C-Prime Anti Fuzzy Bi-ideals in Γ-Near-Rings

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Abstract

The concept of fuzzy sets was introduced by Zadeh in 1965. Γ-near-rings were defined by Sathyanarayana and the ideals in Γ-near-rings studied by Sathyanarayana and Booth. C-prime fuzzy ideals of near-rings were introduced by Kedukodi, Sathyanarayana and Kuncham in 2007. In this paper, we introduce the notion of c-prime anti fuzzy bi-ideals. An anti fuzzy bi-ideal μ of M is called c-prime if for all x ,yε M, γεΓ, μ(xγy)≥ min { μ(x) ,μ(y) }and obtain some of their properties.

Keywords

Fuzzy bi-ideal, anti fuzzy bi-ideal, c-prime anti fuzzy bi - ideal.

1. Introduction

The introduction of fuzzy sets by Zadeh, the fuzzy set theory developed by Zadeh and others has found many applications in the domain of mathematics. Gamma near-rings were defined by Bh. Satyanarayana [5] and the ideal theory in Gamma near rings was studied by Bh. Satyanrayana [5] and G.L. Booth. Fuzzy ideals in Gamma near-rings were introduced by Y.B.Jun and M.A.Ozturk. In this paper, we introduce c-prime anti fuzzy bi-ideals in Gamma near-rings and study their properties. Throughout this paper, we assume that *M* is a zero symmetric Γ-near-ring.

2. Preliminaries

Definition 2.1:

A non-empty set N with two binary operations"+" and"." is called a near-ring, if it satisfies the following axioms:
(i)(N, +) is a group,
(ii)(N, .) is semi group,

(iii)(x+y).z=x.z+y.z for all x, y, zϵN.

Precisely speaking it is a right near-ring, because it satisfies the right distributive law. We will use the word "near-ring" to mean "right near-ring". We denote x y instead of x. y. moreover, a near-ring Nis xϵN, where 0 is the additive identity in N.

Definition 2.2:

Let (M, +) be a group and Γ be a non empty set. Then M is said to be a Γ-near-ring, if there exist a mapping M x Γ x M→ M (The image of (x , α, y) is denoted by ($x\ \alpha\ y$) satisfied the following conditions: (i)(x + y) α z=x α z + y α z,

(ii)(x α y) β z=x α(y α z) for all x ,y ,z ϵ M and α, β ϵ Γ.

Definition 2.3:

A Γ near-ring *M* is said to be zero-symmetric if $m\gamma 0 = 0$ for all $m \epsilon M$ and for all $\gamma\epsilon\Gamma$.

Definition 2.4:

Let A be a non-empty set. A fuzzy subset of A is a function μ: A → [0, 1]. For any t ϵ [0,1], the set μ_t ={x A:μ(x)≥t}is called level subset of μ.For any t € [0,1] the set μ_t={x ϵA: μ(x)≤t}is called anti level subset of μ.

Definition 2.5:

Let M be a Γ –near-ring and μ be a fuzzy subset of M. Then the complement of μ is denoted by μ^c and is defined by $\mu^c(x) = 1 - \mu(x)$, for any $x\epsilon$M.

Definition 2.6:

Let M and N be Γ-near-rings. A map f: M →N is called a Γ-near-ring homomorphism, if (x + y) = f(x)+f(y) and f(x α y)=f(x) α f(y) for all x,y ϵM and α ϵ Γ

Definition 2.7:

Let M be a Γ-near-ring .For an endomorphism f of M and fuzzy set μ in M, we define a new fuzzy set μ^f in M by μ^f (x) =μ (f(x)) or all xϵM.

Definition 2.8:

Let μ be a fuzzy set of a Γ-near-ring M and f be a function defined on M, then the fuzzy set v in f(M) is defined by v(y)= inf {μ(x):xϵf^{-1}(y)) for all yϵf(M) is called the image of μ under.similarly,if v is a fuzzy set in f(M),thenμ=v o f in M(that is, the fuzzy set defined by μ(x)=v(f(x)) for all x ϵM is called the pre-image of v under f.

Definition 2.9:

(i) For a family of anti fuzzy sets $\{\mu_i: i\epsilon\wedge\}$ in a Γ-near-ring M, the union is defined by $\cap_{i\epsilon\wedge}\ \mu_i$(x) =sup{$\mu_i$(x):i ϵ∧}for each xϵM

(ii) For a family of anti fuzzy sets {μ_i: $i\epsilon\wedge$} in a Γ-near-ring M, the intersection $\cap_i\ \mu_i$(x) of {μ_i: iϵ∧} is defined by $\cap_{i\epsilon\wedge}\ \mu_i$(x) = inf {$\mu_i$(x):$i\epsilon\wedge$} for each xϵM.

Definition 2.10:

γ_1	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	a	0	a
c	0	0	b	c

A fuzzy set μ in a Γ-near-ring M is called a fuzzy left(respectively right) ideal of M (i)μ(x-y)≥min{μ(x),μ(y)},(ii)μ(y + x-y)≥μ(x),for all x, yϵM (iii)μ(u α(x +v)-u α v)≥μ(x)(respectively μ(x α u)≥μ(x))for all x, u, vϵM and αΓϵ. A fuzzy set μ in a Γ-near-ring M is called an fuzzy ideal M ,if μ is both fuzzy left and right ideal of M. Note that if μis a fuzzy left (respectively right) ideal of a Γ-near-ring M ,then μ(0)≥μ(x) for all xϵM,where 0 is the zero element of M.

Definition 2.11:

A fuzzy set μ in a Γ-near-ring M is called an anti fuzzy left(respectively right) ideal of M (i)μ(x-y)≤ max{μ(x),μ(y)}, (ii)μ(y +x-y)≤μ(x),for all x, yϵM (iii)μ(u α(x +v)-u α v)≤μ(x)(respectively μ(x α u)≥μ(x))for all x, u, vϵ M and αϵΓ. Afuzzy setμ in a Γ-near-ring M is called an anti fuzzy ideal M, if μ is both anti fuzzy left and right ideal of M. Note that if μis an anti fuzzy left (resp right) ideal of a Γ-near-ring M, then μ(0)≤μ(x) for all xϵM.where 0 is the zero element of M.

Definition 2.12:

A subgroup B of (M, +) is a bi-ideal if and only if BΓMΓB⊆B.

Definition 2.13:

A fuzzy set μ in M is called an anti fuzzy bi-ideal of M if

(i)μ(x-y) ≤ max {μ(x), μ(y)} for all x, yϵM

(ii)μ (x α y β z) ≤ max {μ(x), μ (z)} for all x, y, zϵM and αϵΓ.

Definition 2.14:

A fuzzy bi-ideal μ of M is called c-prime if for all x, y ϵM, γϵΓ, μ(x γ y) ≤ max {μ(x),μ(y)}.

3. C – prime anti fuzzy bi – ideals

Definition 3.1:

An anti fuzzy bi-ideal μ of M is called c-prime if for all x, y ϵM, γϵΓ, μ(x γ y) ≥ min {μ(x),μ(y)}.

Example 3.2:

Consider the Γ-near-ring defined by the ‘Klein’s four group {0,a,b,c} with Γ={γ_1,γ_2}where ‘γ_1’and ‘γ_2’ are given by the schemes 7(0,7,11,1)and 12:(0,7,0,7)

Let μ be an anti fuzzy set on M .Take the anti fuzzy points "0.4, 0.6, 0.4 and 0.5".Then μ is a c-prime anti fuzzy bi-ideal of M.

Lemma 3.3:

Let B be a bi-ideal of a Γ-near-ring M. For any 0<t<1,
There exists an anti fuzzy bi-ideal μof M such that μ_t=M\B.

Proof:

Let B be a bi-ideal of a Γ near-ring M. Define μ:M→[0,1] by

$$\mu(x) = \begin{cases} 0 \ if \ x\epsilon B \\ t \ if \ x \notin B \end{cases}$$

where t is a fixed number in (0,1).clearly μ_t =M\B.
Let x, y ∈M. If
x, y ∈ B then μ (x−y) =0= max{ μ(x), μ(y)}. If at least one of x and y is not in B, then x−y∉Band so μ(x−y)=t=max {μ(x),μ(y)}.
Let x, y, z ∈ M and α, β ∈Γ. If x, z ∈B, then μ(x) =0 ;μ(z) =0. Also
μ (x α y β z) =0= max {μ(x), μ (z)}.
If at least one of x and z is not in B,
Then μ(x α y β z) ≥0=min {μ(x), μ (z)}. Thus μ is an anti fuzzy bi-ideal of M.

Lemma 3.4:

Let B be a non-empty subset of M. Then B is a bi-ideal of M if and only if the characteristic function μ_B of B is an anti fuzzy bi-ideal of M

Proof:

Let B be a bi-ideal of a Γ near-ring M. For, x, y ∈B, x−y∈B.

(i) Let x, y ∈M.
(a) If x, y ∈B, then μ_B(x) = 1 and μ_B(y) = 1. Thus μ_B (x−y) =1=max {μ_B(x), μ_B(y)}
.
(b) If x ∈ B and y ∉ B, then μ_B (x) = 1 and μ_B (y) = 0. Thus
μ_B (x−y)=0≤ max { μ_B (x), μ_B (y)}.
(c) If x ∉Band y ∈B, then μ_B (x) =0; μ_B(y) = 1. Thus
μ_B (x−y)=0≤ max { μ_B (x), μ_B(y)}.
(d) If x ∉ Band y ∉ B, then μ_B(x) =0; μ_B(y) =0. Thus
μ_B(x−y) =0= max {μ_B (x), μ_B (y)}.

(ii) Let x, y, z ∈ M. and α, β ϵΓ
(a) If x ∈ Band z ∈B, then μ_B (x) = 1 and μ_B (z) = 1. Thus
μ_B (x α y β z)=1= max {μ_B (x), μ_B (z) }.
(b) If x ∈ Band z ϵ B, then μ_B (x) = 1 and μ_B (z) = 0. Thus

γ_2	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	0	0
c	0	a	0	a

μ_B (x α y β z)=0≤max {μ_B (x), μ_B (z)}.
(c) If x ∉ Band z ∈ B, then μ_B (x) = 0 and μ_B (z) = 1. Thus
μ_B (x α y β z)=0≤ max {μ_B (x), μ_B (z) }.
(d) If x ∉ Band z ∉ B, then μ_B (x) = 0 and μ_B (z) = 0. Thus
μ_B (x α y β z)=0= max {μ_B (x), μ_B(z)}.
Thus μ_B is a bi-ideal of M. Conversely, suppose μ_B is an anti fuzzy bi-ideal of M. Then by lemma 3.3, μ_B is two valued. Hence B is a bi-ideal of M.

Proposition 3.5:

Let B be a non-empty subset of M. Then B is a c-prime bi-ideal of M if and only if μ_B is a c-prime anti fuzzy bi-ideal of M.

Proof:

Suppose that B is a c-prime bi-ideal of M and μ_B is the characteristic function of B.
By above lemma 3.4, μ_B is an anti fuzzy bi-ideal of M.
Let x, yϵM and γϵΓ.
If x γ yϵB, then μ_B(x γ y) =1
Since B is a c-prime anti fuzzy bi-ideal of M and x γ y ϵB.

(a) If xϵB or yϵB, then μ_B (x)=1 or μ_B(y)=1
μ_B(x γ y)≥min {μ_B (x), μ_B (y)}

(b) If x γ y ∉B then μ_B(x γ y) =0

μ_B(x γ y)≥ min {μ_B (x), μ_B (y)}

Conversely,
Assume that μ_B is a c-prime anti fuzzy bi-ideal of M.
Then μ_B is an anti fuzzy bi-ideal of M.
By lemma 3.4, B is a bi-ideal of M.
Let x, yϵ M be such that xγyϵB
Then μ_B(xγy) =1
μ_B(x γ y)≥ min{μ_B (x), μ_B (y)}
We have min {μ_B (x), μ_B (y)} =1.
Thus μ_B (x) =1 or μ_B (y) =1.Hence x, yϵB.

Proposition 3.6:

Let B be a c-prime bi-ideal of M. for any tϵ(0,1) there exits a c-prime anti fuzzy bi-ideal of M such that μ_t=M\B.

Proof:

Let t∈ (0, 1)
Then by lemma 3.3, there exits an anti fuzzy bi-ideal μ of M defined by

$\mu(x) = \begin{cases} t \text{ if } x \epsilon B \\ 0 \text{ otherwise} \end{cases}$

Such that μ_t =M\B.
Suppose μ is not a c-prime anti fuzzy bi-ideal of M
There exits x, y ∈M and γ ∈ Γ. Such that
μ (xγy) ≤ min {μ(x), μ(y)}.
By definition of μ, we get
μ(x) =0 or μ(y) =0 and μ(x γ y) =t
x γ y∈ B and x, y$\notin B$.
Which is contradiction to
Since B is a c-prime bi-ideal of M.
Hence μ is a c-prime anti fuzzy bi-ideal of M.

Proposition 3.7:

Let f: M→N is a homomorphism. If μ is a c-prime ideal of M, then $f^{-1}(\mu)$ is a c-prime anti fuzzy bi-ideal of M.

Proof:

Let f: M→N be a Γ near -ring homomorphism, v be an anti fuzzy bi-ideal of M and μ be the pre -image of v under f.
Then μ(x-y)=v($\theta(x-y)$)=v($\theta(x) - v(\theta(y)$≤max{v($\theta(x)$, v($\theta(y)$}=max{ μ(x),μ(y)}.
Further μ(x α y β z)=v(θ(xαyβz)=v($\theta(x)$, α($\theta(y)\beta\theta(z)$)≤max{ v($\theta(x)$, v($\theta(z)$}=max{ μ(x),μ(z)}for all x, y, zϵM and α,βϵΓ.
Hence μ is an anti fuzzy bi-deal of M.

Definition 3.8:

Let M be an ordered Γ-near- ring. An anti fuzzy subset μ of M is called an anti fuzzy bi- filter of M if:

(i)x≥y=>μ(x)≥μ(y)

(ii)μ(x-y) ≤ max {μ(x), μ(y)}.

(iii) μ (x α y β z) ≥ min {μ(x), μ (z)}.for all x , y ,zϵM and $\alpha, \beta \epsilon \Gamma$.

Proposition 3.9:
Let M be an ordered Γ near ring and μ be an anti fuzzy subset of M. Then μ is an anti fuzzy bi-filter of M if and only if the complement μ of $\mu^c$is a c-prime anti fuzzy bi-ideal subset of M.

Proof:
Suppose that μ is an anti fuzzy bi-filter of M.
Let x, yϵM, x ≥y
Since μ is an anti fuzzy bi-filter of M, we have μ(x) ≥μ(y)
Then -μ(x) ≤ -μ(y)
=>1- μ(x) ≤1-μ(y)
Thus μ^c (x) ≤ μ^c (y)
Let x, y be any two arbitrary elements of M. since μ is an anti fuzzy bi-filter of M. we have, μ(x-y) ≤max {μ(x), μ(y)}.
Let x, y, zϵM and α, βϵΓ
Since μ is an anti fuzzy bi-filter of M. we have μ (xαyβz) ≥ min {μ(x),μ(z)}
=> μ^c (xαyβz) ≤ max {μ^c (x), μ^c (z)}

Thus $\mu^c$is an anti fuzzy bi-ideal of M.
Let x, yϵM
Since μ is anti fuzzy bi-filter
We have μ (xγy) ≤ max {μ(x), μ(y)}.We get μ^c (xγy) ≥ min {μ^c (x), μ^c (y)}
Hence μ'is a c-prime anti fuzzy bi-ideal subset of M.

Conversely, Assume that μ^c is a c-prime anti fuzzy bi-ideal subset of M

Then $\mu^c$is an anti fuzzy bi-ideal of M

Let x, yϵM, x≥y

Since μ^c is an anti a fuzzy bi-ideal of M

We have μ^c (x) ≤μ^c (y)

Then 1- μ(x) ≤1-μ(y)
=>-μ(x) ≤ -μ(y)
=> μ(x) ≥μ(y)
Since μ^c is a c-prime anti a fuzzy bi-ideal subset of M.

We have μ^c (xγy) ≥ min {μ^c (x), μ^c (y)} for all x, yϵM and $\gamma\epsilon$Γ

=> μ (xγy) ≤ max {μ(x),μ(y)}

Let x, y, z ϵM and $\alpha, \beta\epsilon$Γ

Since, μ^c is an anti fuzzy bi-ideal of M
μ^c (xαyβz) ≤ max {μ^c (x), μ^c (z)}
μ (xαyβz) ≥ min {μ(x),μ(z)}
Thus μ is an anti fuzzy bi-filter of M.

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