Interval Valued Fuzzy weak hyper bi-Γ-ideals in Γ – hypernear – rings

[1]A.Kalaiarasi, [2]N.Meenakumari, [3]M.Navaneetha Krishnan

[1,3] PG & Research Department of Mathematics, Kamaraj College(affiliated to Manonmaniam Sundaranar University,Abishekapatti, Tirunelveli - 627012),Thoothukudi,India.

[2]PG & Research Department of Mathematics, A.P.C.Mahalaxmi College for Women (affiliated to Manonmaniam Sundaranar University,Abishekapatti, Tirunelveli-627012),Thoothukudi,India.

[1]ranikalai1976@gmail.com, [2]meenakumari.n123@gmail.com

Abstract:

In this paper we introduce interval- valued (i-v) fuzzy weak hyper bi-Γ-ideals in Γ-hyper near- rings and obtain some properties. An i-v fuzzy set $\overline{\eta}$ of M is called an i-v fuzzy weak hyper bi-Γ-ideal of M. if

(i) $\inf_{z\in x-y} \overline{\eta}(z) \geq \min^{i}\{\overline{\eta}(x),\overline{\eta}(y)\}$ for all $x, y \in M$.

(ii) $\overline{\eta}(x\alpha y\beta z) \geq \min^{i}\{\overline{\eta}(x),\overline{\eta}(y),\overline{\eta}(z)\}$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Keywords:

Γ- hypernear-ring, i-v fuzzy ideal, i-v weak fuzzy ideal, fuzzy weak hyper bi-Γ-ideals

1.Introduction

Zadeh[7] introduced the concept of interval valued fuzzy subsets, where the values of the membership functions are intervals of numbers, instead of the numbers. Young Ban Jun[6] defined interval valued fuzzy R- subgroups of near-rings. N.Meenakumari and T.Tamizh Chelvam [5] studied the fuzzy bi- ideals in Γ-near- rings .The concept of hyper near-rings was first introduced by Dasic in [2]. Bijan Davvaz[1]defined the fuzzy hyper ideals of Γ-hyper near- rings. In this paper we introduce interval valued fuzzy hyper weak bi-Γ-ideals of Γ-*hypernear- rings.*

2.Preliminaries

Definition 2.1 [Bh.Satyanayana]

A Γ-near- ring is a triple $(M,+,\Gamma)$ where

(i) $(M,+)$ is a (not necessarily abelian) group.

(ii) Γ is a non empty set of binary operators on M such that, for each $\alpha \in \Gamma, (M,+,\alpha)$ is a right near–ring.

(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.2

A Γ-near- ring M is said to be zero-symmetric, if 0 γ m = 0 for all $m \in M$ and for all $\gamma \in \Gamma$.

Definition 2.3

A canonical hyper group is an algebraic structure $(H,+)$ satisfying the following conditions:

(i) for every $x, y, z \in H, x+(y+z) = (x+y)+z$

(ii) there exists a $0 \in H$ such that $0+x = x+0 = x$ for all $x \in H$

(iii) for every $x \in H$ there exists a unique element $x' \in H$. Such that $0 \in (x+x') \cap (x'+x)$, (we call the element x' the opposite of x).

(iv) $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$

Definition 2.4

A hyper near-ring is an algebraic structure $(R, +, \bullet)$ satisfying the following axioms:

(i) $(R, +)$ is a canonical hyper group

(ii) With respect to the multiplication, $(R, \bullet)$ is a semi group

(iii) $x \bullet (y + z) = x \bullet y + x \bullet z$ for all $x, y, z \in R$

Definition 2.5

A Γ- hyper near – ring is a triple $(M, +, \Gamma)$ where

(i) Γ is a non-empty set of binary operators such that $(M, +, \alpha)$ is a hyper near-ring

(ii) for each $\alpha \in \Gamma$, $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

Example 2.6

Let M={0,a,b}and Γ be the non-empty set of binary operators. $\alpha, \beta \in \Gamma$ are defined as follows

+	0	a	b
0	{0}	{a}	{b}
a	{a}	{0, a, b}	{a,b}
b	{b}	{a, b}	{0, a, b}

Then $(M, +, \Gamma)$ is a Γ- hypernear- ring

Throughout this section M to denote a Γ-hyper near-ring unless otherwise specified.

Definition 2.7

A subset A of M is called a left (resp. right) hyper ideal of M if it satisfies

(i) $(A, +)$ is a normal subgroup of $(M, +)$

(ii) $u\alpha x \in A$ (resp., $(u + x)\alpha v - u\alpha v \in A$ for all $x \in A, \alpha \in \Gamma$ and $u, v \in M$

A subset A of M is called a hyper-ideal of M if it is both a left hyper ideal and right hyper ideal.

Definition 2.8

A fuzzy set μ of M is called a fuzzy hyper bi-Γ ideal of M if

(i) $\inf_{z \in x - y} \mu(z) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in$M

Definition 2.9

By Interval number on[0,1] say $\bar{a}$ we mean an interval such that $0 \leq a^- \leq a^+ \leq 1$ and where a^- and a^+ are the lower and upper limits of $\bar{a}$ respectively.

Definition 2.10 :

Let $\bar{\eta}$ be an i.v fuzzy subset of X and $[t_1, t_2] \in D[0,1]$. Then the set $\bar{U}(\bar{\eta} : [t_1, t_2]) = \{x \in X \mid \bar{\eta}(x) \geq [t_1, t_2]\}$ is called the upper level subset of $\bar{\eta}$.

Definition 2.11

An interval numbers $\bar{a} = [a^-, a^+]$ and $\bar{b} = [b^-, b^+]$ on [0, 1], we define

(i) $\bar{a} \leq \bar{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$

(ii) $\bar{a} = \bar{b}$ $\bar{a} = \bar{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$

(iii) $\bar{a} + \bar{b} = [a^- + b^-, a^+ + b^+]$ whenever $a^- + b^- \leq 1$ and $a^+ + b^+ \leq 1$

Definition 2.12

α	0	a	b
0	0	0	0
a	0	a	b
b	0	a	b

β	0	a	b
0	0	0	0
a	0	0	0
b	0	0	0

Let X be any set, A mapping $\bar{A} : X \to D[0,1]$ is called an interval-valued fuzzy subset, [briefly $i - v$ fuzzysub set] of Xwhenever $D[0, 1]$ denotes the family of all closed subintervals of [0,1] and

$\bar{A}(x) = [A^-(x), A^+(x)]$ where $A^-(x)$ and $A^+(x)$are fuzzy subsets of X.

Definition 2.13

A mapping Min^i from $D[0,1] \times D[0,1] \to D[0,1]$ given by

$\min^i(\bar{a},\bar{b}) = [\min(a^-,b^-), \min(a^+,b^+)] \forall \bar{a},\bar{b} \in D[0,1]$ is called an interval Min-norm.

Definition 2.14

Let $\bar{\mu}$, $\bar{\mu}_i$ be an interval valued fuzzy subsets of a set X. The following are defined by

i) $\bigcup_{i\in\Omega} \bar{\mu}(x) = \sup^i\{\bar{\mu}_i(x) / i \in \Omega\}$

ii) $\bigcap_{i\in\Omega} \bar{\mu}(x) = \inf^i\{\bar{\mu}_i(x) / i \in \Omega\}$

Definition 2.15

An i-v fuzzy subset $\bar{\eta}$ of M is called an i-v fuzzy hyper bi-Γ-ideal of M if

i) $\inf_{z\in x-y} \bar{\eta}(z) \geq \min^i\{\bar{\eta}(x), \bar{\eta}(y)\}$ for all $x, y \in M$.

ii) $\bar{\eta}(x\alpha y\beta z) \geq \min^i\{\bar{\eta}(x), \bar{\eta}(z)\}$ for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.

3. Interval valued fuzzy weak hyper bi-Γ-ideal of Γ-hyper near-rings

In this section, we introduce the notion of i-v fuzzy hyper weak bi-Γ-ideal of M and discuss some of the properties.

Definition 3.1

If $\bar{\eta}$ and $\bar{\lambda}$ are i-v fuzzy subset of M. Then $\bar{\eta} \cap \bar{\lambda}, \bar{\eta} \cup \bar{\lambda}$ and $\bar{\eta} * \bar{\lambda}$ are fuzzy subset of M defined by

$$(\bar{\eta} \cap \bar{\lambda})(x) = \min^i\{\bar{\eta}(x), \bar{\lambda}(x)\}$$

$$(\bar{\eta} \cup \bar{\lambda})(x) = \max^i\{\bar{\eta}(x), \bar{\lambda}(x)\}$$

$$(\bar{\eta} + \bar{\lambda})(x) = \begin{cases} \sup^i_{z\in x-y}\{\min^i\{\bar{\eta}(x), \bar{\lambda}(y)\}\} \\ 0, Otherwise \end{cases}$$

$$(\bar{\eta} * \bar{\lambda})(x) = \begin{cases} \sup^i_{x=y\alpha z}\{\min^i\{\bar{\eta}(y), \bar{\lambda}(z)\}\} \\ 0, Otherwise \end{cases}$$

Definition 3.2

An i-v fuzzy set $\bar{\eta}$ of M is called an i-v -fuzzy weak hyper bi-Γ-ideal of M. if

(i) $\inf_{z\in x-y} \bar{\eta}(z) \geq \min^i\{\bar{\eta}(x), \bar{\eta}(y)\}$ for all $x, y \in M$.

(ii) $\bar{\eta}(x\alpha y\beta z) \geq \min^i\{\bar{\eta}(x), \bar{\eta}(y), \bar{\eta}(z)\}$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Example : 3.3

Consider the Γ- hyper near-ring $(M,+,\Gamma)$

Let M=(0,a,b) and Γ be the non-empty set of binary operators such that $\alpha, \beta \in \Gamma$

Then $(M,+)$ is a Γ-hyper near ring. We define a fuzzy set by $\mu(a) = [0.3, 0.4], \mu(b) = [0.3, 0.4]$, $\mu(0) = [0.3, 0.5]$

By routin calculation, we can verify that μ is i-v fuzzy weak hyper bi-Γ-ideals of M.

Theorem : 3.4

Let $\bar{\eta}$ be an i-v fuzzy subset of M. Then $\bar{\eta}$ is an i-v fuzzy weak hyper bi-Γ-ideal of M if and only if $\bar{\eta} * \bar{\eta} * \bar{\eta} \subseteq \bar{\eta}$.

Proof :

Assume that $\bar{\eta}$ is an i-v fuzzy hyper weak bi-Γ-ideals of M.

Let $x, y, z, y_1, y_2 \in M$ and $\alpha, \beta \in \Gamma$. Such that $x = y\alpha z$ and $y = y_1\beta y_2$. Then

$$(\bar{\eta} * \bar{\eta} * \bar{\eta})(x) = \sup^i_{x=y\alpha z}\{\min^i\{\bar{\eta}(y), \bar{\eta}(z)\}\}$$

$$= \sup^i_{x=y\alpha z}\{\min^i\{\sup^i_{y=y_1\beta y_2} \min^i\{(\bar{\eta})(y_1), \bar{\eta}(y_2), \bar{\eta}(z)\}\}$$

$$= \sup^i_{x=y\alpha z} \sup^i_{y=y_1\beta y_2} \sup^i \min^i\{\min^i\{(\bar{\eta})(y_1), \bar{\eta}(y_2), \bar{\eta}(z)\}\}$$

$$= \sup^i_{x=y_1\beta y_2\alpha z} \bar{\eta}(y_1\beta y_2\alpha z)$$

Since $\bar{\eta}$ is an i-v fuzzy weak bi-Γ-ideal of M

$$\bar{\eta}(y_1\beta y_2\alpha z) \geq Min^i\{\bar{\eta}(y_1), \bar{\eta}(y_2), \bar{\eta}(z)\}$$

$$\subseteq \bar{\eta}(y_1\beta y_2\alpha z)$$

If x cannot be expressed as $x = y\alpha z$, then

$(\bar{\eta} * \bar{\eta} * \bar{\eta})(z) = \bar{0} \subseteq \bar{\eta}(x)$ in both cases $(\bar{\eta} * \bar{\eta} * \bar{\eta})(z) \subseteq \bar{\eta}$

Conversely, assume that $(\bar{\eta}*\bar{\eta}*\bar{\eta})(z)\subseteq\bar{\eta}$, For $x,y,z\in z$ and $\alpha,\beta,\alpha_1,\beta_1\in\Gamma$

Let x' be such that $x'=x\alpha y\beta z$

Then $\bar{\eta}(x\alpha y\beta z)=\bar{\eta}(x')\supseteq(\bar{\eta}*\bar{\eta}*\bar{\eta})(x')$

$$=\sup^i_{x'=p\alpha q}\{\min^i\{(\bar{\eta}*\bar{\eta})(p),\bar{\eta}(q)\}$$

$$=\sup^i_{x'=p\alpha_1 q}\left\{\min^i\left\{\sup^i_{p=p_1\beta_1 p_2}\min^i\{\bar{\eta}(P_1),\bar{\eta}(p_2)\},\bar{\eta}(q)\}\right\}\right\}$$

$$=\sup^i_{x'=p_1\beta_1 p_2\alpha_1 q}\{\min^i\{\bar{\eta}(P_1),\bar{\eta}(p_2)\},\bar{\eta}(q)\}$$

$$\geq\min^i(\bar{\eta}(x),\bar{\eta}(y),\bar{\eta}(z)\}$$

Hence $\bar{\eta}(x\alpha y\beta z)\geq\min^i(\bar{\eta}(x),\bar{\eta}(y)\bar{\eta}(z)\}$ $\bar{\eta}$ is an i-v fuzzy weak hyper bi-Γ-ideal of M.

Theorem : 3.5

Let $\bar{\eta}$ and $\bar{\lambda}$ be an i-v fuzzy weak hyper bi-Γ-ideal of M. Then the products $\bar{\eta}*\bar{\lambda}$ and $\bar{\lambda}*\bar{\eta}$ are also an i-v fuzzy weak hyper bi-Γ-ideals of M.

Proof :

Let $\bar{\eta}$ and $\bar{\lambda}$ be an i-v fuzzy weak hyper bi-Γ-ideals of M.

$$\inf_{z\in x-y}(\bar{\eta}*\bar{\lambda})(z)=\inf_{z\in x-y}\left[\sup^i_{z=u\alpha v}\{\min^i\{\bar{\eta}(u),\bar{\lambda}(v)\}\right]$$

$$=\sup^i_{z=u\alpha v}\min^i\left\{\inf_{u\in x-y}\bar{\eta}(u),\inf_{v\in x-y}\bar{\lambda}(v)\}\right\}$$

$$\geq\sup^i_{z=u\alpha v}\min^i\left\{\min^i\{\bar{\eta}(x),\bar{\eta}(y)\},\min^i\{\bar{\lambda}(x),\bar{\lambda}(y)\}\right\}$$

$$=\min^i[\sup^i_{z\in x-y}\min^i\{\bar{\eta}(x),\bar{\eta}(y)\},\sup^i_{z\in x-y}\min^i\{\bar{\lambda}(x),\bar{\lambda}(y)\}]$$

$$=\min^i\{\bar{\eta}*\bar{\lambda}(z),\bar{\eta}*\bar{\lambda}(z)\}$$

$$\inf_{z\in x-y}(\bar{\eta}*\bar{\lambda})(z)\geq\min^i\{\bar{\eta}*\bar{\lambda}(z),\bar{\eta}*\bar{\lambda}(z)\}$$

It follows that $(\bar{\eta}*\bar{\lambda})$ is an i-v fuzzy subgroup of M. Further,

By theorem 3.3, It is enough if we prove $(\bar{\eta}*\bar{\lambda})*(\bar{\eta}*\bar{\lambda})*(\bar{\eta}*\bar{\lambda})\subseteq\bar{\eta}*\bar{\lambda}$

Now $(\bar{\eta}*\bar{\lambda})*(\bar{\eta}*\bar{\lambda})*(\bar{\eta}*\bar{\lambda})\subseteq\bar{\eta}*\bar{\lambda}$

$\subseteq\bar{\eta}*(\bar{\lambda}*\bar{\lambda}*\bar{\lambda})$ Since $\bar{\lambda}$ is an i-v fuzzy weak hyper bi-Γ-ideal of M.

$\subseteq\bar{\eta}*\bar{\lambda}$

$\therefore\bar{\eta}*\bar{\lambda}$ is an i-v fuzzy weak hyper bi-Γ-ideal of M. Similarly $\bar{\lambda}*\bar{\eta}$ is an i-v fuzzy hyper weak bi-Γ-ideal of M.

Theorem : 3.6

Let $\{\bar{\eta}_j / i\in\wedge\}$ be family of i-v fuzzy weak hyper bi-Γ-ideals of M. Then $\bigcap_{i\in\wedge}\bar{\eta}_i$ is also an i-v fuzzy weak hyper bi-Γ-ideal of M. Where $\wedge$ is any index set.

Proof : Straightforward

Theorem 3.7

Let $\bar{\eta}$ be an i-v fuzzy subset of M. then $\bar{\eta}$ is an i-v fuzzy weak hyper bi-Γ- ideal of M iff $\overline{U}(\bar{\eta}:[t_1,t_2])$ is a weak hyper bi-Γ- ideal of M for all $[t_1,t_2]\in D[0,1]$

Proof: Straightforward

Theorem : 3.8

Let $\bar{\eta}=[\eta^-,\eta^+]$ be an i-v fuzzy subset of M.
Then $\bar{\eta}$ is an i-v fuzzy weak hyper bi-Γ-ideal of M if and only if η^-,η^+ are fuzzy weak hyper bi-Γ-ideal of M.

Proof :

Assume that $\bar{\eta}$ is an i-v fuzzy weak hyper bi-Γ-ideal of Γ-hyper near-ring M for any $x,y,z\in M$ and $\alpha,\beta\in\Gamma$

Now

$$\left[\inf_{z\in x-y}\eta^-(z),\inf_{z\in x-y}\eta^+(z)\right]=\inf_{z\in x-y}\bar{\eta}(z)$$

$$\geq \min{}^i\{\bar{\eta}(x),\bar{\eta}(y)\}$$

$$=[\min\{\eta^-(x),\eta^-(y)\},\min\{\eta^+(x),\eta^+(y)\}]$$

It follows that

$$\left[\inf_{z\in x-y}\eta^-(z)\geq\min\{\eta^-(x),\eta^-(y)\}\right]\&$$

$$\inf_{z\in x-y}\eta^+(z)\geq\min\{\eta^+(x),\eta^+(y)\}$$

$$[\eta^-(x\alpha y\beta z),\eta^+(x\alpha y\beta z)]=\bar{\eta}(x\alpha y\beta z)$$

$$\geq \min{}^i\{\bar{\eta}(x),\bar{\eta}(y),\bar{\eta}(z)\}$$

$$=\min\{[\eta^-(x),\eta^+(x)],[\eta^-(y),\eta^+(y)],[\eta^-(z),\eta^+(z)]\}$$

$$=\min\{[\eta^-(x),\eta^-(y),\eta^-(z)],\min[\eta^+(x),\eta^+(y),\eta^+(z)]\}$$

It follows that

$$\eta^-(x\alpha y\beta z)\geq\min\{\eta^-(x),\eta^-(y),\eta^-(z)\}\&$$

$$\eta^+(x\alpha y\beta z)\geq\min\{\eta^+(x),\eta^+(y),\eta^+(z)\}$$

Therefore η^-,η^+ are fuzzy hyper weak bi-Γ-ideal of M.Conversely, Assume that η^-,η^+ are fuzzy hyper weak bi-Γ-ideal of Γ-hyper near-ring M Let $x,y,z\in M$ and $\alpha,\beta\in\Gamma$ Then

$$\inf_{z\in x-y}\eta^-(z)\geq\min\{\eta^-(x),\eta^-(y)\}$$

$$\inf_{z\in x-y}\eta^-(z)\geq\min\{\eta^-(x),\eta^-(y)\}$$

$$\inf_{z\in x-y}\bar{\eta}(z)=\left[\inf_{z\in x-y}\eta^-(z),\inf_{z\in x-y}\eta^+(z)\right]$$

$$\geq[\min\{\eta^-(x),\eta^-(y)\},\min\{\eta^-(x),\eta^-(y)\}]\text{And}$$

$$=\min{}^i\{\bar{\eta}(x),\bar{\eta}(y)\}$$

$$\bar{\eta}(x\alpha y\beta z)=[\eta^-(x\alpha y\beta z),\eta^+(x\alpha y\beta z)]$$

$$[\eta^-(x\alpha y\beta z),\eta^+(x\alpha y\beta z)]=\bar{\eta}(x\alpha y\beta z)$$

$$\geq\min\{[\eta^-(x),\eta^-(y),\eta^-(z)],\min[\eta^+(x),\eta^+(y),\eta^+(z)]\}$$

$$=\min\{[\eta^-(x),\eta^+(x)],[\eta^-(y),\eta^+(y)],[\eta^-(z),\eta^+(z)]\}$$

$$=\min{}^i\{\bar{\eta}(x),\bar{\eta}(y),\bar{\eta}(z)\}$$

Therefore $\bar{\eta}$ is an i-v fuzzy weak hyper bi-Γ-ideal of Γ-hyper near-ring M

References

1. BijanDavvaz, Jianming Zhan, Kyug Ho KIM, fuzzy Γ-hypernear-rings, computers and mathematics with Applications 59 (2010) 2846-2853.
2. V.Chinnadurai et al.,Interval valued fuzzy weak bi-ideals of г-near-rings.J.Linear and Topological Algebra vol.06.No.03.2017,223-236.
3. V. Dasic,Hypernearrings, Fourth int. Congress on AHA (1990). World Scientific (99)
4. Y. B. Jun, K.H. Kim and M.A Oztwrk, On fuzzy ideals of Gamma near-rings. J. fuzzy math., 9 (2001), 51-58
5. K.H.Kim, B.Dawaz. EH.Roh. On hyper R-subgroups of gamma-hypernear-rings Sci. Journal of Algebra and Discrete Structure, Vol 9 (2011) No. 1 & 2. PP 43-52.
6. N.Meenakumari,T.Tamizh chelvam,Fuzzy bi-ideals in gamma –near-rings,Journal of algebra and Discrete structure,vol 9(2011) No1&2.PP43-52.Fuzzy bi-ideals in Γ-near-rings.
7. Young Bae Jun, Interval-valued fuzzy R-subgroups of near-rings, Indian J. Pure Math., 33(1) 71-80 January 2002.
8. L.A.Zadeh, The concept of linguistic variables and its application to approximate reasoning, inform Sci 8(1975) 199-249.