APPLICATION ON sb$\hat{g}$-CLOSEDSETS IN TOPOLOGICAL SPACES

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Abstract:
In 1963, Levine Introduced the concept of semi-open sets in topological spaces. Biwas defined semi-closed sets in 1970. Crossley and Hildebrand defined semi-closure of sets and irresolute functions in 1971. In 1970, Levine defined generalized closed sets. Das defined semi-interior point and semi-limit point of a subset. The semi-derived set of a subset of a topological space was also defined and studied by him in 1973. Following him, now we define sb$\hat{g}$-limit point, sb$\hat{g}$-derived set, sb$\hat{g}$-border, sb$\hat{g}$-Fronterior and sb$\hat{g}$-Exterior of a subset of a topological spaces using the concept of sb$\hat{g}$-Closed sets. A subset A of a topological space (X,τ) is called sb$\hat{g}$-closedset if scl(A)⊆U whenever A⊆U and U is b$\hat{g}$-open in X. Also we defined some of its properties.

Keywords: sb$\hat{g}$-limit point, sb$\hat{g}$-derived set, sb$\hat{g}$-border, sb$\hat{g}$-Fronterior, sb$\hat{g}$-Exterior and sb$\hat{g}$-Closed sets.

1.INTRODUCTION:

In 1973, Das[8] defined semi-interior point and semi-limit point of a subset. The semi-derived set of a subset of a topological space was also defined and studied by him. In 2015, K.Bala Deepa Arasi and S.Navaneetha Krishnan[1] introduced sb$\hat{g}$-closed sets and studied some of its properties. Afterwards, they were introduced sb$\hat{g}$-continuous functions and sb$\hat{g}$-Homeomorphisms[4], contra sb$\hat{g}$-continuous function[5], topological sb$\hat{g}$ quotient mappings[6] and sb$\hat{g}$-connected and sb$\hat{g}$-compact spaces [7].

Now, we define new class of sets namely sb$\hat{g}$-limit points, sb$\hat{g}$-Derived sets, sb$\hat{g}$-border, sb$\hat{g}$-Fronterior, sb$\hat{g}$-Exterior of a subset of a topological space and studied some of their properties. Also, we prove some of the properties of sb$\hat{g}$-closure and sb$\hat{g}$-interior of a subset of a topological space.

2.PREMILINARIES:

Throughout this paper (X,τ) represents topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), Cl(A), Int(A), D(A), b(A) and Ext(A) denote the closure, interior, derived, border and exterior of A respectively.

Definition 2.1: [1] A subset A of a topological space (X,τ) is called **sb$\hat{g}$-closed** set if scl(A)⊆U whenever A⊆U and U is b$\hat{g}$-open in X. The family of all sb$\hat{g}$-closed sets of X is denoted by sb$\hat{g}$-C(X,τ). The complement of sb$\hat{g}$-closed set is called sb$\hat{g}$-open set. The family of all sb$\hat{g}$-open sets of X is denoted by sb$\hat{g}$-O(X,τ).

Definition 2.2: Let A be the subset of a space(X,τ). Then

1) The **border** of A is defined asb(A)=A\ Int(A).

2) The **frontier** of A is defined as Fr (A) = Cl(A) \ Int (A).
3) The **Exterior** of A is defined as Ext (A) = Int (X\A).

Theorem 2.3:[1] Every closed set is sb$\hat{g}$-closed.

3. Properties of sb$\hat{g}$- interior and sb$\hat{g}$-closure

Definition 3.1: The **sb$\hat{g}$-interior** of A is defined as the union of all sb$\hat{g}$-open sets of X contained in A. It is denoted by sb$\hat{g}$ Int(A).

Definition 3.2: A point $x \in X$ is called **sb$\hat{g}$-interior point** of A if A contains a sb$\hat{g}$-open set containing x.

Definition 3.3: The **sb$\hat{g}$-closure** of A is defined as the intersection of all sb$\hat{g}$-closed sets of X containing A. It is denoted by sb$\hat{g}$ Cl(A).

Theorem 3.4: If A is a subset of X, then sb$\hat{g}$ Int(A) is the set of all sb$\hat{g}$-interior points of A.

Proof: If $x \in$ sb$\hat{g}$ Int (A), then x belongs to some sb$\hat{g}$-open subset U of A. That is, x is a sb$\hat{g}$-interior point of A.

Remark 3.5: If A is any subset of X, sb$\hat{g}$ Int (A) is sb$\hat{g}$-open. In fact, sb$\hat{g}$ Int(A) is the largest sb$\hat{g}$-open set contained in A.

Remark 3.6: A subset A of X is sb$\hat{g}$-open$\Leftrightarrow$sb$\hat{g}$ Int (A)=A.

Result 3.7: For the subset A of a topological space (X,τ),Int(A)$\subseteq$sb$\hat{g}$ Int(A).

Proof: We know that, Int(A) is the union of open sets. From theorem 2.3(2), Int (A) is sb$\hat{g}$-open. Hence from the definition 3.1, Int(A) $\subseteq$sb$\hat{g}$ Int (A).

Theorem 3.8: Let A and B be the subsets of a topological space (X,τ), then the following result holds:

1) sb$\hat{g}$ Int (ϕ) = ϕ;
2) sb$\hat{g}$Int (X) = X;
3) sb$\hat{g}$ Int(A) $\subseteq$ A;
4) A $\subseteq$ B $\Rightarrow$sb$\hat{g}$ Int (A) $\subseteq$sb$\hat{g}$ Int(B)
5) sb$\hat{g}$ Int(A$\cup$B) $\supseteq$sb$\hat{g}$ Int(A) $\cup$sb$\hat{g}$ Int(B);
6) sb$\hat{g}$ Int(A$\cap$B)$\subseteq$sb$\hat{g}$ Int (A)$\cap$sb$\hat{g}$ Int(B);
7) sb$\hat{g}$ Int(Int(A)) = Int(A);
8) Int(sb$\hat{g}$ Int(A))$\subseteq$ Int(A);
9) sb$\hat{g}$ Int(sb$\hat{g}$ Int(A))=sb$\hat{g}$ Int(A);

Proof:(1),(2),(3) holds from definition 3.1.

(4) By definition 3.1 we have, sb$\hat{g}$Int(A)$\subseteq$A. Since A$\subseteq$B, sb$\hat{g}$ Int(A)$\subseteq$B. Using remark 3.5, sb$\hat{g}$ Int(A)$\subseteq$sb$\hat{g}$ Int(B).

(5) Since have A$\subseteq$A$\cup$B; B$\subseteq$A$\cup$B and using(4),sb$\hat{g}$ Int(A)$\subseteq$sb$\hat{g}$ Int(A$\cup$B) and sb$\hat{g}$ Int(B)$\subseteq$sb$\hat{g}$ Int(A$\cup$B). Hence, sb$\hat{g}$ Int(A) $\cup$sb$\hat{g}$ Int(B)$\subseteq$sb$\hat{g}$ Int(A$\cup$B).

(6) Since A$\cap$B$\subseteq$A; A$\cap$B$\subseteq$B and by (4) we have, sb$\hat{g}$ Int(A$\cap$B)$\subseteq$sb$\hat{g}$ Int(A) and sb$\hat{g}$ Int(A$\cap$B)$\subseteq$sb$\hat{g}$ Int(B). Hence, sb$\hat{g}$ Int(A$\cap$B)$\subseteq$sb$\hat{g}$Int(A) $\cap$sb$\hat{g}$ Int(B).

(7) Since Int(A) is an open set and by theorem 2.3(2), Int(A) is sb$\hat{g}$-open. By remark 3.6, sb$\hat{g}$ Int(Int(A))=Int(A).

(8) Using definition 3.1 we have, sb$\hat{g}$ Int(A)$\subseteq$A. Clearly, Int(sb$\hat{g}$Int(A))$\subseteq$Int(A).

(9) Follows from remark 3.6 and 3.5.

Remark 3.9: If A is any subset of X, sb$\hat{g}$Cl(A) is sb$\hat{g}$-closed. Infact sb$\hat{g}$ Cl(A) is the smallest sb$\hat{g}$-closed set containing A.

Remark 3.10: A subset of A of X sb$\hat{g}$-closed $\Leftrightarrow$sb$\hat{g}$ Cl(A) = A.

Theorem 3.11: Let A and B be the subsets of a topological space (X,τ), then the following result holds:

1. sb$\hat{g}$ Cl(ϕ) = ϕ;
2. sb$\hat{g}$ Cl(X) = X;
3. A $\subseteq$sb$\hat{g}$ Cl(A);
4. A $\subseteq$ B $\Rightarrow$sb$\hat{g}$ Cl(A) $\subseteq$sb$\hat{g}$ cl(B);
5. sb$\hat{g}$ Cl(sb$\hat{g}$ Cl(A)) = sb$\hat{g}$ Cl(A);
6. sb$\hat{g}$ Cl(A$\cup$B)$\supseteq$sb$\hat{g}$ Cl(A)$\cup$ sb$\hat{g}$ Cl(B);
7. sb$\hat{g}$ Cl(A$\cap$B) $\subseteq$sb$\hat{g}$ Cl(A) $\cap$ sb$\hat{g}$ Cl(B);
8. sb$\hat{g}$ Cl(Cl(A)) = Cl (A);
9. Cl(sb$\hat{g}$ Cl(A)) = Cl (A);

Proof:

(8) We know that Cl(A) is a closed set. From theorem 2.3 (1), Cl(A) is sb$\hat{g}$-

closed set. Hence, by remark 3.10, sb$\hat{g}$ Cl(Cl(A)) = Cl(A).

(9) Follows from remark 3.9 and 3.10.

Result 3.12: Let A be a subset of a topological space X. Then,

a) sb$\hat{g}$ Cl(X\A) = X \ sb$\hat{g}$ Int(A)
b) sb$\hat{g}$ Int(X\A) = X \ sb$\hat{g}$ Cl(A)

4. Applications of sb$\hat{g}$-Open sets

Definition 4.1: Let A be a subset of a topological space X. A point x∈X is said to be **sb$\hat{g}$-limit point** of A is for every sb$\hat{g}$-open set U containing x, U ∩ (A\ {x}) ≠ϕ. The set of all sb$\hat{g}$-limit points of A is called an **sb$\hat{g}$-derived set** of A and is denoted by sb$\hat{g}$ D(A).

Theorem 4.2: For subsets A,B of a space X, the following statements holds:

1) D(A)⊆ sb$\hat{g}$ D(A), where D(A) is the derived set of A;
2) sb$\hat{g}$ D(Φ)=Φ;
3) If A⊂B, then sb$\hat{g}$ D(A)⊆sb$\hat{g}$ D(B);
4) sb$\hat{g}$D(A∪B)⊇sb$\hat{g}$D(A)∪sb$\hat{g}$ D(B);
5) sb$\hat{g}$ D(A∩B)⊆sb$\hat{g}$D(A)∩sb$\hat{g}$D(B);
6) sb$\hat{g}$ D(A)⊆sb$\hat{g}$ D(A\{x});

Proof: (1) Let x ∈ (A). By the definition of D(A), there exist an open set U containing x such that U∩(A\{x})≠Φ. By theorem 2.3(2), U is an sb$\hat{g}$-open set containing x such that U ∩ (A\{x}≠Φ. Therefore, x ∈ sb$\hat{g}$ D(A). Therefore, D(A)⊆sb$\hat{g}$(A).

(2) For all sb$\hat{g}$-open set U and for all x ∈ X, U ∩ (Φ\{x})=Φ. Therefore, sb$\hat{g}$ D(Φ)=Φ.

(3) Let x ∈sb$\hat{g}(A)$. Then for each sb$\hat{g}$-open set U containing x, U∩(A\{x})≠Φ. Since A⊆B, U∩(B\{x})≠Φ. This implies that x ∈ sb$\hat{g}$ D(B). Therefore, sb$\hat{g}$ D(A)⊆sb$\hat{g}$ D(B).

(4) Let x∈ sb$\hat{g}$D(A) ∪sb$\hat{g}$ D(B). Then x∈sb$\hat{g}$ D(A) or x∈sb$\hat{g}$ D(B). If x∈sb$\hat{g}$ D(A), then for each sb$\hat{g}$-open set U containing x, U∩(A\{x}) ≠Φ. Since A ⊆ A∪B, U∩(A∪B\{x}) ≠Φ. This implies that x∈sb$\hat{g}$D(A∪B).Therefore sb$\hat{g}$ $D(A)$ ⊆sb$\hat{g}$ D(A ∪ B)…………..(1). Otherwise, if x∈sb$\hat{g}$ D(B), then for each sb$\hat{g}$-open set U containing x, U∩(B\{x}) ≠Φ. Since B ⊆ A∪B, U∩(A∪B)\{x}) ≠Φ. This implies that x∈sb$\hat{g}$ D(A∪B). Hence, sb$\hat{g}$ D(B) ⊆sb$\hat{g}$ D(A∪B)……..…..(2). From (1) and (2),we get sb$\hat{g}$ D(A)∪sb$\hat{g}$ D(B) ⊆sb$\hat{g}$ D(A∪B).

(5) Let x∈sb$\hat{g}$(A∩B). Then for each sb$\hat{g}$-open set U containing x, U∩(A∩B\{x}) ≠Φ. Since A∩B ⊆ A, U∩(A\{x}) ≠Φ. Hence x ∈sb$\hat{g}$ D(B). Therefore, x ∈sb$\hat{g}$ D(A)∩sb$\hat{g}$ D(B). Thus, sb$\hat{g}$D(A∩B) ⊆sb$\hat{g}$ D(A)∩sb$\hat{g}$ D(B).

(6) Let x ∈sb$\hat{g}$ D(A). Then for each sb$\hat{g}$-open set U containing x. U ∩(A\{x}) ≠Φ. This implies that U ∩ ((A\{x})\{x})≠Φ. Hence, x ∈ sb$\hat{g}$ D(A\{x}. Therefore, sb$\hat{g}$ D(A) ⊆sb$\hat{g}$ D(A\{x}).

Definition 4.3: If A is a subset of X, then the **sb$\hat{g}$-border** of A is defined by sb$\hat{g}$ b(A) = A\sb$\hat{g}$ Int(A).

Theorem 4.4: For a subset A of a space X, the following statements holds:

1) sb$\hat{g}$ b(Φ) = Φ;
2) sb$\hat{g}$ b(X) = Φ;
3) sb$\hat{g}$ b(A) ⊆ A;
4) sb$\hat{g}$ b(A) ⊆ b(A), where b(A) denotes the border of A;
5) sb$\hat{g}$ Int(A) ∪sb$\hat{g}$ b(A) = A;
6) sb$\hat{g}$ Int(A) ∩sb$\hat{g}$ b(A) = Φ;
7) sb$\hat{g}$ b(sb$\hat{g}$ Int(A)) = Φ;
8) sb$\hat{g}$ Int(sb$\hat{g}$ b(A)) = Φ;
9) sb$\hat{g}$ b(sb$\hat{g}$ b(A)) = sb$\hat{g}$ b(A);
10) sb$\hat{g}$ b(sb$\hat{g}$ Cl(A)) = Φ;

Proof: (1), (2) and (3) holds from definition 4.3.

(4) Let x ∈sb$\hat{g}$ b(A). Then by definition 4.3, x ∈ A\sb$\hat{g}$ Int(A). This implies that x ∈ A and x ∉sb$\hat{g}$ Int(A). By result 3.7, x ∈ A and x∉Int(A). Hence, x ∈ A\ Int(A). This implies that x ∈ b(A). Hence, sb$\hat{g}$ b(A) ⊆ b(A).

(5) and (6) holds from definition 4.3.

(7) sb$\hat{g}$ b(sb$\hat{g}$ Int(A)) = sb$\hat{g}$ Int(A)\sb$\hat{g}$ Int(sb$\hat{g}$ Int(A)) = sb$\hat{g}$

Int(A)\s $b\hat{g}$ Int(A) (by theorem 3.8(9)) which is Φ. Hence, s$b\hat{g}$(s$b\hat{g}$ Int(A)) = Φ.

(8) Let x ∈s$b\hat{g}$ Int(s$b\hat{g}$ b(A)). By theorem 3.8(3), x ∈s$b\hat{g}$ b(A). On the other hand, since s$b\hat{g}$ b(A) ⊆ A, we have x ∈s$b\hat{g}$ Int (A). Hence, x ∈ s$b\hat{g}$ b(A) ∩s$b\hat{g}$ Int(A), which is a contradiction to (6). Hence, s$b\hat{g}$ Int(s$b\hat{g}$ b(A)) = Φ.

(9) s$b\hat{g}$ b(s$b\hat{g}$ b(A)) = s$b\hat{g}$ b(A)\ s$b\hat{g}$ Int(s$b\hat{g}$ b(A)) = s$b\hat{g}$ b(A)\Φ = s$b\hat{g}$ b(A) (using (8)). Hence, s$b\hat{g}$ (s$b\hat{g}$ b(A)) = s$b\hat{g}$ b(A).

(10)s$b\hat{g}$ b(s$b\hat{g}$ Cl(A)) = s$b\hat{g}$ Cl(A)\ s$b\hat{g}$Int(s$b\hat{g}$ Cl(A) ⊆s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Cl(A) (using (6)) = Φ.

Definition 4.5: If A is a subset of X, then the **s$b\hat{g}$-frontier** of A is defined by s$b\hat{g}$ Fr(A) = s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Int(A).

Theorem 4.6: Let A be a subset of a space X. Then the following statement holds:

1) s$b\hat{g}$ Fr(Φ) = Φ;
2) s$b\hat{g}$ Fr(X) = Φ;
3) s$b\hat{g}$ Fr(A) ⊆s$b\hat{g}$ Cl(A);
4) s$b\hat{g}$ Cl(A) = s$b\hat{g}$ Int(A) ∪s$b\hat{g}$ Fr(A);
5) s$b\hat{g}$ Int(A) ∩s$b\hat{g}$ Fr(A) = Φ;
6) s$b\hat{g}$ b(A) ⊆s$b\hat{g}$ Fr(A);
7) s$b\hat{g}$ Fr(s$b\hat{g}$ Int(A)) ⊆s$b\hat{g}$ Fr(A)
8) s$b\hat{g}$ Cl(s$b\hat{g}$ Fr(A)) ⊆s$b\hat{g}$ Cl(A);
9) s$b\hat{g}$ Int(A) ⊆s$b\hat{g}$ Cl(A);
10) s$b\hat{g}$ Int(s$b\hat{g}$ Fr(A)) ⊆s$b\hat{g}$ Cl(A);
11) X = s$b\hat{g}$ Int(A) ∪s$b\hat{g}$ Int(X\A) ∪ s$b\hat{g}$ Fr(A);
12) s$b\hat{g}$ Fr(A) = s$b\hat{g}$ Cl(A) ∩s$b\hat{g}$ Cl(X\A);
13) s$b\hat{g}$ Fr(A) = s$b\hat{g}$ Fr(X\ A).

Proof: (1), (2), (3) and (4) holds from definition 4.5.

(5) s$b\hat{g}$ Int(A) ∩s$b\hat{g}$ Fr(A) = s$b\hat{g}$ Int(A) ∩ (s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Int(A)) ⊆ A ∩(s$b\hat{g}$ Cl(A)\A) (by theorem 3.8(3)). s$b\hat{g}$ Int(A) ∩s$b\hat{g}$ Fr(A) ⊆s$b\hat{g}$ Cl(A) ∩ (s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Cl(A)) (by theorem 3.11(3). s$b\hat{g}$ Int(A) ∩s$b\hat{g}$ Fr(A) = s$b\hat{g}$ Cl(A) ∩Φ = Φ. Hence, s$b\hat{g}$ Int(A) ∩ s$b\hat{g}$ Fr(A) = Φ.

(6) Let x ∈s$b\hat{g}$ Int(A). Then x ∈ A\ s$b\hat{g}$ Int(A). By theorem 3.11(3), x ∈s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Int(A) = s$b\hat{g}$ Fr(A). Hence, s$b\hat{g}$ b(A) ⊆s$b\hat{g}$ Fr(A).

(7)s$b\hat{g}$ Fr(s$b\hat{g}$ Int(A)) =s$b\hat{g}$ Cl(s$b\hat{g}$ Int(A)\ s$b\hat{g}$Int(s$b\hat{g}$ Int(A)) ⊆s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Int(A) (By theorem 3.8(3),(9)) which is s$b\hat{g}$ Fr(A). Hence, s$b\hat{g}$ (s$b\hat{g}$ Int(A)) ⊆s$b\hat{g}$ Fr(A).

(8) From (3) we have, s$b\hat{g}$ (s$b\hat{g}$ Fr(A)) ⊆s$b\hat{g}$ Cl(s$b\hat{g}$ Cl(A)) = s$b\hat{g}$ Cl(A) (by theorem 3.11(5)). Hence, s $b\hat{g}$ (s$b\hat{g}$Fr(A)) ⊆s$b\hat{g}$ Cl(A).

(9) Holds from(4).

(10)From (9), s$b\hat{g}$ Int(s$b\hat{g}$ Fr(A)) ⊆s$b\hat{g}$ Cl(s$b\hat{g}$ Fr(A)) ⊆s$b\hat{g}$ Cl(A) (from (8)). Hence, s$b\hat{g}$ Int(s$b\hat{g}$ Frs(A)) ⊆s$b\hat{g}$ Cl(A).

(11)s$b\hat{g}$ Fr(s$b\hat{g}$ Fr(A)) = s$b\hat{g}$ Cl(s$b\hat{g}$ Fr(A)\ s$b\hat{g}$ Int(s$b\hat{g}$ Fr(A)) ⊆s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Cl(A) = ϕ (from (8), (10)). Hence, s$b\hat{g}$ (s$b\hat{g}$ Fr(A)) = ϕ.

(12) s$b\hat{g}$ Int(A) ∪s$b\hat{g}$ Int(X\A) ∪s$b\hat{g}$ Fr(A) = s$b\hat{g}$ Cl(A) ∪s$b\hat{g}$ Int(X\ A) (from (4)) = s$b\hat{g}$ Cl(A) ∪{X\s$b\hat{g}$ Cl(A)} (by result 3.12(ii)) which is X. Hence, x = s$b\hat{g}$ Int(A) ∪ Int(X\ A) ∪s$b\hat{g}$ Fr(A).

(13) s$b\hat{g}$ Cl(A) ∩s$b\hat{g}$ Cl(X\ A) = s$b\hat{g}$ Cl(A) ∩ (X\ s$b\hat{g}$ Int(A)) (by result 3.12(i)) = s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Int(A) (from(9))= s$b\hat{g}$ Fr(A).

(14) s$b\hat{g}$ Fr(X\A) = s$b\hat{g}$ Cl(X\ A) s$b\hat{g}$ Int(X\ A)= (X\ s$b\hat{g}$ Int(A))\ (X\ s$b\hat{g}$ Cl(A)) (by result 3.12). s$b\hat{g}$ Fr(X\A)= s$b\hat{g}$ Cl(A)\ s$b\hat{g}$ Int(A) = s$b\hat{g}$ Fr(A).

Definition 4.7: Let A be a subset of X, then the **s$b\hat{g}$-exterior** of A is defined by s$b\hat{g}$ Ext(A) = s$b\hat{g}$Int(X\A).

Theorem 4.8: Let A be a subset of a space X. Then the following statement holds:

1) s$b\hat{g}$ Ext(ϕ)=X;
2) s$b\hat{g}$ Ext(X) = ϕ;
3) Ext(A) ⊆ s$b\hat{g}$ Ext(A);
4) s$b\hat{g}$ Ext(A) = X\ s$b\hat{g}$ Cl(A);
5) A is s$b\hat{g}$-closed iff s$b\hat{g}$ Ext(A) = X\A;
6) If A⊆B, then s$b\hat{g}$ Ext(A) ⊇s$b\hat{g}$ Ext(B);
7) s$b\hat{g}$ Ext(A∪B) ⊆s$b\hat{g}$ Ext(A) ∩ s$b\hat{g}$Ext(B);

8) $sb\hat{g}$ Ext(A ∩ B) ⊇ $sb\hat{g}$ Ext(A) ∪ $sb\hat{g}$Ext(B);
9) $sb\hat{g}$ Ext(A) is $sb\hat{g}$-open;
10) $sb\hat{g}$ Ext(X\ $sb\hat{g}$ Ext(A)) = $sb\hat{g}$ Ext(A);
11) $sb\hat{g}$ Ext($sb\hat{g}$ Ext(A)) = $sb\hat{g}$ Int($sb\hat{g}$ Cl(A));
12) $sb\hat{g}$ Int(A) ⊆ $sb\hat{g}$ Ext($sb\hat{g}$ Ext(A));
13) X= $sb\hat{g}$Int(A) ∪ $sb\hat{g}$ Int(X\A) ∪ $sb\hat{g}$ Fr(A).

Proof: (1) $sb\hat{g}$ (ϕ) = $sb\hat{g}$ Int(X\ϕ) = $sb\hat{g}$ Int(X) = X (by theorem 3.8 (2)).

(2) $sb\hat{g}$ Ext(X) = $sb\hat{g}$ Int(X\X) = $sb\hat{g}$ Int(ϕ) = ϕ (by theorem 3.8 (1)).

(3) Let x ∈ Ext(A). Then by definition 2.2 (3), x ∈ Int(X\A). By theorem 3.7, x ∈ $sb\hat{g}$ Int(X\A) = $sb\hat{g}$ Ext(A). Hence, Ext(A) ⊆ $sb\hat{g}$ Ext(A).

(4) Let x ∈ $sb\hat{g}$ Ext(A) ⇔ x ∈ $sb\hat{g}$ (X\A) ⇔ x ∈ X\ $sb\hat{g}$ Cl(A) (by result 3.12 (ii)). Hence, $sb\hat{g}$ Ext(A) = X\ $sb\hat{g}$ Cl(A).

(5) Let A be $sb\hat{g}$-closed. Then X\A is $sb\hat{g}$- open. Using remark 3.6, $sb\hat{g}$ Int(X\A) = X\A. This implies that $sb\hat{g}$ Ext(A) = X\A. On the other hand, let $sb\hat{g}$Ext(A) = X\A. Then $sb\hat{g}$ Int(X\A) = X\A. Again by remark 3.6, X\A is $sb\hat{g}$-open. Hence, A is $sb\hat{g}$ closed.

(6) $sb\hat{g}$ Ext(A) = $sb\hat{g}$ Int(X\A) = X\ $sb\hat{g}$ Cl(A) (using result 3.12) ⊇ X\ $sb\hat{g}$ Cl(B) (since A ⊆ B and by theorem 3.11(4)) = $sb\hat{g}$ Int(X\B) = $sb\hat{g}$Fr(B) (by definition 4.7). Hence, $sb\hat{g}$ Ext(A) ⊇ $sb\hat{g}$ Ext(B).

(7) Since A ⊆ A∪B and by (6), $sb\hat{g}$ Ext(A∪B) ⊆ $sb\hat{g}$ Ext(A). Similarly since B ⊆ A∪B and by (6), $sb\hat{g}$ Ext(A∪B) ⊆ $sb\hat{g}$ Ext(B). Hence, $sb\hat{g}$ Ext(A∪B) ⊆ $sb\hat{g}$ Ext (A) ∩ $sb\hat{g}$ Ext (B).

(8) Since A∩B ⊆ A and by(6), $sb\hat{g}$ Ext(A) ⊆ $sb\hat{g}$ Ext(A∩B). Similarly since A ∩ B ⊆ B and by (6), $sb\hat{g}$ Ext(B) ⊆ $sb\hat{g}$ (A ∩ B). Hence, $sb\hat{g}$ Ext(A) ∪ $sb\hat{g}$ Ext(B) ⊆ $sb\hat{g}$ (A∪B).

(9) holds from definition 4.7 and theorem 3.8(2).

(10) $sb\hat{g}$ Ext(X\ $sb\hat{g}$ Ext(A)) = $sb\hat{g}$ Ext(X\ $sb\hat{g}$Int(X\A)) = $sb\hat{g}$ Int(X\ {X\ $sb\hat{g}$ Int(X\A)})= $sb\hat{g}$ Int($sb\hat{g}$ Int(X\ A)) = $sb\hat{g}$ Int(X\A) (by theorem 3.8(9)) which is $sb\hat{g}$Ext(A). Hence, $sb\hat{g}$ (X\ $sb\hat{g}$ Ext(A)) = $sb\hat{g}$ Ext(A).

(11) $sb\hat{g}$ Ext($sb\hat{g}$ Ext(A)) = $sb\hat{g}$ Int(X\ $sb\hat{g}$ Ext(A)) = $sb\hat{g}$ Int(X\ $sb\hat{g}$ Int(X\A)) = $sb\hat{g}$ Int($sb\hat{g}$ Cl(X\(X\A)) = $sb\hat{g}$ Int($sb\hat{g}$ Cl(A)) (by result 3.12(i)). Hence, $sb\hat{g}$ Ext($sb\hat{g}$ Ext(A)) = $sb\hat{g}$ Int($sb\hat{g}$ Cl(A)).

(12) Since A ⊆ $sb\hat{g}$ Cl(A), $sb\hat{g}$ Int(A) ⊆ $sb\hat{g}$ Int($sb\hat{g}$ Cl(A)) = $sb\hat{g}$ Ext($sb\hat{g}$ Ext(A)) (from (11)).

(13) $sb\hat{g}$ Int(A) ∪ $sb\hat{g}$ Ext(A) ∪ $sb\hat{g}$ Fr(A) = $sb\hat{g}$ Int(A) ∪ $sb\hat{g}$ Int(X\ A) ∪ $sb\hat{g}$ Fr(A) = X (from theorem 4.6 (12)).

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