FUZZY PRE CONVERGENCE OF FUZZY SEQUENCE

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Abstract

In a metric space the closure of a set can be characterized using convergent sequences. The continuity of a function from one metric space to another can be studied using convergent sequences. Recently fuzzy sequences are introduced and convergence is studied. In this paper we introduce fuzzy pre convergence of fuzzy sequence in a metric space.

1. Introduction

1. N.Levine introduced semi open sets in a topological space. A set A⊂X is called a semi open set if A⊂cl intA. Then pre open sets are introduced. A set A⊂X is called pre open if A⊂X is called pre open if A⊂int clA.

Zadeh introduced fuzzy sets. In the year 2015, fuzzy convergence of fuzzy sequences are studied in a metric space. Let X be a non empty set. A set A:N×X→[0,1] is called a fuzzy sequence. Let X be topological space. Let A be a fuzzy sequence in X. Let a∈X. Let α∈(0,1). The fuzzy sequence fuzzy converges to a at level α if given any open set U containing a, There exists n_0 ∈N, such that x∈U for all n≥n_0 and A(n,x)≥α.

2. pre convergence of sequences

2.1 Definition

Let X be a topological space and let (x_n) be a sequence in X. let a∈X. Sequence (x_n) is said to pre converges to a if given any pre open set U contains a, There exists n_0 ∈N such that x_n ∈U for all n≥n_0.

2.2 Example of fuzzy convergence sequence

Let X=Q with usual metric topology. Consider $(\frac{1}{n})$ in X. We claim that $(\frac{1}{n})$ pre converge to 0.

Let U be a pre open set contains 0. Let $(-a,a)_Q$= Set of all rationals in (-a,a). Since U is pre open. Clearly there existsn_0 ∈N such that $\frac{1}{n} \in (-a, a)_Q$ for all n≥n_0. Hence $\frac{1}{n}$ ∈U for all n≥n_0. Hence $(\frac{1}{n})$ pre convergence to 0 in Q.

2.3 Result

If a sequence (x_n) pre converge to α then it converges to α.

Proof:-

Let X be a topological space and let (x_n) be a sequence in X. Let a∈X. Let (x_n) pre converges to a. Now we claim that (x_n) converges to a. Let U be an open set contains a. Then U is a pre open set contains a. Since (x_n) pre converges to a, there exists n_0 ∈N such that x_n ∈U ∀ n≥n_0. Hence (x_n) converges to a.

2.4 Result:

Converse is not true.

Consider R with usual metric topology. Consider $(\frac{1}{n})$ in R. Clearly $(\frac{1}{n})$→0. Now we claim that $(\frac{1}{n})$ does not pre converge to 0. Let U= all irrational numbers in (-1,1)∪{0} cl U= [-1,1] int clU= (-1,1). Now U⊂int clU. Hence U is pre open set containing 0. All the elements of U expect 0 are irrational numbers. All the elements of the sequence are rational numbers. Therefore there cannot exists n_0 such that x_n ∈U ∀ n≥n_0. Hence $(\frac{1}{n})$ does not pre converges to 0.

3. Fuzzy Pre Convergence

3.1 Definition

Let X be a topological space. Let A be a fuzzy sequence in X. Let a∈ X. Let ∝∈ (0,1] A is said to fuzzy pre converges to a at level α if given any pre open set U containing a, there exists n_0 ∈N such that x∈U ∀ n≥ n_0 and A(n,x)≥∝. Also A(n,x)≥∝ for atleast one x for each n∈N.

3.2 Example

Consider Q with usual metric topology. Consider A:N×Q→(0,1].

A(n,x)=$\begin{cases} 1 & if\ x=\frac{1}{n} \\ 0 & otherwise \end{cases}$. Take α=$\frac{1}{2}$. Let U be a pre open set contains 0. Then U contains $(-a, a)_Q$. Choose n_0 ∈N such that $\frac{1}{n_0} < a$.

Now n≥n_0 and A(n,x)≥α

⇒n≥n_0 and A(n,x)=1

⇒ n≥n_0 and x=$\frac{1}{n}$.

⇒ x=$\frac{1}{n}$≤$\frac{1}{n_0}$ <a

⇒ x=$\frac{1}{n}$ <a

⇒ x∈ $(-a, a)_Q$.

⇒ x∈U.

Hence there exists n_0 such that x∈U ∀ n≥n_0 and A(n,x)≥$\frac{1}{2}$.

Hence A pre converges to 0 at level α=$\frac{1}{2}$.

3.3 Theorem

Let (x_n) be a crisp sequence in X. Let A be the corresponding fuzzy sequence. If (x_n) pre converges to a in X, then A fuzzy pre converges to a at level α.

Proof:-

X is a topology space. (x_n) is a crisp sequence in X. Define A;N×X→(0,1] as A(n,x)=$\{^{1\ if\ x=x_n}_{0\ otherwise}$.

A is the corresponding fuzzy sequence. Now, (x_n) pre converge to a. We claim that A fuzzy converges to a. Let U be a pre open set contain a. Since (x_n) pre converges to a, there exists n_0 ∈N such that x_n ∈U ∀ n≥n_0. Now Take α∈(0,1]. n≥n_0 and A(n,x)≥α ⇒n≥n_0 and A(n,x)=1. ⇒$n \geq n_0$ and x=x_n⇒x∈U. Hence A fuzzy pre converges to a at level α.

3.4 Theorem

Let X be a topological space. Let (x_n) be a sequence in X. Let A be the corresponding fuzzy sequence. If A fuzzy pre converge to a at level α then (x_n) pre converge to a.

Proof:-

X is a topological space. (x_n) is a sequence in X. We define A:N×X→[0,1] as A(n,x)=$\{^{1\ if\ x=x_n}_{0\ otherwise}$.Take α∈[0,1]. Let U be a pre open set contains a. Since A pre converge to a, there existsn_0 ∈N such that n≥n_0 and A(n,x)≥α implies that x∈ U.

Hence n≥ n_0 and x= x_n implies that x ∈ U. Hence x_n ∈ U ∀ n≥ n_0. Hence (x_n)→a.

3.5 Theorem

Let X be a topological space. Let (x_n) be a sequence in X. Let A be the corresponding fuzzy sequence. Then A fuzzy pre converge to a at level α∈(0,1] iff (x_n) pre converges to a.

Proof:-

Follow from 3.3 and 3.4.

Note:

Let A be a fuzzy sequence in X. which fuzzy pre converges to a at level α. Then for each n∈N, there exists atleast one x with A(n,x)≥α.

Let x_n = one values of x such that A(n,x)≥α. We get a crisp sequence (x_n) called α induced crisp sequence.

3.6 Theorem

If A fuzzy pre converges to a at level α then α induced crisp sequence pre converges to α.

Proof:-

A fuzzy pre converge to a at level α. Consider (x_n) induced crisp sequence. Let U be a pre open set contains a. A fuzzy pre converges to a at level α. Hence there exists n_0 ∈N such that n≥n_0 and A(n,x)≥α ⇒x∈ U.

Now A(n,x)≥α. Hence x_n ∈U ∀ n≥ n_0.

Therefore (x_n)→a.

References:

1. C.L.Chang, "Fuzzy topological spaces", J.Math.Anal.Appl.24(1968),182-190.
2. L.A.Zadeh, "Fuzzy sets", Information and control, 8(1965)338-353.
3. M.Muthukumari, A.Nagarajan, M. Murugalingam, "Fuzzy Sequences in Metric Spaces" Int. Journal of Math. Analysis, Vol.8, 2014, no 15, 699-706.