Interval valued anti fuzzy ideals in Boolean like semi-rings

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Abstract

In this paper, we introduce the notion of interval valued anti fuzzy ideals of Boolean like semi-rings. An i-v fuzzy subset $\tilde{\mu}$ in a Boolean like semi-ring M is called an i-v anti fuzzy left (resp. right) ideal of M if

(i) $\tilde{\mu}(x-y) \leq max^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$

(ii) $\tilde{\mu}(ra) \leq \tilde{\mu}(a)$

(iii) $\tilde{\mu}\big((r+a)s+rs\big) \leq \tilde{\mu}(a)$

We also investigate some of its properties and illustrate with examples of interval valued anti fuzzy ideals of Boolean like semi-rings.

Keywords

Boolean like semi-rings, fuzzy ideals, anti-fuzzy ideals, Interval valued anti fuzzy ideals.

1.Introduction

The fundamental concept of fuzzy set was introduced by Zadeh in 1965. Again he introduced the notion of interval valued fuzzy subsets in 1975 where the values of the membership functions are closed intervals of numbers instead of a single value. Boolean like semi-rings were introduced in role by K. Venkatesawarlu, B.V.N. Murthy and N. Amaranth during 2011. Boolean like rings of A.L. Foster arise naturally from general ring dulity considerations and preserve many of the formal properties of Boolean ring. A Boolean like ring is a commutative ring with unity and is of characteristic 2. It is clear that every Boolean ring is a Boolean like ring but not conversely. The concept of a fuzzy subset of a nonempty set was introduced by Ziu, and it has been studied by several authors. The notion of fuzzy ideals and its properties were applied to various areas: Semi groups, Bck-algebras and semi-rings. R. Biswas introduced the concept of anti fuzzy subgroups and K.H. Kim and Y.B. Jun studied the notion of anti fuzzy ideals in near rings. Fuzzy ideals in Boolean like semi - rings was introduced by N.Meenakumari and R.Rajeswari[5]. In this paper we introduce the concept of interval valued anti fuzzy ideals in Boolean like semi - rings and study the some properties of anti fuzzy ideals.

2.Preliminaries

Definition:2.1

A system (R,+,.) a **Boolean** semi-ring iff the following properties hold

(i) (R,+) is an additive (abelian) group (whose 'zero' will denoted by '0')
(ii) (R,.) is a semigroup of idempontents in the sense aa=a, for all a∈R
(iii) a(b+c)=ab+ac and
(iv) abc=bac, for all a,b,c∈R

Definition:2.2

A non empty set R together with two binary operations + and . satisfying the following conditions is called a **Boolean like semi - ring**

(i) (R,+) is an abelian group
(ii) (R,.) is a semi group
(iii) a.(b+c)=a.b+a.c for all a,b,c∈R
(iv) a+a=0 for all a in R
(v) ab(a+b+ab)=ab for all a,b∈R

Definition:2.3

A non empty subset I of R is said to be an **ideal** if

(i) (I,+) is a subgroup of (R,+),(ie)., for a,b ∈ R implies a+b ∈ R
(ii) ra∈R for all a∈I, r∈R (ie).,RI ⊂ I
(iii) (r+a)s+rs∈I for all r,s∈R,a∈I

Definition:2.4

Let μ be a fuzzy set defined on R. Then μ is said to be a **fuzzy ideal** of R if

(i) μ(x-y) ≥ min{ μ(x), μ(y)},x,y∈R
(ii) μ(ra) ≥ μ(a) for all r,a∈R
(iii) μ((r+a)s+rs) ≥ μ(a) for all r, a,s∈R

Definition:2.5

A fuzzy set μ in a Boolen like semi - ring R is called **anti fuzzy ideal** of R, if

(i) μ(x-y) ≤ max{ μ(x), μ(y)},x,y∈R
(ii) μ(ra) ≤ μ(a),for all r,a∈R
(iii) μ((r+a)s+rs) ≤ μ(a),for all r,a,s∈R

Definition:2.6

Let R and S be Boolean like semi rings. A map f:R→S is called a **Boolean like semi-ring homorphism** if f(x+y)= f(x)+f(y) and f(xy)=f(x)f(y) for all x,y∈ R

Notation:2.7

An **interval valued number $\tilde{a}$** on [0,1] is a closed sub interval of [0,1] , that is $\tilde{a}$ =$[a^-,a^+]$ such that $0\le a^-\le a^+\le 1$ where a^- and a^+ are lower and upper limits of $\tilde{a}$ respectively. The set of all closed sub intervals of [0,1] is denoted by D[0,1]. In this notation $\tilde{0}=[0^-,0^+]$ *and* $\tilde{1}=[1^-,1^+]$. We also identify the interval [a,a] by the number a∈[0,1]. For any two interval numbers $\tilde{a}=[a^-,a^+]$ and $\tilde{b}=[b^-,b^+]$ on [0,1] . We define

(i) $\tilde{a}\le\tilde{b}\Leftrightarrow a^-\le b^-$ and $a^+\le b^+$

(ii) $\tilde{a}=\tilde{b}\Leftrightarrow a^-=b^-$ and $a^+=b^+$

(iii) $\tilde{a}<\tilde{b}\Leftrightarrow \tilde{a}\le\tilde{b}$ and $\tilde{a}\ne\tilde{b}$

(iv) $k\tilde{a}=[ka^-,ka^+]$, for $0\le k\le 1$

Definition:2.8

A mapping $min^i: D[0,1]\times[0,1]\to D[0,1]$ defined by $min^i(\tilde{a},\tilde{b})=[\min(a^-,b^-),\min(a^+,b^+)]$ for all $\tilde{a},\tilde{b}\in D[0,1]$ is called an **interval min-norm**.

Definition:2.9

A mapping $max^i: D[0,1]\times[0,1]\to D[0,1]$ defined by $max^i(\tilde{a},\tilde{b})=[\max(a^-,b^-),\max(a^+,b^+)]$ for all $\tilde{a},\tilde{b}\in D[0,1]$ is called an **interval max-norm.**

Remark:2.10

Let min^i and max^i be the interval min-norm and interval max-norm on D[0,1] respectively. Then the following are true:

(1) $min^i\{\tilde{a},\tilde{a}\}=\tilde{a}$ *and* $max^i\{\tilde{a},\tilde{a}\}=\tilde{a}\ \forall\tilde{a}\in D[0,1]$.
(2) $min^i\{\tilde{a},\tilde{b}\}=min^i\{\tilde{b},\tilde{a}\}$ *and* $max^i\{\tilde{a},\tilde{b}\}=max^i\{\tilde{b},\tilde{a}\}\ \forall\ \tilde{a},\tilde{b}\in D[0,1]$.

(3) If $\forall \tilde{a}, \tilde{b}, \tilde{c} \in D[0,1]$, $\tilde{a} \geq \tilde{b}$, then $min^i\{\tilde{a}, \tilde{c}\} \geq min^i\{\tilde{b}, \tilde{c}\}$ and $max^i\{\tilde{a}, \tilde{c}\} \geq max^i\{\tilde{b}, \tilde{c}\}$.

Definition:2.11

Let f: X→ Y be a function. For a fuzzy set μ in Y, we define $\mathbf{f^{-1}(\mu)}$(x)=μ(f(x)) for every x∈ X.For a fuzzy set λ in X, **f(λ)** is defined by(f(λ))(y)= sup λ(x), if f(x)=y, x in X where y in Y.

3. Interval valued anti fuzzy ideals in Boolean like semi-rings

Definition:3.1

An interval valued fuzzy subset μ̃ in a Boolean like semi-ring R is called an interval valued **anti fuzzy ideal** of R. if,

- (i) $\tilde{\mu}(x-y) \leq max^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}\ \forall\ x, y \in R$
- (ii) $\tilde{\mu}(ra) \leq \tilde{\mu}(a)$, ∀ r,a ϵ R
- (iii) $\tilde{\mu}\big((r+a)s + rs\big) \leq \tilde{\mu}(a)$, ∀ r,a, $s \in$ R

Example :3.2

Consider the Boolean like semi-ring (R,+,·), where '+' and '·' are defined as follows,

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	0	b	b
c	0	a	c	c

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	a	b	a	0

Let μ̃ be an interval valued anti fuzzy ideal defined on R by μ̃(0) = [0.2, 0.3], μ̃(a) = [0.6,0.7], μ̃(b) =[0.6,0.7], μ̃(c) =[0.8,0.9] . Then μ̃ is an interval valued anti fuzzy ideal in Boolean like semi-ring R.

Theorem:3.3

Let R be a Boolean like semi-ring and $\{\tilde{\mu}_i /\ i \epsilon I\}$ a non-empty family of subset of R. If $\{\tilde{\mu}_i /\ i \epsilon I\}$is an interval valued anti fuzzy ideals of R. Then, $\bigcup_{i\epsilon I} \tilde{\mu}_i$ is an interval valued anti fuzzy ideal of R.

Proof:

Let $\{\tilde{\mu}_i /\ i \epsilon I\}$ be an interval valued anti fuzzy ideal of R.

Let x, y, z ϵ R.

Then we have,

$(\bigcup_{i\epsilon I}\widetilde{\mu_i})$(x-y) = sup^i $\{\{\widetilde{\mu_i}(x-y)$ / iϵI}

= sup^i $\{\{\widetilde{\mu_i}(x-y)$ / iϵI}

$\leq sup^i$ $\{max^i\{\widetilde{\mu_i}(x),\widetilde{\mu_i}(y)\}$/ i$\epsilon$I}

= $max^i\{sup^i\{\widetilde{\mu_i}(x)$/ i$\epsilon$I},$sup^i\{\widetilde{\mu_i}(y)$/ i$\epsilon$I}

= $max^i\{(\bigcup_{i\epsilon I}\widetilde{\mu_i})$(x),$(\bigcup_{i\epsilon I}\widetilde{\mu_i})$(y)}

$(\bigcup_{i\epsilon I}\widetilde{\mu_i})(ra)$

= $sup^i\{\widetilde{\mu_i}(ra)$/ i$\epsilon$I}

$\leq sup^i\{\widetilde{\mu_i}(a)$/ i$\epsilon$I}

= $(\bigcup_{i\epsilon I}\widetilde{\mu_i})$(a)

$(\bigcup_{i\epsilon I}\widetilde{\mu_i})$((r+a)s+rs)

= sup^i $\{\widetilde{\mu_i}((r+a)s+rs)$/ i$\epsilon$I}

$\leq sup^i\{\widetilde{\mu_i}(a)$/ i$\epsilon$I}

= $(\bigcup_{i\epsilon I}\widetilde{\mu_i})$(a)

Therefore $\bigcup_{i\epsilon I}\widetilde{\mu_i}$ is an interval valued anti fuzzy ideal of R.

Theorem:3.4

Intersection of a non-empty collection of interval valued anti fuzzy ideal of a Boolean like semi-ring R is an interval valued anti fuzzy ideal of R.

Proof:

Let R be a Boolean like semi ring.

Let $\{\widetilde{\mu_i}$/ iϵI} be family of interval valued anti fuzzy ideals of R.

Let x, y, z ϵ R.

Then we have,

$(\bigcap_{i\epsilon I}\widetilde{\mu_i})$(x-y)

= inf^i $\{\{\widetilde{\mu_i}(x-y)$ / iϵI}

$\leq inf^i$ $\{max^i\{\widetilde{\mu_i}(x),\widetilde{\mu_i}(y)\}$/ i$\epsilon$I}

= $max^i\{inf^i\{\widetilde{\mu_i}(x)$/ i$\epsilon$I},$inf^i\{\widetilde{\mu_i}(y)$/ i$\epsilon$I}

= $max^i\{(\bigcap_{i\epsilon I}\widetilde{\mu_i})$(x),$(\bigcap_{i\epsilon I}\widetilde{\mu_i})$(y)}

$(\bigcap_{i\epsilon I}\widetilde{\mu_i})(ra)$

= $inf^i\{\widetilde{\mu_i}(ra)$/ i$\epsilon$I}

$\leq inf^i\{\widetilde{\mu_i}(a)$/ i$\epsilon$I}

= $(\bigcap_{i\epsilon I}\widetilde{\mu_i})$(a)

$(\bigcap_{i\epsilon I}\widetilde{\mu_i})$((r+a)s+rs)

= inf^i $\{\widetilde{\mu_i}((r+a)s+rs)$/ i$\epsilon$I}

$\leq inf^i\{\widetilde{\mu_i}(a)$/ i$\epsilon$I}

= $(\bigcap_{i\epsilon I}\widetilde{\mu_i})$(a)

Therefore $\bigcap_{i\epsilon I}\widetilde{\mu_i}$ is an interval valued anti fuzzy ideal of R.

Theorem:3.5

Let R and S be two Boolean like semi-rings and f: R→S be a homomorphism. If $\tilde{v}$ is an interval valued anti fuzzy ideal of a Boolean like semi-ring S then $f^{-1}(\tilde{v})$ is an interval valued anti fuzzy ideal of R.

Proof:

Let $\tilde{v}$ be an interval valued anti fuzzy ideal of S.

Let x, y, z ϵ R.Then,

$f^{-1}(\tilde{v})$ (x-y)

= $\tilde{v}\left(f(x-y)\right)$

=$\tilde{v}$(f(x)-f(y))

$\leq max^i$ {$\tilde{v}$(f(x)), $\tilde{v}$(f(y))}

=max^i\{ $f^{-1}(\tilde{v}(x))$, $f^{-1}(\tilde{v}(y))$}

$f^{-1}(\tilde{v}(ra))$

= $\tilde{v}$ (f(ra))

$\leq \tilde{v}$(f(a))

= $f^{-1}(\tilde{v}(a))$

$f^{-1}(\tilde{v})$ ((r+a)s+rs)

= $\tilde{v}$ (f(r) f(a))

$\leq \tilde{v}$ (f(a))

= $f^{-1}(\tilde{v}(a))$

$f^{-1}(\tilde{v})$ ((r+a)s+rs)

= $\tilde{v}$(f((r+a)s+rs))

$= \tilde{\nu}(f((r+a)s)+f(rs))$

$= \tilde{\nu}(((f(r)+f(a))f(s)) + f(r)f(a))$

$= \tilde{\nu}(f(r)f(s)+f(a)f(s)+f(r)f(s))$

$\leq \tilde{\nu}(f(a))$

$= f^{-1}(\tilde{\nu}(a))$

Hence $f^{-1}(\tilde{\nu})$ is an interval valued anti fuzzy ideal of R.

Theorem:3.6

Let $\tilde{\mu}$ be an interval valued fuzzy subset of a Boolean like semi-ring R. Then $\tilde{\mu} = [\mu^-, \mu^+]$ is an interval valued anti fuzzy ideal of a Boolean like semi-ring R if and only if μ^-, μ^+ are anti fuzzy ideals of R.

Proof:

Let $\tilde{\mu}$ be an interval valued anti fuzzy ideal of a Boolean like semi-ring R for x, y, z ϵ R.

Then, we have

$[\mu^-(x-y), \mu^+(x-y)]$

$= \tilde{\mu}(x-y)$

$\leq max^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$

$= [\max\{\mu^-(x), \mu^-(y)\}, \max\{\mu^+(x), \mu^+(y)\}]$

$\Rightarrow \mu^-(x-y) \leq \max\{\mu^-(x), \mu^-(y)\}$,

and $\mu^+(x-y) \leq \max\{\mu^+(x), \mu^+(y)\}$

$[\mu^-(ra), \mu^+(ra)]$

$= \tilde{\mu}(ra)$

$\leq \tilde{\mu}(a)$

$= [\mu^-(a), \mu^+(a)]$

$\Rightarrow \mu^-(ra) \leq \mu^-(a)$

and $\mu^+(ra) \leq \mu^+(a)$

$[\mu^-((r+a)s+rs), \mu^+((r+a)s+rs)]$

$= \tilde{\mu}((r+a)s+rs)$

$\leq \tilde{\mu}(a)$

$= [\mu^-(a), \mu^+(a)]$

$\Rightarrow \mu^-((r+a)s+rs) \leq \mu^-(a)$

and $\mu^+((r+a)s+rs) \leq \mu^+(a)$

Hence μ^- and μ^+ are anti fuzzy ideals of R.

Conversely,

Suppose that μ^- and μ^+ are anti fuzzy ideals of R

Let x, y, zϵ R.Then,

$\tilde{\mu}(x-y)$

$= [\mu^-(x-y), \mu^+(x-y)]$

$\leq [\max\{\mu^-(x), \mu^-(y)\}, \max\{\mu^+(x), \mu^+(y)\}]$

$= max^i\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)]\}$

$= max^i\{\tilde{\mu}(x), \tilde{\mu}(y)\}$

$\tilde{\mu}(ra)$

$= [\mu^-(ra), \mu^+(ra)]$

$\leq [\{\mu^-(a)\}, \{\mu^+(a)\}]$

$= \tilde{\mu}(a)$

$\tilde{\mu}((r+a)s+rs)$

$= [\mu^-((r+a)s+rs), \mu^+((r+a)s+rs)]$

$\leq [\mu^-((r+a)s+ra), \{\mu^+((r+a)s+rs)]$

$\leq [\{\mu^-(a)\}, \{\mu^+(a)\}]$

$= \tilde{\mu}(a)$

Hence $\tilde{\mu}$ is an interval valued anti fuzzy ideal of a Boolean like semi-ring R.

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