A Study on Idempotent Commutative Gamma Semigroup

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ABSTRACT

The concept of semigroup theory was introduced by A.H. Clifford et. al and extended by many authors like Mchean, David et.al. The algebraic theory of commutative semigroup was studied and extended by M. A. Taiclin, A. P. Biryukov. The concept of idempotent commutative semigroup was studied by D. Radha and P. Meenakshi with some structures like semi-regular, normal, quasi-normal on it. In this paper, we have introduced the notion of Idempotent Commutative Gamma Semigroup and the definition of semi gamma regular, semi gamma normal and gamma quasi-normal semigroups. Also it is proved that, an idempotent commutative gamma semigroup is gamma left(right) normal if and only if it is gamma left(right) quasi normal. Any idempotent commutative gamma semigoup is gamma regular if and only if it is gamma normal and gamma left(right) seminormal if and only if it is gamma left(right) semi regular. And any gamma quasi normal is both gamma semi regular, gamma semi normal and converse is also proved as a characterization theorem.

Keywords: Idempotent element, commutativity, normal, gamma quasi -normal, gamma semi normal, gamma regular, gamma quasi - regular, gamma semi- regular.

1.Introduction

Γ-Semigroup was introduced by Sen and Saha[15] as a generalization of semigroup. The formal study of semigroups began in the early 20th century. Early results include a Cayley theorem for semigrops realizing any semigroup as transformation semigroup, in which arbitrary functions replace the role of bijections from group theory. Other fundamental techniques of studying semigroups like Green's relations do not imitate anything in group theory though. In other areas of applied mathematics, semigroups are fundamental models for linear time-invariant systems. As a generalization of a semigroup Sen[15] introduced the notion of Γ-semigroup in 1981 and developed some theory on Γ-semigroup. Dutta.T.K, Chatterje[11] generalized the green's relations in semigroups to Γ-semigroups. Lee[12] in 1996, and it has been studied by several authors. In this paper we introduce the notions of idempotent commutative Γ-semigroup and characterization theorems.

Let S and Γ be two non-empty sets. Then S is called a Γ-semigroup if there exists a mapping from $S\times\Gamma\times S$ to S which maps $(a,\alpha,b)\rightarrow a\alpha b$ satisfying the condition: $(a\gamma b)\mu c = a\gamma(b\mu c)$ for all $a,b,c \in S$ and $\gamma,\mu\in\Gamma$. It is denoted by (S,Γ). In this paper we have proved that Idempotent Gamma Semigroup satisfies some properties.

2. Structure of Idempotent Commutative Γ-Semigroup

In this section, we see some basic definition on Γ-Semigroup and we have showed that idempotent commutative semigroup satisfies some properties.

Definition:2.1

A Γ-semigroup S is said to be commutative provided $a\gamma b= b\gamma a$ for all $a,b\in S$ and $\gamma\in\Gamma$.

Definition:2.2

An element a of Γ-semigroup S said to be an idempotent or Γ-idempotent if $a\alpha a=a$ for all $\alpha\in\Gamma$.

Definition:2.3

A Γ-semigroup (S,Γ) is said to be left(right) gamma singular if it satisfies the identity $a\alpha b=a(a\alpha b=b)$ for all $a,b\in S$ and for all $\alpha\in\Gamma$.

Definition:2.4

A Γ-semigroup (S,Γ) is gamma rectangular if it satisfy the identity aαbβa=a for all a,b∈S for all α,β∈Γ.

Definition:2.5

A Γ-Semigroup (S, Γ) is called gamma regular if it satisfies the identity $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a$ for all a,b,c in S, for all $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \epsilon \Gamma$.

Definition:2.6

A Γ-Semigroup (S, Γ) is said to be gamma total if every element of S can be written as the product of two elements of S. i.e, SαS = S for all α∈Γ.

Definition:2.7

A Γ-Semigroup (S, Γ) is said to be gamma left(right) normal if aαbβc = aαcβb(aαbβc = bαaβc) for all a, b, c in S and for all α,β∈Γ.

Definition:2.8

A Γ-Semigroup (S, Γ) is said to be gamma normal if it satisfies the identity $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 a$ for all a, b, c in S and for all $\gamma_1, \gamma_2, \gamma_3 \epsilon \Gamma$.

Definition:2.9

A Γ-Semigroup (S, Γ) is said to be gamma left(right) quasi-normal if it satisfies the identity $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c (a\gamma_1 b\gamma_2 c = a\gamma_1 b\gamma_2 a\gamma_{13} c)$ for all a, b, c in S and for all $\gamma_1, \gamma_2, \gamma_3 \epsilon \Gamma$.

Definition:2.10

A Γ-Semigroup (S, Γ) is said to be gammaleft(right) semi-regular if it satisfies the $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a (a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a)$ for all a, b, c in S and for all $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 \epsilon \Gamma$.

Theorem: 2.11

An Idempotent Commutative Γ-Semigroup(S,Γ) is gamma left(right) normal iff it is gamma left(right) quasi-normal

Proof:

Let (S, Γ) be an Idempotent Commutative Γ-Semigroup. Now let (S, Γ) be gamma left normal, then $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_3 c \Rightarrow a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c$ ($c\gamma_3 c = c) \ \forall \gamma_1, \gamma_2, \gamma_3 \epsilon \Gamma$. Therefore (S, Γ) is gamma left quasi-normal. Conversely. Let (S, Γ) be gamma left quasi-normal, then $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c \Rightarrow a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 c\gamma_3 b \ (b\gamma_3 c = c\gamma_3 b) \Rightarrow a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_1 b (c\gamma_2 c = c) \ \forall \gamma_1, \gamma_2, \gamma_3 \epsilon \Gamma$.

Theorem 2.12

An idempotent commutative Γ- semigroup (S,Γ) is gamma regular iff it is gamma normal

Proof:

Let (S, Γ) be an idempotent Commutative Γ-Semigroup.Assume that (S, Γ) is gamma regular then $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 a \ (a\gamma_3 c = c\gamma_3 a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 a \ (b\gamma_1 c = c\gamma_2 b)$

$\forall \gamma_1, \gamma_2, \gamma_3, \gamma_4 \epsilon \Gamma$. Conversely, assume that (S, Γ) is gamma normal then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 a \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a$

$(c\gamma_2 b = b\gamma_2 c) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_4 a\gamma_1 b\gamma_2 c\gamma_3 a \ (a = a\gamma_4 a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_4 b\gamma_1 a\gamma_2 c\gamma_3 a \ (a\gamma_1 b = b\gamma_1 a) \forall \gamma_1, \gamma_2, \gamma_3, \gamma_4 \epsilon \Gamma$. Hence (S, Γ) is gamma regular.

Theorem 2.13

An Idempotent Commutative Γ-Semigroup S is gamma left(right) semi-normal iff it is gamma left(right) semi-regular.

Proof:

Let (S, Γ) be an idempotent Commutative Γ-Semigroup. Assume that (S, Γ) is gamma left semi-normal then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_5 a\gamma_1 c\gamma_2 b\gamma_6 b\gamma_3 c\gamma_4 a (a\gamma_5 a = a \ \& \ b\gamma_6 b = b) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_5 a\gamma_1 b\gamma_2 c\gamma_6 b\gamma_3 c\gamma_4 a \ (b\gamma_2 c = c\gamma_2 b)$

$\Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 = a\gamma_5 b\gamma_1 a\gamma_2 c\gamma_6 b\gamma_3 c\gamma_4 a$

$(a\gamma_1 b = b\gamma_1 a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_5$

$b\gamma_1 c\gamma_2 a\gamma_6 b\gamma_3 c\gamma_4 a \ (c\gamma_2 a = a\gamma_2 c) \Rightarrow a\gamma_1 b$

$\gamma_2 c\gamma_3 a = a\gamma_7 a\gamma_5 b\gamma_1 c\gamma_2 a\gamma_6 b\gamma_3 c\gamma_4 a \ (a\gamma_7 a = a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a =$

$a\gamma_7 b\gamma_5 a\gamma_1 c\gamma_2 a\gamma_6 b\gamma_3 c\gamma_4 a$

$(a\gamma_5 b = b\gamma_5 a) \forall \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 \epsilon \Gamma$.

Hence (S, Γ) is gamma left semi-regular. Conversely, assume that (S, Γ) is gamma left semi-regular then, $a\gamma_1 b\gamma_2 c\gamma_3 a =$

$a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a$
$= a\gamma_1 b\gamma_2 a\gamma_3 a\gamma_4 c\gamma_5 b\gamma_6 c\gamma_7 a\ (a\gamma_4 c = c\gamma_4 a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a$
$= a\gamma_1 b\gamma_2 a\gamma_4 c\gamma_5 b\gamma_6 c\gamma_1 a$
$(a\gamma_3 a = a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a$
$= a\gamma_1 a\gamma_2 b\gamma_4 c\gamma_5 b\gamma_6 c\gamma_1 a$
$(a\gamma_2 b = b\gamma_2 a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3$
$= a\gamma_1 a\gamma_2 b\gamma_4 c\gamma_5 b\gamma_6 c\gamma_7 a$
$(a\gamma_1 a = a \&\ c\gamma_4 b = b\gamma_4 c) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a$
$= a\gamma_2 c\gamma_4 b\gamma_6 c\gamma_7 a\ (b\gamma_5 b = b)$
$\Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_2 c\gamma_4 b\gamma_6 c\gamma_7 a$
$\forall \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 \epsilon \Gamma$. Hence (S, Γ) isgamma left semi-normal.

Theorem:2.14

An Idempotent Commutative Γ- Semigroup (S,Γ) is gamma left(right) quasi-normal iff it is gamma left(right) semi-regular.

Proof:

Let (S, Γ) be an idempotent Commutative Γ Semigroup. Assume that (S, Γ) isgamma left quasi-normal then, $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_5 a\gamma_1 c\gamma_2 b\gamma_6 b\gamma_3 c\gamma_4 a\ (a\gamma_5 a = a \&\ b\gamma_6 b = b) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 = a\gamma_5 a\gamma_1 b\gamma_2 c\gamma_6 b\gamma_3 c\gamma_4 a\ (b\gamma_2 c = c\gamma_2 b)$
$\Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_5 b\gamma_1 a\gamma_2 c\gamma_6 b\gamma_3 c\gamma_4 a$
$(a\gamma_1 b = b\gamma_1 a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_5 b\gamma_1 c\gamma_2 a\gamma_6 b\gamma_3 c\gamma_4 a (c\gamma_2 a = a\gamma_2 c) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a$
$= a\gamma_7 a\gamma_5 b\gamma_1 c\gamma_2 a\gamma_6 b\gamma_3 c\gamma_4 a (a\gamma_7 a = a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a$
$= a\gamma_7 b\gamma_5 a\gamma_1 c\gamma_2 a\gamma_6 b\gamma_4 c\gamma_3 a$
$(b\gamma_5 a = a\gamma_5 b) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_3 a$
$= a\gamma_7 b\gamma_5 a\gamma_1 c\gamma_2 a\gamma_6 b\gamma_4 c\gamma_3 a$
$\forall \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 \epsilon \Gamma$.

Hence (S, Γ) is gamma left semi-regular. Conversely, assume (S, Γ) is gamma left semi-regular then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a \Rightarrow a\gamma_1 b\gamma_2 a\gamma_3 c = a\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a$
$(a\gamma_3 c = c\gamma_3 a\ \&a\gamma_2 b = b\gamma_2 a) \Rightarrow a\gamma_1 b\gamma_2 a\gamma_3 c$
$= a\gamma_2 c\gamma_3 b\gamma_4 a\gamma_5 b\gamma_6 c\gamma_7 a$
$(a\gamma_3 b = b\gamma_3 a\&\ a\gamma_1 a = a\ \&\ b\gamma_3 c = c\gamma_3 b)$
$a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 b\gamma_6 a\gamma_7 c$
$(a\gamma_2 b = b\gamma_2 a\ \&a\gamma_7 c = c\gamma_7 a) \Rightarrow a\gamma_2 b\gamma_3 c$
$= a\gamma_2 c\gamma_3 a\gamma_4 b\gamma_6 a\gamma_7 c$
$(b\gamma_5 b = b\&a\gamma_1 a = a) \Rightarrow a\gamma_2 b\gamma_3 c$
$= a\gamma_2 c\gamma_3 a\gamma_4 a\gamma_6 b\gamma_7 c$
$(a\gamma_6 b = b\gamma_6 a) \Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_3 a\gamma_6 b\gamma_7 c$
$(a\gamma_4 a = a) \Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_2 a\gamma_3 c\gamma_6 b\gamma_7 c$
$(a\gamma_3 c = c\gamma_3 a) \Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_3 c\gamma_6 b\gamma_7 c$
$(a\gamma_2 a = a)\ \forall\ \gamma_{1,}\gamma_{2,}\gamma_{3,}\gamma_{4,}\gamma_{5,}\gamma_{6,}\gamma_7.$

Hence (S, Γ) is gamma left quasi-normal.

Theorem:2.15

An Idempotent Commutative Γ- Semigroup (S, Γ) is gamma left(right) quasi-normal iffgamma left(right) semi-normal.

Proof:

Let (S, Γ) be an idempotent Commutative Γ-Semigroup. Assume (S, Γ) is gamma left quasi-normal then, $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a$. Hence (S, Γ) is gamma left semi-normal. Conversely assume that (S, Γ) is gamma left semi-normal then, $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a \Rightarrow a\gamma_1 b\gamma_2 a\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_3 a\gamma_4 c (a\gamma_3 c = c\gamma_3 a) \Rightarrow a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 a\gamma_3 b\gamma_4 c$
$(a\gamma_3 b = b\gamma_3 a) \Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_1 a\gamma_2 c\gamma_3 b\gamma_4 c$
$(a\gamma_1 a = a\ \&a\gamma_2 c = c\gamma_2 a)$
$a\gamma_1 b\gamma_2 c = a\gamma_2 c\gamma_3 b\gamma_4 c$
$(a\gamma_1 a = a)\forall\ \gamma_1, \gamma_2, \gamma_3, \gamma_4 \epsilon \Gamma.$

Hence (S, Γ) is gamma left quasi- normal.

Theorem:2.16

An Idempotent Commutative Semigroup (S, Γ) is gamma left(right) quasi-normal iff it is gamma right(left) semi-normal.

Proof:

Let (S, Γ) be an idempotent Commutative Γ-Semigroup. Assume that (S, Γ) is gamma left quasi-normal then $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_1 b\gamma_2 c\gamma_3 c\gamma_4 a(b\gamma_2 c = c\gamma_2 b) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_1 b\gamma_2 c\gamma_4 a(c\gamma_3 c = c) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_1 b\gamma_5 b\gamma_2 c\gamma_4 a(b\gamma_5 b = b)$
$\Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_1 b\gamma_5 c\gamma_2 b\gamma_4 a(b\gamma_2 c = c\gamma_2 b)\forall \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 \epsilon \Gamma$. Hence (S, Γ) is gamma right semi-normal.Conversely assume that (S, Γ) is gamma right semi-normal then $a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 b\gamma_4 a \Rightarrow a\gamma_1 b\gamma_2 a\gamma_3 = a\gamma_1 c\gamma_2 b\gamma_3 b\gamma_4 a(a\gamma_3 c = c\gamma_3 a\ \&\ b\gamma_2 c = c\gamma_2 b) \Rightarrow$

$a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_4 a(b\gamma_3 b = b \,\&\, a\gamma_2 b = b\gamma_2 a) \Rightarrow a\gamma_2 b\gamma_3 c$

$= a\gamma_1 c\gamma_2 a\gamma_4 b(a\gamma_1 a = a \,\&\, a\gamma_4 b = b\gamma_4 a)$

$\Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_1 a\gamma_2 c\gamma_4 b(a\gamma_1 c = c\gamma_1 a)$

$\Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_4 b$

$(a\gamma_1 a = a)$

$\Rightarrow a\gamma_2 b\gamma_3 c\gamma_6 c = a\gamma_2 c\gamma_4 b\gamma_6 c \Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_4 b\gamma_6 c(c\gamma_6 c = c)\forall \gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5,\gamma_6 \epsilon \Gamma$.

Hence (S, Γ) is gamma left quasi-normal.

Theorem:2.17

An Idempotent Commutative Γ- semigroup (S, Γ) is gammaleft(right) quasi-normal iff it is gamma right(left) semi-regular

Proof:

Let (S, Γ) be an idempotent Commutative Γ-Semigroup.Assume that (S, Γ) is gamma left quasi-normal then, $a\gamma_1 b\gamma_2 c = a\gamma_1 c\gamma_2 b\gamma_3 c \Rightarrow$

$a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_5 a\gamma_1 c\gamma_2 b\gamma_6 b\gamma_3 c\gamma_4 a\gamma_7 a(a\gamma_5 a = a \,\&\, b\gamma_6 b = b) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a = a\gamma_5 a\gamma_1 b\gamma_2 c\gamma_6 b\gamma_3 a\gamma_4 c\gamma_7 a$

$(b\gamma_2 c = c\gamma_2 b \,\&\, a\gamma_4 c = c\gamma_4 a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a$

$= a\gamma_5 b\gamma_1 a\gamma_2 c\gamma_6 b\gamma_3 a\gamma_4 c\gamma_7 a(a\gamma_1 b$

$= b\gamma_1 a) \Rightarrow a\gamma_1 b\gamma_2 c\gamma_4 a$

$= a\gamma_5 b\gamma_1 c\gamma_2 a\gamma_6 b\gamma_3 a\gamma_4 c\gamma_7 a$

$(a\gamma_2 c = c\gamma_2 a)\forall \gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5,\gamma_6 \epsilon \Gamma$. Therefore (S, Γ) is gamma right semi-regular.Conversely assume that (S, Γ) is gamma right semi-regular then,

$a\gamma_1 b\gamma_2 c\gamma_3 a = a\gamma_1 b\gamma_2 c\gamma_3 a\gamma_4 b\gamma_5 a\gamma_6 c\gamma_7 a$

$\Rightarrow a\gamma_1 b\gamma_2 a\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_3 b\gamma_4 a\gamma_5 a\gamma_6 c\gamma_7 a$

$(a\gamma_3 c = c\gamma_3 a \,\&\, b\gamma_2 c = c\gamma_2 b \,\&\, a\gamma_4 b = b\gamma_4 a)$

$\Rightarrow a\gamma_1 a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_4 a\gamma_6 c\gamma_7 a$

$(a\gamma_2 b = b\gamma_2 a \,\&\, b\gamma_3 b = b \,\&\, a\gamma_5 a = a)$

$\Rightarrow a\gamma_2 b\gamma_3 = a\gamma_1 c\gamma_2 b\gamma_4 a\gamma_6 a\gamma_7 c$

$(a\gamma_7 c = c\gamma_7 a)$

$\Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 b\gamma_4 a\gamma_7 c(a\gamma_6 a = a)$

$\Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_1 c\gamma_2 a\gamma_4 b\gamma_7 c$

$(a\gamma_4 b = b\gamma_4 a) \Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_1 a\gamma_2 c\gamma_4 b\gamma_7 c$

$(a\gamma_2 c = c\gamma_2 a) \Rightarrow a\gamma_2 b\gamma_3 c = a\gamma_2 c\gamma_4 b\gamma_7 c$

$(a\gamma_1 a = a)$

$\forall \gamma_1,,\gamma_2,\gamma_3,\gamma_4,\gamma_5,\gamma_6,\gamma_7 \epsilon \Gamma$. Therefore (S, Γ) is gamma left quasi-normal.

References

[1]. A. P. Biryukov, *Some algorithmic problems for finitely defined commutative semigroups*, Siberian Math.J.8(1967), 384-391.

[2]. A. H. Clifford and G. B. Preston, *The algrbraic theory of semigroup* Math.surveys7;vol.I Amer.math.soc 1961.

[3].D. Radha and V. Selvi, *Pseudo Stable Gamma Near Rings*, Proceedings on National Conference on Recent Trends in Pure and Applied Mathematics, Septemberr 2017, ISBN: 978-81-935198-1-3.

[4]. D. Radha and C. Raja Lakshmi, *A study on semicentral seminear – rings*, Proceedings; National Seminar on New Dimensions in Mathematics and its Applications, October 17, 2018, ISBN No: (yet to receive).

[5]. D.Radha and P. Meenakshi, *Some Structures of Idempotent Commutative Semigroup*, International Journal of Science, Engineering and Management (IJSEM) Vol 2, Issue 12, December 2017, ISSN(Online)2456-1304.

[6]. D. Radha and C. Raja Lakshmi, *On Innvations in Mathematics* (NCIM-2018), Feburary 2018, ISBN: 978-81-935198-5-1, Page(10-18).

[7]. D. Radha and M.Kavitha, *Pseudo Symmetric Temary – Semiring*, Proceedings on National Conference on Innovations in Mathematics (NCIM -2018), Feburary 2018, ISBN: 978-81-935198-5-1, Page(19-24).

[8]. D. Radha and C. Raja Lakshmi, *On Zero – Symmetric – Near Rings*, International Journal of Science, Engineering and Management (IJSEM) Vol 3, Issue 4, April 2018, ISSN(Online) 2456-1304.

[9]. D. Radha and S. Suguna, *Normally in Idempotent Commutative – Semigroup*, International Journal of Science, Engineering and Management(IJSSM) Vol 3, Issue 4, April 2018, ISSN (Online) 2456-1304.

[10]. D. Radha and M. Parvathi Banu, *Left Singularity and Left Regularity in Near Idempotent – Semigroup*, International Journal of Science, Engineering and Management (IJSEM) Vol 3, Issue 4, April 2018, ISSN (Online) 2456-1304.

[11]. Dutta.T.K, Chatterje,T.K, *Green's equivalences on Γ-semigroup* Ball col.soc.80(1987),30-35.

[12]. Kwon.Y.I. and Lee.S.K., *Some special elements in ordered Γ-semigroups*,

kyungpookmathematicalJournal,35(1996), 679-685.

[13]. Naoki. Kimura *Note on idempotent semigroups III*, Proc. Japan Acad.,34.113(1958)

[14]. Naoki. Kimura *Note on idempotent semigroups IV identities of three variables*, Proc. Japan Acad.,34,121-123(1958)

[15]. Sen.M.K., Saha.N.K:, *On Γ-semigroup I*, Bull.col.math.Soc.80,Saha.N.K.,(1988)pp.180.

[16].Sen.M.K. and Saha.N.K., *On Γ-semigroupsI*, Bull.Calcutta Maths.soc. 78(1986),No.3,180-186.