CONTINUOUS FUZZY MAPS

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Abstract:

In this paper we introduce continuity of fuzzy maps

1. Introduction

1.1 The concept of continuous function in case of metric spaces and topological spaces is an important study in Analysis and Topology. In the year 1965 L.A.Zadeh introduced the concept of fuzzy sets. In the year 2015, myself and others introduced fuzzy maps.

1.2 Preliminaries

1.2.1 Let X and Y be two non empty sets. A function F: X × Y ⟶ [0,1] is called a fuzzy map from X to Y.

1.2.2 Let F be a fuzzy map from X to Y. Let $x_0 \in X$. Then $F(x_0) = \{ y \in Y / F(x_0, y)=1\}$. $F(x_0)$ is called the image of x_0

1.2.3 Let F be a fuzzy map from X to Y. Let A be a non empty subset of X. F(A) = { y ϵ Y / F(x ,y)=1 for some x ϵ A }

1.2.4 Let F be a fuzzy map from X to Y. Let $y_0 \in Y$. $F^{-1}(y_0) = \{ x \in X / F(x, y_0)=1\}$.

1.2.5 Let F be a Fuzzy map from X to Y. Let B ⊂ Y. $F^{-1}(y_0) = \{ x \in X / F(x, y)=1 \text{ and } y \in B\}$

2. Continuity

2.1 Definition Let X and Y be metric spaces. Let F be a fuzzy map from X to Y. Let a ϵ X F is continuous at x if for every ε>0, for every b ϵ F(a), ∃δ >0 such that x ϵ B(a,δ) implies atleast one of the elements of F(x) belongs to B(a,δ).

2.2 Example Take X=Y=R

Define F:X x Y ⟶ [0,1] as

$$F(x,y) = \begin{cases} 1 & if\ y = 2x\ or\ 3x \\ 0 & otherwise \end{cases}$$

take 1 ϵ X. F(1)= {2,3}. Consider 2 ϵ F(1). Let ε >0 be given . Take δ=ε/2. x ϵB(1,δ). F(x)={2x,3x}. |x-1|< δ

⇒2|x-1|<2δ ⇒ |2x-2|<2δ ⇒ |2x-2|<ε

⇒one value of F(x) ϵ B(2,δ). Now consider 3 ϵ F(1).

Take δ=ε/3. |x-1|< δ ⇒ 3|x-1|<3δ ⇒ |3x-3|<2δ ⇒ |3x-3|<ε ⇒one value of F(x) ϵ B(3,δ). Hence F is continuous at x=1.

2.3 Definition Let X and Y be metric spaces. Let F be a fuzzy map from X to Y. F is continuous if F is continuous at each point of X.

2.4 Example Take X=Y= R.

$$F(x,y) = \begin{cases} 1 & if\ y = 5x \\ 0.5\ if\ y = x^3 \\ 0\ otherwise \end{cases}$$

Take a ϵ X, F(a)={5a}. Consider 5a for x ϵ X, F(x)={5x}. Let ε>0 be given. Take δ=ε/5. x ϵ B(a,δ) ⇒|x-a| < δ ⇒|5x-5|<5δ ⇒|5x-5a| <ε ⇒5x ϵ B(5a,ε) ⇒ one value of F(x) ϵ B(5a,δ). F is continuous at a ϵ X. This is true for all a ϵ X. Hence F is Continuous.

2.4 Theorem Let X and Y be two metric spaces. Let f: X →Y be a crisp map. Let F be the Corresponding fuzzy map. Let a ϵ X. f is continuous at a implies F is continuous at a.

Proof : X and Y are metric spaces. a ϵ X. f:X→Y is a crisp map. The corresponding fuzzy map is defined as follows

$$F(x,y) = \begin{cases} 1\ if\ y = f(x) \\ 0\ otherwise \end{cases}$$

Now f is continuous at a.

Claim F is continuous at a. Now F(a)={f(a)}. Letε>0 be given. Since f is continuous at a, ∃δ>0 such that x ϵ B(a,δ)⇒ f(x) ϵ B(f(a),ε). Clearly F(x) = {f(x)}. Therefore one value of F(x) ϵ B(f(a),ε). Hence F is Continuous at a.

2.5 Theorem Let X and Y be two metric spaces. Let f: X→Y be a crisp map. Let F be the corresponding fuzzy map. Let a ϵ X. F is continuous at a implies f is continuous at a.

Proof: F is defined as in theorem 2.4. F(a)={f(a)}. Let ε>0 be given. F is continuous at a. Hence ∃δ>0 such that x ϵ B(a,δ)⇒one value of F(x) ϵ B(f(a),ε). Clearly F(x)={f(x)}. Hence f(x) ϵ B(f(a),ε). Hence f is continuous at a.

2.6 Theorem Let X and Y be two metric spaces. Let f: X →Y be a crisp map. Let F be the Corresponding fuzzy map. f is continuous at a if and only if F is continuous at a.

Proof : Follows from above theorems.

2.7 Theorem Let X and Y be two metric spaces. Let f: X →Y be a crisp map. Let F be the Corresponding fuzzy map. f is continuous if and only if F is continuous.

References:

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