ON QUASI-IDEALS AND K-REGULAR IN NEAR SUBTRACTION SEMIGROUPS

[1]K.MUMTHA, [2]V.MAHALAKSHMI and [3]S.USHA DEVI

[1]M.Phil scholar,
Department of Mathematics,
A.P.C.Mahalaxmi college for women, Thoothukudi.
mamthakasi8696@gmail.com

[2]Assintant Professor of Mathematics,
A.P.C.Mahalaxmi college for women, Thoothukudi.
maha.krishna86@gmail.com

[3]Assistant Professor of Mathematics,
Sri Parasakthi College for Women, Coutrallam.
ushadevinathan@gmail.com

Abstract:

In this paper, with a new idea, we define quasi-ideal, K-regular and investigate some of its properties. We characterize quasi-ideal by subalgebra of near subtraction semigroup and K-regular by quasi-ideal. A characterization of quasi-ideals of near subtraction semigroup is given. In this paper we shall introduce K-regular near subtraction semigroups and obtain equivalent conditions for K-regular near subtraction semigroups using quasi-ideals. This concept motivates the study of different kinds of new ideals in algebraic graph theory especially ideals in subtraction bialgebra and fuzzy algebra.

Key words: Quasi-ideal, s-near subtraction semigroup, property (α), K- regular.

1. Introduction

B.M. Schein [10] considered systems of the form (X; o; -), where X is a set of functions closed under the composition "o" of function (and hence (X; o) is a function semigroup) and the set theoretic subtraction "-" (and hence (X; -) is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible function. B.Zelinka [11] discussed a problem proposed by B.M.Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Y.B.Jun et al. [4] introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. For basic definition one may refer to Pilz[8]. In ring theory the notation of quasi-ideal introduced by O. Steinfiled in [9].

2. Preliminaries on near Subtraction Algebra

Definition:2.1

A non empty set X together with binary operation "$-$" is said to be a **subtraction algebra** if it satisfies the following:

i. $x-(y-x)=x$.

ii. $x-(x-y)=y-(y-x)$.

iii. $(x-y)-z=(x-z)-y$. for every x, y, z ϵ X.

Definition:2.2

A non empty set X together with binary operations "$-$" and "$\cdot$" is said to be a **near subtraction semigroup** if it satisfies the following:

i. (X, $-$) is a subtraction algebra.

ii. (X,·) is a semigroup.

iii. $(x-y)z = xz-yz$, for every x, y, z ϵ X.

Definition:2.3

A non empty set X together with binary operations " $-$ " and " $\cdot$ " is said to be a **near subtraction semigroup (right)** if it satisfies the following:

i. (X, $-$) is a subtraction algebra.

ii. (X,·) is a semigroup

iii. $(x-y)z=xz-yz$, for every x, y, z ϵ X.

Definition:2.5

A non empty subset s of a subtraction algebra X is said to be **subalgebra** of X, if $x-x' \epsilon X$ wheneever $x, x' \epsilon S$.

Definition:2.6

Let (X, $-$) be a near subtraction semigroup. A non empty subset I is called

i. A **left ideal** if I is a sub algebra of (X, $-$)and $XI \subseteq I$.

ii. A **right ideal** if I is a subalgebra of (X, $-$) and $IX \subseteq I$.

iii. An **ideal** if I is both left and right ideal.

Definition:2.8

We say that X is an $\boldsymbol{s}(\boldsymbol{s}')$**near subtraction semigroup** $if\ a \in Xa(aX)$, for all a ∈ X.

Definition: 2.9

A s-near subtraction semigroup X is said to be $\bar{\boldsymbol{s}}$ $-$**near subtraction semigroup** if $a \in aX$, for a ∈ X.

Definition:2.10

Let x be a near subtraction semigroup. Given two subsets A and B of X, $AB = \{ab/a \in A, b \in B\}$. Also we define another operation "*" by, $A * B = \{ab - a(a' - b), a, a' \in A, b \in B\}$.

Definition:2.1

A near subtraction semigroup X is said to be **subcommutative** if $aX = Xa$, for every $a \in X$.

Definition:2.14

An element aX is said to be **regular** if for each $a \in X, a = aba$, for some b ∈ X.

Definition:2.15

A near subtraction semigroup X is said to have **property(α)** if x is a subalgebra of (X,−), for every x ∈ X.

Definition: 2.16

A sub algebra S of (X,−) is called an left(right)X-subalgebra of X if $XS \subseteq S(SX \subseteq S)$.

-	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	b
c	c	0	c	0

•	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	0	0	0
c	0	a	b	c

Definition:2.17

A near subtraction semigroup X is said to be two sided if every left X- subalgebra is right X-subalgebra and viceversa.

3.Quasi-ideals and K-regular near subtraction semigroups

Definition:3.1

A subalgebra Q of (X,−) is said to be quasi-ideal of X if $QX \cap XQ \cap X * Q \subseteq Q$.

Example:3.1.1

Let X = {0, a, b, c} in which " − " and "." are defined by,

clearly {0, a} is quasi- ideal of X.

Definition:3.2

A near subtraction semigroup X is called K-regular near subtraction semigroup if a $\in <a>_r X <a>_l$, for any $a \in X$, Where $<a>_r$ ($<a>_l$) is the right (left) X-subalgebra generated by a ∈ X.

Note:3.5

Every regular near subtraction semigroup is K-regular. But the converse is not true.

Example:3.3

Let X = {0,1,2,3} in which "−" and "·"are defined by,

-	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

•	0	1	2	3
0	0	0	0	0
1	1	1	1	0
2	2	2	2	0
3	3	3	3	0

Clearly X is K-regular. But not regular, since 3∉3.X. 3={0}.

Proposition:3.4

The set of all quasi-ideals of a near subtraction semigroup X form a moore system on X.

Proof:

Let Q {i ∈ I} be a set of all quasi-ideals in X. Let Q=∩ Q. Then $QX \cap QX \cap X * Q \subseteq Q_i X \cap XQ_i \cap XQ_i \cap\subseteq Q_i$, for all i∈ I.there for $QX \cap XQ \cap X * Q \subseteq Q$. (ie) Q is a quasi-idealof X.

Proposition:3.5

If Q is a Quasi-ideal of a near subtraction semigroup X and S is a semigroup of X, then Q ∩ S is a qusai-ideal of S.

Proof:

Since Q is a quasi-ideal of X, $QX \cap XQ \cap X * Q \subseteq Q$.

Let C=Q∩ S. Then,

$CS \cap SC \cap S * C = (Q \cap S)S \cap S(Q \cap S) \cap S * (Q \cap S) \subseteq QS \cap SQ \cap S * Q \cap S \subseteq QX \cap XQ \cap X * Q \cap S \subseteq Q \cap S = C$. There fore $CS \cap SC \cap S * C \subseteq C$.

(ie) B ∩ S is a qusai-ideal of S.

Proposition:3.6

Let X be zero-symmetric near subtraction semigroup. A subalgebra Q of X is a quasi-ideal $QX \cap XQ \subseteq Q$.

Proof:

Let Q be a quasi-ideal of X Then $QX \cap XQ \cap X * Q \subseteq Q$. since X is zero-symmetric, XQ ⊆ X ∗ Q. Therefore $QX \cap XQ = (QX \cap XQ) \cap (QX \cap XQ) = (QX \cap XQ) \cap XQ \subseteq QX \cap XQ \cap X * Q \subseteq Q$. Conversely, $QX \cap XQ \subseteq Q$. There fore $QX \cap XQ \cap X * Q \subseteq QX \cap XQ \subseteq Q$.(ie) Q is a quasi-ideal of X.

Proposition:3.8

Let X be a near subtraction semigroup. Then the following are equivalent:

(i) X is k-regular

(ii) $RL = R \cap L$, for every right X-subalgebra R of X and for every left X-subalgebra L of X.

(iii) For every pair of element a, b of X, $< a >_r \cap < b >_l = < a >_r < b >_l$

(iv) For any element a of X, $< a >_r \cap < b >_l = < a >_r < b >_l$

Proof :

(i)⇒(ii):

Clearly $RL \subseteq R \cap L$. If $x \in R \cap L,\ then\ x \in < x >_r X < x >_l \subseteq RXL \subseteq RL$. Thus $R \cap L = RL$.

(ii)⇒(iii) and **(iii)⇒(iv)** are trivial.

(iv) ⇒(i):

Let a ∈ X. then $< a >_r \cap < a >_l = s < a >_r < a >_l$. Since $a \in < a >_r \cap < a >_l, a = bc$, for some $b \in < a >_r$ and $c \in < a >_l$. Similarly $b = de$, for some $d \in < b >_l,\ e \in < b >_l$.Thus $a = dec \in < a >_r X < a >_l$. (ie) X is K-regular.

Proposition:3.9

Let X be a s-near subtraction semigroup with property (α). Then the following are equivalent :

(i) X is K-regular.

(ii) X is regular.

(iii) For every quasi-ideal Q, $QPQ = Q$, for some subset P of X.

Proof:

(i)$\Rightarrow$ **(ii)**:

Let x ∈ X. Then we have $x \in < x >_r X < x >_l$. Since $x \in < x >_r, xX$ is a right X-subalgebra of X and $xX \subseteq < x >_r$. since X is a $\bar{s}$-near subtraction semigroup $x \in xX$ and so $< x >_r \subseteq xX$. Thus $< x >_r = xX$. Similarly $< x > = Xx$.

Hence $x \in < x >_r X < x >_l \subseteq xXx$. (ie)X is regular.

(ii)$\Rightarrow$ **(iii):**

Since X is regular, $Q \subseteq QXQ$,for every Quasi-ideal Q of X. Also $QXQ \subseteq Q$.

Thus $Q = QXQ$.

(iii)$\Rightarrow$ **(i):**

Since X is a $\bar{s}$-near subtraction semigroup. $< x >_r = xX$ and $< x >_l = Xx$, for x∈X. Thusfor $x \in X$, $< x >_r \cap < x >_l = < x >_r \cap < x >_l P < x >_r \cap < x >_l \subseteq < x >_r X < x >_l$. $(i.e)\ x \in X < x >_r < x >$. Hence X is K-regular.

References:

[1] J.C.Abbott, Sets, Lattices, and Boolean Algebras, Allyn and Bacon, Mass.1969.

[2] P.Dheena and G.Satheesh Kumar On Strongly Regular near subtraction semigroups, Commun. Korean Math. Soc. 22(2007), No.3, pp.323-330.

[3] Y.B.Jun and H.S.Kim, On ideals in subtraction algebras, Sci. Math.Jpn. 65(2007), no.1,129-134.

[4] Y.B.Jun, H.S.Kim and E.H.Roh, Ideal theory of subtraction lgebras, Sci. Math.Jpn.01(2005) no.3, 459-464.

[5] K.H.Kim, On subtraction semigroups, Scientiate Mathematicae Japonicae 62(2005), no.2, 273-280.

[6] C.V.L.N Murty, Generalised –near field, croc. Edinburgh Math.soc. 27(1984), 21-24.

[7] S.Maharasi, V.Mahalakshmi, On Bi-ideals of Near-Subtraction Semigroup, Research India Publications, New Delhi ISSN 0973-6964 Volume 6, Number 1(2013)

[8] Pilz Gunter,. Near-Rings, North Holland, Amsyterdam, 1983.

[9] O.Steinfield. Acta.Sci. Math. (Szeged) 17(1956), 170-80.

[10] B.M.Schein,Difference semigroups, Comm Algebra 20(1992), no8, 2153-2169.

[11] B.Zelinika, subtraction semigroups, Math.Bohem. 120(1995), no.4, 445-447.