RATIO CUM MEDIAN BASED MODIFIED RATIO ESTIMATORS WITH KNOWN QUARTILES AND SKEWNESS

Dr.R. Srija[1] and Dr.P.Vetri Selvi[2]

[1] *Department of Mathematics, Fatima College, Madurai, Tamilnadu,* Email: srijaramaraj@gmail.com

[2]*Department of Statistics, Fatima College, Madurai, Tamilnadu,* Email: vetrichandran@yahoo.com

ABSTRACT

In this paper, some ratio cum median based modified ratio estimators with known quartiles and skewness of the auxiliary variable have been proposed. The performance of the proposed class of estimators is assessed with that of simple random sampling without replacement (SRSWOR) sample mean, ratio estimator and modified ratio estimators in terms of variance/mean squared errors. The performance of proposed class of estimators is illustrated with the help of certain natural population available in the literature. The percentage relative efficiency of the proposed class of estimators with respect to SRSWOR sample mean, ratio estimator and some of the existing modified ratio estimators are also obtained.

Keywords: Auxiliary variable; Bias; Mean squared error; Natural population; Percentage relative efficiency; Simple random sampling.

INTRODUCTION

The main objective of sampling is to estimate the population mean of the study variable on the basis of selecting a random sample of size n from the population of sizeN. In this connection, a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N distinct and identifiable units has been considered for the estimation of the finite population mean. Let $Y(X)$ denote the study (auxiliary) variable taking values $Y_i(X_i), i = 1,2,\dots,N$ and is measured onU_i. Ratio estimator is used to improve the precision of the estimator based on SRSWOR sample mean by making use of the information of auxiliary variable which is positively correlated with that of the study variable. For a detailed discussion on the ratio estimator and its related problems the readers are referred to the text books by Cochran (1977) and Murthy (1967). The efficiency of the ratio estimator can be improved further with the help of known parameters of the auxiliary variable such as, correlation coefficient, coefficient of variation, Skewness, Kurtosis, Quartiles etc. The resulting estimators are called in literature as modified ratio estimators. See for example Adepoju and Shittu (2013), Das and Tripathi (1978), Diana, Giordan and Perri (2011), Gupta and Shabbir(2008), Kadilar and Cingi (2004), Kadilar and Cingi (2006a, 2006b), Koyuncu (2012), Koyuncu and Kadilar (2009), Shittu and Adepoju (2013), Singh and Agnihotri (2008), Singh and Tailor(2003), Sisodia and Dwivedi (1981), Subramani and Kumarapandiyan (2012a, 2012b) and the references cited there in.

Recently a new median based ratio estimator that uses the population median of the study variable Y has been introduced by Subramani (2013). From the median based ratio estimator, the median based modified ratio estimators are developed by Subramani and Prabavathy (2014a, 2014b, 2015). Recently Jayalakshmi et.al (2016), Srijaet.al. and Subramani et.al (2016) have introduced some ratio cum median based modified ratio estimators for estimation of finite population mean with known parameters of the auxiliary variable such as kurtosis, skewness, coefficient of variation and correlation coefficient and their linear combinations. In this paper, some more ratio cum median based modified ratio estimators with known quartiles and skewness of the auxiliary variable and their linear combinations are introduced. Before discussing about the proposed estimators, we present the notations to be used and are as follows:

1.1 Notations to be used

$N -$ Population size

$n -$ Sample size

$f = n/N$, Sampling fraction

- **$\delta = \frac{1-f}{n}$, finite population correction**
- **$\bar{X}, \bar{Y}$ –Population means**
- **$\bar{x}, \bar{y}$ –Sample means**
- **$S_X, S_y -$ Population standard deviations**
- **$S_{xy} -$ Population covariance between X and Y**
- **$C_X(C_y) -$ Co-efficient of variation of X (Y)**

- $\rho = \frac{S_{xy}}{S_x S_y}$ −Co-efficient of correlation between X and Y
- $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$, Skewness of the auxiliary variable
- $\beta_2 = \frac{\mu_4}{\mu_2^2}$, Kurtosis of the auxiliary variable where $\mu_r = \frac{1}{N}\sum_{i=1}^{N}(X_i - \overline{X})^r$
- Q_1 – First(lower) quartile of the auxiliary variable
- Q_3 - Third(upper) quartile of the auxiliary variable
- $M\ (m)$ − Population (sample) Median of the study variable
- $B(.)$ −Bias of the estimator
- $MSE(.)$ −Mean squared error of the estimator
- $V(.)$ −Variance of the estimator
- $\bar{y}$ −Simple random sampling without replacement (SRSWOR) sample mean
- $\widehat{\overline{Y}}_R$ − Ratio estimator
- $\widehat{\overline{Y}}_M$ −Median Based Ratio Estimator
- $\widehat{\overline{Y}}_{p_j}$ − j^{th}Proposed median based modified ratio estimator of $\overline{Y}$
- **Existing Estimators**

In case of SRSWOR, the sample mean $\bar{y}$is used to estimate population mean$\overline{Y}$which is an unbiased estimator. The SRSWOR sample mean together with its variance is given below:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad (1)$$

$$V(\bar{y}) = \frac{(1-f)}{n} S_y^2 \qquad (2)$$

where $f = \frac{n}{N}, S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^2$

The ratio estimator for estimating the population mean $\overline{Y}$of the study variable Y is defined as

$$\widehat{\overline{Y}}_R = \frac{\bar{y}}{\bar{x}}\overline{X} = \widehat{R}\overline{X} \qquad (3)$$

The mean squared error of $\widehat{\overline{Y}}_R$ is given below:

$$MSE\left(\widehat{\overline{Y}}_R\right) = \overline{Y}^2\{C'_{yy} + C'_{xx} - 2C'_{yx}\} \qquad (4)$$

where $C'_{yy} = \frac{V(\bar{y})}{\overline{Y}^2}, C'_{xx} = \frac{V(\bar{x})}{\overline{X}^2}, \ C'_{yx} = \frac{Cov(\bar{y},\bar{x})}{\overline{X}\overline{Y}}$

The modified ratio estimator$\widehat{\overline{Y}}_i$ with known parameter λ_i of the auxiliary variable for estimating the finite population mean $\overline{Y}$ is defined as

$$\widehat{\overline{Y}}_i = \bar{y}\left[\frac{\overline{X}+\lambda_i}{\bar{x}+\lambda_i}\right] \qquad (5)$$

The mean squared error of $\widehat{\overline{Y}}_i$ is as follows:$MSE\left(\widehat{\overline{Y}}_R\right) = \delta\overline{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\rho\theta_i C_x C_y\right)$ (6)

PROPOSED ESTIMATORS

In this section, some more ratio cum median based modified ratio estimators with known linear combinations of the known parameters of the auxiliary variable like First Quartile Q_1 and Third Quartile Q_3 and skewness in line with the ratio cum median based modified ratio estimators by Jayalakshmi et.al (2016), Subramani et.al (2016) and Srija et.al (2016). The proposed estimators together with their mean squared errors are given below:

Case i: The proposed estimator with known First Quartile $Q_1$and the skewness $\beta_1$are

$$\widehat{\overline{Y}}_{P_1} = \bar{y}\left\{\alpha_1\left(\frac{\beta_1 M+Q_1}{\beta_1 m+Q_1}\right) + \alpha_2\left(\frac{\beta_1\overline{X}+Q_1}{\beta_1\overline{X}+Q_1}\right)\right\} \qquad (7)$$

Case ii: The proposed estimator with known Third Quartile Q_3 and the skewness $\beta_1$are

$$\widehat{\overline{Y}}_{P_2} = \bar{y}\left\{\alpha_1\left(\frac{\beta_1 M+Q_3}{\beta_1 m+Q_3}\right) + \alpha_2\left(\frac{\beta_1\overline{X}+Q_3}{\beta_1\bar{x}+Q_3}\right)\right\} \qquad (8)$$

Theorem 2.1:In SRSWOR, ratio cum median based modified ratio estimator

$\widehat{\overline{Y}}_{P_1} = \bar{y}\left\{\alpha_1\left(\frac{\beta_1 M+Q_1}{\beta_1 m+Q_1}\right) + \alpha_2\left(\frac{\beta_1\overline{X}+Q_1}{\beta_1\bar{x}+Q_1}\right)\right\}$where $\alpha_1 + \alpha_2 = 1$, for the known parameter Q_1 and $\beta_1$is not an unbiased estimator for its population mean $\overline{Y}$ and its bias and MSE are respectively given as:

$$B\left(\widehat{\bar{Y}}_{p_1}\right) = \bar{Y}\left\{\alpha_1\left(\theta_i{}^2 C'_{mm} - \theta_i C'_{ym} - \theta_i \frac{B(m)}{M}\right) + \alpha_2\left(\varphi_i{}^2 C'_{xx} - \varphi_i C'_{yx}\right)\right\}$$

$$MSE\left(\widehat{\bar{Y}}_{p_1}\right) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\},$$

where $\theta_i = \frac{\boldsymbol{\beta_1} M}{\boldsymbol{\beta_1} M + Q_1}, \varphi_i = \frac{\boldsymbol{\beta_1}\bar{X}}{\boldsymbol{\beta_1}\bar{X} + Q_1}$

Proof: By replacing $T_i = Q_1 / \boldsymbol{\beta_1}$ in Theorem 2.0 the proof follows (**Srija et.al.(2016**)).

Theorem 2.2:In SRSWOR, ratio cum median based modified ratio estimator $\widehat{\bar{Y}}_{P_2} = \bar{y}\left\{\alpha_1\left(\frac{\beta_1 M + Q_3}{\beta_1 m + Q_3}\right) + \alpha_2\left(\frac{\beta_1\bar{X} + Q_3}{\beta_1\bar{x} + Q_3}\right)\right\}$ where $\alpha_1 + \alpha_2 = 1$, for the known parameter Q_3 and β_1 is not an unbiased estimator for its population mean $\bar{Y}$ and its bias and MSE are respectively given as:

$$B\left(\widehat{\bar{Y}}_{P_2}\right) = \bar{Y}\left\{\alpha_1\left(\theta_i{}^2 C'_{mm} - \theta_i C'_{ym} - \theta_i \frac{B(m)}{M}\right) + \alpha_2\left(\varphi_i{}^2 C'_{xx} - \varphi_i C'_{yx}\right)\right\}$$

$$MSE\left(\widehat{\bar{Y}}_{P_2}\right) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\},$$

where $\theta_i = \frac{\boldsymbol{\beta_1} M}{\boldsymbol{\beta_1} M + Q_3}, \varphi_i = \frac{\boldsymbol{\beta_1}\bar{X}}{\boldsymbol{\beta_1}\bar{X} + Q_3}$

Proof: By replacing $T_i = Q_3 / \beta_1$ in Theorem 2.0 the proof follows (**Srija et.al.(2016**)).

NOTE 2.1: The proposed estimators are written into a class of estimators with the population parameter T_i is

$$\widehat{\bar{Y}}_{P_i} = \bar{y}\left\{\alpha_1\left(\frac{M + T_i}{m + T_i}\right) + \alpha_2\left(\frac{\bar{X} + T_i}{\bar{x} + T_i}\right)\right\} \quad (9)$$

where $\alpha_1 + \alpha_2 = 1, \quad i = 1, 2$

The mean squared error of proposed estimator is given as

$$\mathbf{MSE}\left(\widehat{\bar{Y}}_{p_i}\right) = \bar{Y}^2\{C'_{yy} + \alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} - 2\alpha_1\theta_i C'_{ym} - 2\alpha_2\varphi_i C'_{yx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm}\} \quad (10)$$

where $\theta_i = \frac{M}{M + T_i}, \varphi_i = \frac{\bar{X}}{\bar{X} + T_i}, \quad C'_{xm} = \frac{Cov(\bar{x}, m)}{M\bar{X}},$

$$T_1 = Q_1 / \beta_1, T_2 = Q_3 / \beta_1$$

The detailed derivation of the above expression of the mean square error is given in Srija et.al.(2016)

3. EFFICIENCY COMPARISON

In this section, the efficiencies of proposed estimators given in (9) are assessed with that of SRSWOR sample mean ratio estimator and modified ratio estimators in terms of variance/mean squared error. The results are as follows:

3.1Comparison with that of SRSWOR sample mean

Comparing (10) and (2), it is noticed that the proposed estimators perform better than the SRSWOR sample mean if $MSE\left(\widehat{\bar{Y}}_{P_i}\right) \leq V(\bar{y})$ i.e.

$$\alpha_1^2\theta_i^2 C'_{mm} + \alpha_2^2\varphi_i^2 C'_{xx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm} \leq 2(\alpha_1\theta_i C'_{ym} + \alpha_2\varphi_i C'_{yx}) \quad (11)$$

3.2Comparison with that of Ratio Estimator

Comparing (10) and (4), it is noticed that the proposed estimators perform better than the ratio estimator if $MSE\left(\widehat{\bar{Y}}_{P_i}\right) \leq MSE\left(\widehat{\bar{Y}}_R\right)$ i.e.

$$\alpha_1^2\theta_i^2 C'_{mm} + (\alpha_2^2\varphi_i^2 - 1)C'_{xx} + 2\alpha_1\alpha_2\theta_i\varphi_i C'_{xm} \leq 2\left[\alpha_1\theta_i C'_{ym} + (\alpha_2\varphi_i - 1)C'_{yx}\right] \quad (12)$$

4. NUMERICAL COMPARISON

In the section3, the conditions for the efficiency of proposed estimators given in (9) with that of existing estimators have been derived algebraically. To support it by means of numerical comparison, data of a natural population from Singh and Chaudhary (1986, page.177) has been considered.

Population Description

X= Area under Wheat in 1971 and Y= Area under Wheat in 1974

The population parameters computed for the above population is given below:

N= 34	**n= 3**	**$\bar{Y}$= 856.4118**
β_1 = 0.8732	**M= 767.5**	**$\bar{X}$= 208.8824**
Q_1= 94.25	**Q_3= 254.75**	

The variance/mean squared error of the existing and proposed estimators at different values of α_1 and α_2 are given in the following table

Table 4.1: Mean Squared Errors for different values of α_1 and α_2

Existing Estimators			
SRSWOR Sample mean		$\bar{y}$	**163356.41**
Ratio Estimator		$\hat{\bar{Y}}_R$	**155579.71**
Proposed Estimators			
α_1	α_2	$\hat{\bar{Y}}_{P_1}$	$\hat{\bar{Y}}_2$
0.1	**0.9**	**123500.92**	**122866.82**
0.2	**0.8**	**114994.56**	**115398.67**
0.3	**0.7**	**107432.36**	**108716.18**
0.4	**0.6**	**100932.25**	**102766.36**
0.5	**0.5**	**95483.48**	**97575.73**
0.6	**0.4**	**91027.69**	**93133.22**
0.7	**0.3**	**87627.18**	**89458.89**
0.8	**0.2**	**85277.94**	**86529.56**
0.9	**0.1**	**83977.87**	**84369.77**

Page4

From Table 4.1, it is observed that the proposed estimators discussed in (9) have less mean squared errors than the SRSWOR sample mean, ratio estimator and the modified ratio estimators.

The percentage relative efficiencies (PRE) of the proposed estimators with respect to the existing estimators are obtained by using the formula $\mathbf{PRE(e,p)} = \frac{\mathbf{MSE(e)}}{\mathbf{MSE(p)}} * \mathbf{100}$ and are given in the following table:

Table 4.2: PRE of proposed estimators with respect to SRSWOR sample mean

α_1	α_2	$\hat{\bar{Y}}_{P_1}$	$\hat{\bar{Y}}_2$
0.1	**0.9**	**132.27**	**132.95**
0.2	**0.8**	**142.06**	**141.56**
0.3	**0.7**	**152.06**	**150.26**
0.4	**0.6**	**161.85**	**158.96**
0.5	**0.5**	**171.08**	**167.42**
0.6	**0.4**	**179.46**	**175.40**
0.7	**0.3**	**186.42**	**182.61**
0.8	**0.2**	**191.56**	**188.79**
0.9	**0.1**	**194.52**	**193.62**

Table 4.3: PRE of proposed estimators with respect to Ratio Estimator

α_1	α_2	$\hat{\bar{Y}}_{P_1}$	$\hat{\bar{Y}}_2$

0.1	**0.9**	**125.97**	**126.63**
0.2	**0.8**	**135.29**	**134.82**
0.3	**0.7**	**144.82**	**143.11**
0.4	**0.6**	**154.14**	**151.39**
0.5	**0.5**	**162.94**	**159.45**
0.6	**0.4**	**170.91**	**167.05**
0.7	**0.3**	**177.54**	**173.91**
0.8	**0.2**	**182.44**	**179.80**
0.9	**0.1**	**185.26**	**184.40**

From Tables 4.2 and 4.3, it is observed that the PRE values of the proposed estimators with respect to SRSWOR sample mean, ratio estimator and modified ratio estimators are greater than 100 and hence we conclude that the proposed estimators are efficient estimators.

- **In fact the PREs are ranging from**
  - **132.27 to 194.52 for the case of SRSWOR sample mean**
  - **125.97 to 185.26 for the case of ratio estimator**

1. SUMMARY

In this paper we have proposed some more ratio cum median based modified ratio estimators with the known parameters such as quartiles Q_1 ,Q_3 and β_1 of the auxiliary variable and their linear combinations. The efficiencies of the proposed ratio cum median based modified ratio estimators are assessed algebraically as well as numerically with that of SRSWOR sample mean, ratio estimator and some of the modified ratio estimators. Further it is shown from the numerical comparison that the PREs of proposed ratio cum median based modified ratio estimators with respect to the existing estimators are more than 100. Hence the proposed ratio cum median based modified ratio estimators with known quartiles and skewness may be recommended for the use of practical applications.

REFERENCES

[1]. ADEPOJU, K.A. and SHITTU, O.L. (2013): On the Efficiency of Ratio Estimator Based on Linear Combination of Median, Coefficients of Skewness and Kurtosis, American Journal of Mathematics and Statistics, 3(3), 130-134

[2]. COCHRAN, W. G. (1977): Sampling techniques, Third Edition, Wiley Eastern Limited

[3]. DAS, A.K. and TRIPATHI, T.P. (1978): Use of auxiliary information in estimating the finite population variance, Sankhya, 40, 139-148

[4]. DIANA, G., GIORDAN, M. and PERRI, P.F. (2011): An improved class of estimators for the population mean, Stat Methods Appl., 20, 123-140

[5]. JAYALAKSHMI, S.SUBRAMANI, J. and SRIJA, R. (2016): Ratio cum Median Based Modified Ratio Estimators for the Estimation of Finite Population Mean with Known Coefficient of Variation and Correlation Coefficient, International Journal of Computer and Mathematical Sciences, Intern. Jour. Computers and Math. Sci., 5(6), 122-127

[6]. KADILAR, C. and CINGI, H. (2004): Ratio estimators in simple random sampling, Applied Mathematics and Computation, 151, 893-902

[7]. KADILAR, C. and CINGI, H. (2006a): An improvement in estimating the population mean by using the correlation co-efficient, Hacettepe Journal of Mathematics and Statistics, 35 (1), 103-109

[8]. KADILAR, C. and CINGI, H. (2006b): Improvement in estimating the population mean in simple random sampling, Applied Mathematics Letters, 19, 75-79

[9]. KOYUNCU, N. (2012): Efficient estimators of population mean using auxiliary attributes, Applied Mathematics and Computation, 218, 10900-10905

[10]. SINGH, D. AND CHAUDHARY, F.S (1986): Theory and analysis of sample survey designs, New Age International Publisher

[11]. SINGH, H.P. and AGNIHOTRI, N. (2008): A general procedure of estimating population mean using auxiliary information in sample surveys, Statistics in Transition, 9(1), 71–87

[12]. SINGH, H.P. and TAILOR, R. (2003): Use of Known Correlation Co-efficient in Estimating the Finite Population Means, Statistics in Transition, Vol. 6 (4), 555-560

[13]. SISODIA, B.V.S. and DWIVEDI, V.K. (1981): A modified ratio estimator using Co-efficient of Variation of Auxiliary Variable, Journal of the Indian Society of Agricultural Statistics, Vol. 33(2), 13-18

[14]. SRIJA, R, SUBRAMANI, J. and JAYALAKSHMI, S. (2016): Ratio cum Median Based Modified Ratio Estimators for the Estimation of Finite Population Mean with Known Skewness, Kurtosis and Correlation Coefficient, International Journal of Computer and Mathematical Sciences, Intern. Jour. Computers and Math. Sci., 5(7), 29-34

[15]. SUBRAMANI J (2013): A New Median Based Ratio Estimator for Estimation of the Finite Population Mean, Statistics- in Transition – Accepted for publication

[16]. SUBRAMANI, J. and KUMARAPANDIYAN, G. (2012a): Modified Ratio Estimators for Population Mean using function of Quartiles of auxiliary variable, Bonfring International Journal of Industrial Engineering and Management Science, Vol. 2(2), 19-23

[17]. SUBRAMANI, J. and KUMARAPANDIYAN, G. (2012b): Modified ratio estimators using known median and co-efficient of kurtosis, *American Journal of Mathematics and Statistics*, Vol. 2(4), 95-100

[18]. SUBRAMANI, J. and PRABAVATHY, G. (2014a): Some modified ratio estimators with known coefficient of variation and correlation coefficient, Proceedings of International Conference on recent developments in statistical theory and practice (ICRDSTAP), 192-200, ISBN 978-93-83459-13-1

[19]. SUBRAMANI, J. and PRABAVATHY, G. (2014b):Median Based Modified Ratio Estimators with Linear Combinations of Population Mean and Median of an Auxiliary Variable, Journal of Reliability and Statistical Studies, Vol. 7(1), 01-10.

[20]. SUBRAMANI, J. and PRABAVATHY, G. (2015):Median Based Modified Ratio Estimators with known Skewness and Correlation Coefficient of an Auxiliary Variable, Journal of Reliability and Statistical Studies, Vol. 8(1), 15-23.

[21]. SUBRAMANI, J. SRIJA, R. and JAYALAKSHMI, S. (2016): Ratio cum Median Based Modified Ratio Estimators for the Estimation of Finite Population Mean with Known Skewness and Kurtosis, International Journal of Computer and Mathematical Sciences, Intern. Jour. Computers and Math. Sci., 5(6), 115-121

[22]. SRIJA, R and SUBRAMANI, J. (2018):Median Based Modified Ratio Estimators with known Kurtosis and Coefficient of Variation, IJMTT, Vol. 59(3).