A STUDY ON NEUTROSOPHIC GENERALIZED SEMI-CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract : The purpose of this paper is to introduce and study the concepts of neutrosophic generalized semi-closed sets and neutrosophic generalized semi-open sets in neutrosophic topological space.

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INTRODUCTION

The fuzzy set was introduced by Zadeh [7] in 1965, where each element had a degree of membership. The intuitionistic fuzzy set on a universe X was introduced by K. Atanassov [1, 2, 3] in 1983 as a generalization of fuzzy set, where each element had the degree of membership and the degree of non- membership. The neutrosophic set was introduced by Smarandache [4] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. In 2012, Salama, Alblowi [6] was introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. For the past few years, many researchers were going on in neutrosophic topological spaces and many concepts in intuitionistic fuzzy topology were extended to neutrosophic topology.

The concepts of neutrosophic semi-open sets, neutrosophic semi-closed sets, neutrosophic semi-interior and neutrosophic semi-closure in neutrosophic topological spaces were introduced by P. Iswarya et. al. [5] in 2016. In this paper, we introduce the definitions of neutrosophic generalized semi-closed sets and neutrosophic generalized semi-open sets in neutrosophic topological spaces. We study some of their basic properties in neutrosophic topological spaces with examples.

I . PRELIMINARIES

Before entering to our work, we recall the following definitions and theorems as given by P. Iswarya [5] and A. A. Salama et. al. [6].

Definition 1.1 [6] Let X be a non-empty fixed set. A neutrosophic set [*NS* for short] A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Definition 1.2 [6] Let $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ be a *NS* on X. Then the complement of the set *A* [C (*A*) for short] may be defined as three kinds of complements :

(C_1) $C(A) = \{ \langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x) \rangle : x \in X \}$

(C_2) $C(A) = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$

(C_3) $C(A) = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$

Definition 1.3 [6] Let X be a non-empty set and neutrosophic sets *A* and *B* in the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$. Then we may consider two possible definitions for subsets ($A \subseteq B$).

$A \subseteq B$ may be defined as :

(1) $A \subseteq B \Leftrightarrow \{ \langle x, \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x)$ and $\gamma_A(x) \ge \gamma_B(x) \rangle : x \in X \}$

(2) $A \subseteq B \Leftrightarrow \{ \langle x, \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x)$ and $\gamma_A(x) \ge \gamma_B(x) \rangle : x \in X \}$

Definition 1.4 [5] Let *A* be neutrosophic set of a neutrosophic topological space [*NTS* for short] X. Then *A* is said to be neutrosophic semi-open set [*NSO* set for short] of X if there exists a neutrosophic open set NO such that NO $\subseteq A \subseteq$ *NCl* (NO).

Theorem 1.5 [5] A subset *A* in a *NTS* X is *NSO* set if and only if $A \subseteq$ *NCl* (*NInt* (*A*)).

Definition 1.6 [5] A *NS A* is called neutrosophic semi-closed set [*NSC* set for short] if the complement of C (A) is a *NSO* set.

That is, Let *A* be *NS* of a *NTS* X. Then *A* is said to be neutrosophic semi-closed set of X if there exists a neutrosophic closed set NC such that *NInt* (NC) $\subseteq A \subseteq$ NC.

That is, A subset *A* in a *NTS* X is *NSC* set if and only if *NInt* (*NCl* (*A*)) $\subseteq A$.

II . NEUTROSOPHIC GENERALIZED SEMI-CLOSED SETS

In this section, we introduce the concepts of neutrosophic generalized semi-closed sets and study some of their basic properties.

Definition 2.1 A neutrosophic set *A* of a neutrosophic topological space (X, τ) is called a neutrosophic generalized semi-closed set [NGSC set for short] if *NSCl* (*A*) ⊆ U, whenever *A* ⊆ U and U is a neutrosophic open set.

Example 2.2 Let X = { *a* } with $\tau = \{ 0_N, A, B, C, D, 1_N \}$ where A = ⟨ (0.2, 0.5, 0.6) ⟩, B = ⟨ (0.5, 0.1, 0.4) ⟩, C = ⟨ (0.5, 0.5, 0.4) ⟩ and D = ⟨ (0.2, 0.1, 0.6) ⟩. Let us take S = ⟨ (0.3, 0.1, 0.6) ⟩. Then *NCl* (*NInt* (S)) = C (C). Therefore S ⊆ C (C). Hence S is neutrosophic semi-open set. So we have some neutrosophic semi-open sets are 0_N, A, B, C, D, S, C (A), C (C), 1_N. Now, S ⊆ C and C is neutrosophic open set. Then *NSCl* (S) = C (C) ⊆ C. Therefore *NSCl* (S) ⊆ C, S ⊆ C and C is neutrosophic open set. Hence S is neutrosophic generalized semi-closed set.

Definition 2.3 A Neutrosophic set *A* in X is called neutrosophic generalized semi-open set [NGSO set for short] in X if C (*A*) is neutrosophic generalized semi-closed set in X.

That is, U ⊆ *NSInt* (*A*) , whenever U ⊆ *A* and U is a neutrosophic closed set.

Definition 2.4 Let (X, τ) be a neutrosophic topological space. Then a neutrosophic subset *A* of the neutrosophic topological space X is said to be neutrosophic regular open if *A* = *NInt* (*NCl* (*A*)) and neutrosophic regular closed if *NCl* (*NInt* (*A*)) = *A*.

Theorem 2.5 Every neutrosophic closed set in neutrosophic topological space (X, τ) is a neutrosophic generalized semi-closed set.

Proof : Let *A* be a neutrosophic closed set in neutrosophic topological space X. Let *A* ⊆ U and U be a neutrosophic open set in X. Then by Definition 1.16 (b) [5], *A* = *NCl* (*A*) and by Proposition 6.4 [5], *NSCl* (*A*) ⊆ *NCl* (*A*) we get *NSCl* (*A*) ⊆ *NCl* (*A*) = *A* ⊆ U. Hence *A* is a neutrosophic generalized semi-closed set in X.

The converse of the above theorem is not true as shown by the following example.

Example 2.6 From Example 2.2, S is neutrosophic generalized semi-closed set but not neutrosophic closed set.

Theorem 2.7 Every neutrosophic semi-closed set in the neutrosophic topological space (X, τ) is a neutrosophic generalized semi-closed set.

Proof : Let *A* be a neutrosophic semi-closed set in the neutrosophic topological space X. Let *A* ⊆ U and U be a neutrosophic open set in X. Since *A* is neutrosophic semi-closed set, *NInt* (*NCl* (*A*)) ⊆ *A*. This implies that *NSCl* (*A*) = *A* ∪ *NInt* (*NCl* (*A*)) ⊆ *A*. Since *A* ⊆ U, *NSCl* (*A*) ⊆ U. Therefore *NSCl* (*A*) ⊆ U, *A* ⊆ U and U is a neutrosophic open set. Hence *A* is a neutrosophic generalized semi-closed set in X.

The converse of the above theorem is not true as shown by the following example.

Example 2.8 From Example 2.2, S is neutrosophic generalized semi-closed set but not neutrosophic semi-closed set.

Theorem 2.9 If *A* and *B* are neutrosophic generalized semi-closed sets, then *A* ∩ *B* is also a neutrosophic generalized semi-closed set.

Proof : Let *A* and *B* are neutrosophic generalized semi-closed sets. If *A* ∩ *B* ⊆ U and U is neutrosophic open set, then *A* ⊆ U and *B* ⊆ U. Since *A* and B are neutrosophic generalized semi-closed sets, *NSCl* (*A*) ⊆ U and *NSCl* (*B*) ⊆ U. Hence *NSCl* (*A*) ∩ *NSCl* (*B*) ⊆ U. By Proposition 6.5 (ii) [5], *NSCl* (*A* ∩ *B*) ⊆ *NSCl* (*A*) ∩ *NSCl* (*B*) ⊆ U. This implies that *NSCl* (*A* ∩ *B*) ⊆ U. Therefore *NSCl* (*A* ∩ *B*) ⊆ U, *A* ∩ *B* ⊆ U and U is neutrosophic open set. Thus *A* ∩ *B* is neutrosophic generalized semi-closed set.

Remark 2.10 Union of any two neutrosophic generalized semi-closed sets in (X, τ) need not be a neutrosophic generalized semi-closed set, as seen from the following example.

Example 2.11 Let X = { *a* } with τ = { 0_N, A, B, C, D, E, F, G, H, 1_N } where A = ⟨ (0.6, 0.3, 0.7) ⟩, B = ⟨ (0.4, 0.9, 0.2) ⟩, C = ⟨ (0.5, 0.2, 0.8) ⟩, D = ⟨ (0.6, 0.9, 0.2) ⟩, E = ⟨ (0.5, 0.9, 0.2) ⟩, F = ⟨ (0.4, 0.3, 0.7) ⟩, G = ⟨ (0.4, 0.2, 0.8) ⟩ and H = ⟨ (0.5, 0.3, 0.7) ⟩. Also some neutrosophic semi-open sets are 0_N, A, B, C, D, E, F, G, H, C (A), 1_N. Now we consider the two neutrosophic generalized semi-closed sets C and F. Their intersection G is neutrosophic generalized semi-closed set but their union H is not neutrosophic generalized semi-closed set.

Theorem 2.12 If A is a neutrosophic generalized semi-closed set in X and $A \subseteq B \subseteq NSCl\ (A)$, then B is a neutrosophic generalized semi-closed set in X.

Proof : Let U be a neutrosophic generalized semi-open set in X such that $B \subseteq$ U. Since $A \subseteq B$, $A \subseteq$ U. Again since A is a neutrosophic generalized semi-closed set, $NSCl\ (A) \subseteq$ U. By hypothesis, $B \subseteq NSCl\ (A)$. By Proposition 6.3 (iii) [5], $NSCl\ (B) \subseteq NSCl\ (\ NSCl\ (A)\ ) = NSCl\ (A)$. That is $NSCl\ (B) \subseteq NSCl\ (A)$. This implies that $NSCl\ (B) \subseteq$ U. Hence B is a neutrosophic generalized semi-closed set in X.

Theorem 2.13 A neutrosophic set A of a neutrosophic topological space (X, τ) is a neutrosophic generalized semi-closed set if and only if $NSCl\ (A) \subseteq B$ where B is a neutrosophic open set and $A \subseteq B$.

Proof : Assume that A is a neutrosophic generalized semi-closed set in X. Let B be a neutrosophic open set in X such that $A \subseteq B$. Then C (B) is a neutrosophic closed set in X such that C $(B) \subseteq$ C (A). Since C (A) is a neutrosophic generalized semi-open set, C $(B) \subseteq NSInt$ (C (A)). By Proposition 6.2 (ii) [5], $NSInt$ (C (A)) = C ($NSCl\ (A)$). Therefore C $(B) \subseteq$ C ($NSCl\ (A)$) implies that $NSCl\ (A) \subseteq B$. Conversely, assume that $NSCl\ (A) \subseteq B$ where B is a neutrosophic open set and $A \subseteq B$. Then C $(B) \subseteq$ C ($NSCl\ (A)$) where C (B) is a neutrosophic closed set and C $(B) \subseteq NSInt$ (C (A)). Therefore C (A) is a neutrosophic generalized semi-open set. This implies that A is a neutrosophic generalized semi-closed set.

Theorem 2.14 A neutrosophic set A of a neutrosophic topological space (X, τ) is a neutrosophic generalized semi-closed set if and only if $NInt\ (NCl\ (A)) \subseteq B$ whenever B is a neutrosophic open set and $A \subseteq B$.

Proof : Assume that A is a neutrosophic generalized semi-closed set in X. Let B be a neutrosophic open set in X and $A \subseteq B$. Then C (B) is a neutrosophic closed set in X such that C $(B) \subseteq$ C (A). Since C (A) is a neutrosophic generalized semi-open set, C $(B) \subseteq NSInt$ (C (A)). Therefore C $(B) \subseteq NCl\ (NInt$ (C (A))). Hence C $(B) \subseteq$ C ($NInt\ (NCl\ (A)$)). This implies that $NInt\ (NCl\ (A)) \subseteq B$. Conversely, let A be neutrosophic set of X and $NInt\ (NCl\ (A)) \subseteq B$ whenever B is neutrosophic open set and $A \subseteq B$. Then C $(B) \subseteq$ C (A) and C (B) is neutrosophic closed set. By hypothesis C $(B) \subseteq$ C ($NInt\ (NCl\ (A)$)). Hence C $(B) \subseteq NCl\ (NInt$ (C (A))) implies that C $(B) \subseteq NSInt$ (C (A)). So that C (A) is neutrosophic generalized semi-open set. Hence A is neutrosophic generalized semi-closed set of X.

III . NEUTROSOPHIC GENERALIZED SEMI-OPEN SETS

In this section, we study the concepts of neutrosophic generalized semi-open sets and some of their basic properties.

Definition 3.1 The family of all NGSC set (resp, NGSO set) of a neutrosophic topological space (X, τ) will be denoted by NGSC (X) (resp. NGSO (X)).

Example 3.2 From Example 2.2, C (C) ⊆ C (D), C (C) is neutrosophic closed. Therefore C (C) ⊆ $NSInt$ (C (D)), C (C) ⊆ C (D) and C (C) is neutrosophic closed set. Hence C (D) is neutrosophic generalized semi-open set.

Theorem 3.3 Every neutrosophic open set in neutrosophic topological space (X, τ) is a neutrosophic generalized semi-open set.

Proof : Let A be a neutrosophic open set in neutrosophic topological space X. Then by Definition 1.16 (a) [5], $A = NInt\,(A)$. Again by Proposition 6.4 [5], $NInt\,(A) \subseteq NS\,Int\,(A) \subseteq A$. Therefore $A = NSInt\,(A)$. Hence A is a neutrosophic generalized semi-open set in X.

The converse of the above theorem is not true as shown by the following example.

Example 3.4 From Example 3.2, C (D) is neutrosophic generalized semi-open set but not neutrosophic open set.

Theorem 3.5 Every neutrosophic semi-open set in neutrosophic topological space (X, τ) is a neutrosophic generalized semi-open set.

Proof : Let A be a neutrosophic semi-open set in neutrosophic topological space X. Let $U \subseteq A$ and U be a neutrosophic open set in X. Since A is neutrosophic semi-open set, $A \subseteq NCl\,(NInt\,(A))$. This implies that $A \subseteq NSInt\,(A) = A \cap NCl\,(NInt\,(A))$. Since $U \subseteq A$, $U \subseteq NSInt\,(A)$. Therefore $U \subseteq NSInt\,(A)$, $U \subseteq A$ and U is a neutrosophic closed set. Hence A is a neutrosophic generalized semi-open set in X.

The converse of the above theorem is not true as shown by the following example.

Example 3.6 From Example 3.2, C (D) is neutrosophic generalized semi-open set but not neutrosophic semi-open set.

Theorem 3.7 If A and B are neutrosophic generalized semi-open sets, then $A \cup B$ is also a neutrosophic generalized semi-open set.

Proof : Let A and B are neutrosophic generalized semi-open sets. If $U \subseteq A \cup B$ and U is a neutrosophic closed set, then $U \subseteq A$ and $U \subseteq B$. Since A and B are neutrosophic generalized semi-open sets, $U \subseteq NSInt\,(A)$ and $U \subseteq NSInt\,(B)$. Hence $U \subseteq NSInt\,(A) \cup NSInt\,(B)$. By Theorem 5.3 (ii) [5], $NSInt\,(A \cup B) \supseteq NSInt\,(A) \cup NSInt\,(B) \supseteq U$. This implies that $U \subseteq NSInt\,(A \cup B)$. Therefore $U \subseteq NSInt\,(A \cup B)$, $U \subseteq A \cup B$ and U is neutrosophic closed set. Thus $A \cup B$ is neutrosophic generalized semi-open set.

Remark 3.8 Intersection of any two neutrosophic generalized semi-open sets in (X, τ) need not be a neutrosophic generalized open set, as seen from the following example.

Example 3.9 From Example 2.11, we consider the two neutrosophic generalized semi-open sets C (C) and C (F). Their union C (G) is neutrosophic generalized semi-open set but their intersection C (H) is not neutrosophic generalized semi-open set.

Theorem 3.10 If A is a neutrosophic generalized semi-open set in X and if $NSInt\,(A) \subseteq B \subseteq A$, then B is a neutrosophic generalized semi-open set in X.

Proof : Let A be a neutrosophic generalized semi-open set in X. Since $NSInt\,(A) \subseteq B \subseteq A$ and by Proposition 6.2 (i) [5], we have $C\,(A) \subseteq C\,(B) \subseteq C\,(NSInt\,(A)) = NSCl\,(C\,(A))$. Again since $C\,(A)$ is a neutrosophic generalized semi-closed set and by Theorem 2.12, we have $C\,(B)$ is a neutrosophic generalized semi-closed set in X. Hence B is a neutrosophic generalized semi-open set in X.

Theorem 3.11 A neutrosophic set A of a neutrosophic topological space (X, τ) is a neutrosophic generalized semi-open set if and only if $B \subseteq NSInt\,(A)$ where B is a neutrosophic closed set and $B \subseteq A$.

Proof : Assume that A is a neutrosophic generalized semi-open set in X. Let B be a neutrosophic closed set in X such that $B \subseteq A$. Then C (B) is a neutrosophic open set in X such that C (A) $\subseteq$ C (B). Since C (A) is a neutrosophic generalized semi-closed set, *NSCl* (C (A)) $\subseteq$ C (B). By Proposition 6.2 (i) [5], *NSCl* (C (A)) = C (*NSInt* (A)). Therefore C (*NSInt* (A)) $\subseteq$ C (B) implies that $B \subseteq$ *NSInt* (A). Conversely, assume that $B \subseteq$ *NSInt* (A) where B is a neutrosophic closed set and $B \subseteq A$. Then C (*NSInt* (A)) $\subseteq$ C (B) where C (B) is a neutrosophic open set and *NSCl* (C (A)) $\subseteq$ C (B). Therefore C (A) is a neutrosophic generalized semi-closed set. This implies that A is a neutrosophic generalized semi-open set.

Theorem 3.12 A neutrosophic set A of a neutrosophic topological space (X, τ) is a neutrosophic generalized semi-open set if and only if $B \subseteq$ *NCl* (*NInt* (A)) whenever B is a neutrosophic closed set and $B \subseteq A$.

Proof : Assume that A is a neutrosophic generalized semi-open set in X. Let B be a neutrosophic closed set in X and $B \subseteq A$. Then C (B) is a neutrosophic open set in X such that C (A) $\subseteq$ C (B). Since C (A) is a neutrosophic generalized semi-closed set, *NSCl* (C (A)) $\subseteq$ C (B). Therefore *NInt* (*NCl* (C (A))) $\subseteq$ C (B). Hence C (*NCl* (*NInt* (A))) $\subseteq$ C (B). This implies that $B \subseteq$ *NCl* (*NInt* (A)). Conversely, let A be neutrosophic set of X and $B \subseteq$ *NCl* (*NInt* (A)) whenever B be neutrosophic closed set and $B \subseteq A$. Then C (A) $\subseteq$ C (B) and C (B) is neutrosophic open set. By hypothesis C (*NCl* (*NInt* (A))) $\subseteq$ C (B). Hence *NInt* (*NCl* (C (A))) $\subseteq$ C (B) implies that *NSCl* (C (A)) $\subseteq$ C (B). So that C (A) is neutrosophic generalized semi-closed set. Hence A is neutrosophic generalized semi-open set of X.

CONCLUSION

In this paper, we studied the concepts of neutrosophic generalized semi-closed sets, neutrosophic generalized semi-open sets and their properties in neutrosophic topological spaces. In future, we extend this neutrosophic topology concepts by neutrosophic generalized semi-continuous, neutrosophic semi-generalized continuous in neutrosophic topological spaces. Also we extend this neutrosophic concepts by nets, filters and borders.

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