposition 3.13: Every Every sbĝ-T_2 space is sbĝ-T_1 space.

Proof : The proof follows from definition 3.3 and 3.5.

Definition 3.14: A subset A of a topological space X is called sbĝ-difference set (briefly sbĝ-D set) if there are U,V ∈ sbĝ-O(X) such that U ≠ X and A = U\V.It is true that every sbĝ-open set different from X is an sbĝ-D set if A = U and V = Φ. So, we can observe the following.

Remark 3.15: Every proper sbĝ-open set is an sbĝ-D set.

The following example shows that the converse of the above remark need not be true

Example 3.16: Let X = {a,b,c,d} with a topology τ = {X,ϕ,{a},{a,c},{a,b,d}}.Here sbĝ-O(X) ={X,ϕ,{a},{a,b},{a,c}, {a,d}, {a,b,c}, {a,b,d}, {a,c,d}}. Consider, U = {a,b,d} ≠ X and V = {a,b,c} are sbĝ-open sets in X. Then, we have A = U\V = {d} is an sbĝ-D set but not sbĝ-open set.

Definition 3.17: A topological space (X,τ) is said to be sbĝ-symmetric if for x,y in X, x ∈ sbĝ-cl({y}) implies y ∈ sbĝ-cl({x}) (or x ∉ sbĝ-cl({y}) implies y ∉ sbĝ-cl({x})).

Example 3.18: Let X = {a,b,c} with a topology τ = {X,ϕ,{a},{b},{a,b}}. Here sbĝ -C(X)={X,ϕ,{a},{b},{c}, {a,c},{b,c}}. Here for any pair of distinct points x,y in X, x ∉ sbĝ-cl({y}) implies y∉sbĝ-cl({x}) So, X is an sbĝ-symmetric space.

Proposition 3.19: Let (X,τ) be a topological space. Then (X,τ) is an sbĝ-symmetric space if and only if {x} is sbĝ-closed for each x ∈ X.

Proof: Assume that {x} is sbĝ-open in X. Then there exists sbĝ-open set U such that {x} ⊆ U, but sbĝ-cl({x}) ⊈ U. Then sbĝ-cl({x}) ∩ (X\U)≠ϕ. Now, we take y ∈ sbĝ-cl({x}) ∩ (X\U), then by hypothesis x ∈ sbĝ-cl({y}) and also sbĝ-cl({y}) ⊆ (X\U). Therefore, x ∉ U, which is a contradiction. Hence,{x}is sbĝ-closed for each x ∈ X. Conversely, suppose singleton sets are sbĝ-closed in X. We claim that X is an sbĝ-symmetric space. Assume that x ∈ sbĝ-cl({y}) but y ∉ sbĝ-cl({x}). Then {y} ⊆ X\sbĝ-cl({x}). So, sbĝ-cl({y}) ⊆ X\sbĝ-cl({x}). Thus, x ∈ X\sbĝ-cl({x}). Therefore, x ∉ sbĝ-cl({x}) which is a contradiction. Hence, y ∈ sbĝ-cl({x}).

Corollary 3.20: If a topological space (X,τ) is an sbĝ-T_1 space, then it is sbĝ-symmetric.

Proof: Since X is an sbĝ-T_1 space and by proposition 3.10, singleton sets are sbĝ-closed. Therefore by Proposition 3.19, (X,τ) is an sbĝ-symmetric space.

Corollary 3.21: If a topological space (X,τ) is sbĝ-symmetric and sbĝ-T0 space, then (X,τ) is sbĝ-T1 space.

Proof: Let x ≠ y and (X,τ) be an sbĝ-T0 space. We may assume that x ∈ U □ X\{y} for some U ∈ sbĝ-O(X). Then x ∉ sbĝ-cl({y}). Since X is an sbĝ-symmetric space, y ∉ sbĝ-cl({x}). Then there exists an sbĝ-cl({y}). Since X is an sbĝ-symmetric space, y ∉ sbĝ-cl({x}). Then there exists an sbĝ-open set V such that y ∈ V ⊂ X\{x}. Hence, (X,τ) is an sbĝ-T1 space.

Definition 3.22: Let (X,τ) be a topological space and A ⊆ X. Then the sbĝ-kernel of A is denoted by sbĝ- ker(A) and is defined to be sbĝ-ker(A)= ∩{U ∈ sbĝ-O(X) : A ⊆ U}.

Example 3.23: Let X = {a,b,c} with a topology τ = {X,ϕ,{a},{a,b},{a,c}}. Here sbĝ O(X) = {X,ϕ, {a},{a,b},{a,c}}. sbĝ-ker({a}) = {a}; sbĝ-ker({b}) = {a,b};sbĝ-ker({c}) = {a,c}; sbĝ-ker({a,b}) = {a,b}; sbĝ-ker({a,c}) = {a,c}; sbĝ-ker({b,c}) = X.

Proposition 3.24: Let (X,τ) be a topological space and x ∈ X. Then y ∈sbĝ-ker({x}) if and only if x ∈ sbĝ-cl({y}).

Proof : Suppose that y ∉ sbĝ-ker({x}). Then there exists an sbĝ-open set V containing x such that y ∉ V. By proposition 2.10, x ∉ sbĝ-cl({y}). Conversely, assume that x ∉ sbĝ-cl({y}). Then there exists an sbĝ-open set U containing x such that y ∉ U. By the definition of sbĝ-kernel, y ∉ sbĝ-ker({x}).

Proposition 3.25: Let (X,τ) be a topological space and A be a subset of (X,τ). Then sbĝ-ker(A) = {x ∈ X : sbĝ-cl({x}) ∩ A ≠ ϕ}.

Proof: Let x ∈ sbĝ-ker(A). Suppose x ∉ {x ∈ X : sbĝ-cl({x}) ∩ A ≠ ϕ}. Then sbĝ-cl({x}) ∩ A = ϕ. So, X\sbĝ-cl({x})is an sbĝ-open set containing A and x ∉ X \ sbĝ-cl({x}). Therefore by definition 3.22 x∉sbĝ-ker(A). It contradicts x ∈ sbĝ-ker(A). Conversely, if sbĝ-cl({x}) ∩ A ≠ ϕ. We claim that, x ∈ sbĝ-ker(A). Suppose that x ∉ sbĝ-ker(A). Then there exists an sbĝ-open set V containing A such that x ∉ V. Now, let y ∈ sbĝ-cl({x}) ∩ A. Then y ∈ sbĝ-cl({x}) and y ∈ A. By proposition 2.10, y ∈ sbĝ-cl({x}) implies V ∩ {x} ≠ ϕ for every open set V containing y. Hence, x ∈ V. By this contradiction, x ∈ sbĝ-ker(A).

Proposition 3.2: The following properties hold for the subsets A, B of a topological space (X,τ):

1)A ⊆ sbĝ-ker(A).

2)A⊆B implies sbĝ-ker(A) ⊆ sbĝ- ker(B).

3)If A is sbĝ-open in (X,τ), then A = sbĝ-ker(A).

4) sbĝ-ker(sbĝ-ker(A)) = sbĝ-ker(A).

Proof: 1)Suppose that A is any subset of X. If x ∉ sbĝ-ker(A), then there exist U ∈ sbĝ-O(X) such that A ⊆ U and x ∉ U. Therefore, x ∉ A. Hence A ⊆sbĝ-ker(A) .

2) Let A ⊆B. Suppose sbĝ-ker(A) ⊈ sbĝ-ker(B). Then x ∈ sbĝ-ker(A) but x ∉ sbĝ-ker(B). By the definition of sbĝ-kernel, there exists an sbĝ-open set U such that B ⊆U and x ∉ U. Since A⊆B⊆ U, x ∉ sbĝ-ker(A). By this contradiction, sbĝ-ker(A) ⊆ sbĝ-ker(B).

3) Obvious from the definition of sbĝ-ker(A).

4) From (1) and (2), we have, sbĝ-ker(A) ⊆ sbĝ-ker(sbĝ-ker(A)). To prove the other implication, if x ∉ sbĝ-ker(A), then there exists U ∈ sbĝ-O(X) such that A ⊆Uand x∉U. Therefore, sbĝ-ker(A)⊆U and so x∉sbĝ-ker(sbĝ-ker(A)). Hence, sbĝ-ker(A) = sbĝ-ker(sbĝ-ker(A)).

Proposition 3.27 : If a singleton set {x} is an sbĝ-D set of (X,τ), then sbĝ-ker({x}) ≠ X.

Proof: since {x} is an sbĝ-D set of (X,τ), there exist two sbĝ –open subsets U ,V such that {x}=U\V.So that {x}⊆U and U≠X. By proposition 3.26(2) and(3),sbĝ-ker({x})⊆U ≠ X. Hence sbĝ-ker({x}) ≠ X.

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