CHARACTERIZATION OF WEAK BI-IDEALS IN NEAR SUBTRACTION SEMIGROUPS

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Abstract: In this paper with a new idea, we define weak bi-ideal and investigate some of its properties. We characterize weak bi-ideal by bi-ideals of near subtraction semigroups.

Introduction:

B.M. Schein , cosidered systems of the form (X; o; -), where X is a set of functions closed under the composition "o" of function (and hence(X; o) is a function semigroup) and the set theoretic subtraction"-"(and hence (X; -) is a subtraction algebra in the sense of). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible function. B.Zelinka discussed a problem proposed by B.M.Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. For basic definition one may refer to Pilz. This concept motivates the study of different kinds of new ideals in algebraic theory especially ideals in subtraction bialgebra and Fuzzy algebra.

Preliminaries

Definition: 2.1

A nonempty set X together with binary operations "-" and is said to be subtraction algebra if it satisfies the following:

(1) x - (y-x) = x.

(2) x - (x-y) = y-(y-x).

(3) (x - y) – z = (x-z) – y, for every x, y, z ϵ X.

Definition: 2.2

A nonempty set X together with two binary operations **"-"and"."** is said to be a subtraction semigroup if it satisfies the following:

(1) (X,-) is a subtraction algebra.

(2) (X, .) is a semigroup.

(3) x(y-z) = xy - xz and (x-y)z = xz – yz , for every x,y,z **ϵ** X.

Definition:2.3

A nonempty set X together with two binary operations **"-"and"."** is said to be a near subtraction semigroup (right) if it satisfies the following:

(1) **(X,-)** is a subtraction algebra.

(2) **(X,.)** is a semigroup.

(3) (x-y)z = xz-yz, for every x, y, z ϵX.

Definition: 2.4

A nonempty subset A of X is called

1. a left X-subalgebra of X if A is a subalgebra of (X,-) and XA ⊆ A.
2. a right X-subalgebra of X if A is a subalgebra of (X,-) and AX ⊆ A.

Result: 1

A near subtraction semigroup X has no non-zero nilpotent elements if and only if x^2 =0 which implies x=0, for all x in X.

Definition: 2.5

For A⊆X, we define the radical √A of A to be {a ϵ X / a^k ϵ A. for some positive integer k}. Obviously A⊆√A.

Definition: 2.6

A near subtraction semigroup X is said to be stable if for all a in X. aX=aXa=Xa.

Definition: 2.7

A near subtraction semigroup X is regular if for every a in X there is some y in in X such that a=aya.

Definition:2.8 A map f: X→X is called a mate function if for all a in X, a=af(a)a. This f(a) is called a mate of a.

Definition: 2.9

A near subtraction semigroup X is said to have the K(m,n) if a^mX = Xa^n, for all a in X , where m and n are positive integers.

Lemma: 2.10

If a near subtraction semigroup X has the condition, eX= eXe =Xe, for all e ϵ E, then E⊆C(X).

Definition: 2.11

A near subtraction semigroup X is called a P_k near subtraction semigroup ($P_k{}'$ near subtraction semigroup) if there exists a positive k such that a^kX= aXa (Xa^k=aXa), for all a in X.

Definition: 2.12

If a ϵ Xa(aϵaX) for all a ϵX, then X is called an s(s′)-near subtraction semigroup.

Definition: 2.13

A near subtraction semigroup X is called left bi-potent if Xa=Xa^2, for all a ϵ X.

Definition: 2.14

A near subtraction semigroup X is called a generalized near-field (GNF) if for each a ϵ X there exists a unique bϵX such that a=aba and b=bab.

Definition: 2.15

A near subtraction semigroup X is called strongly regular if for each a ϵ X there exists b ∈ X such that a=ba^2.

Definition: 2.16

A near subtraction semigroup X is said to be Left permutable if abc=bac, for all a,b,c ϵ X

Lemma:2.17

Let X be a zero-symmetric near subtraction semigroup , if L={0}, then en=ene,for 0≠ eϵE and nϵX.

Lemma: 2.18

If X is a zero-symmetric near subtraction semigroup with E⊆X_d and L={0},then ne=ene, for all eϵE and nϵX..

Result:2

Let X be a left self-distributive s-near subtraction semigroup. Then $B^3 = B,$ for every weak bi-ideal B of X iff X is strongly regular.

Definition: 2.19

A sub algebra B of $(X,-)$ is said to be bi-ideal of X if BXB ∩ (BX)*B⊆B.

Definition: 2.20

A subalgebra B of $(X,-)$ is said to be a generalized (m,n) bi-ideal of X if $B^m X B^n$ ⊆B, where m and n are positive integers.

Definition:2.21

A near subtraction semigroup X is said to be Left self-distributive near subtraction semi group if abc=abac, for all a,b,c ∈ X.

Definition: 2.21

A near subtraction semigroup X is said to be sub commutative if Xa=aX, for all a in X. **Theorem: 1**

The following conditions are equivalent:

i. X is strict weakly regular.
ii. For every a ∈X, a ∈ $(Xa)^2$.
iii. For any two left X-subalgebras S_1&S_2 such that $S_1 \subseteq S_2$, we have S_2S_1=S_1.

Theorem:2
Let X be a near subtraction semigroup with a mate function. Then the following statement are equivalent:

i. X is $P_k{}'$ for any positive integer k.
ii. E ⊆ C(X).
iii. X is a K(r, m), for all positive integers r,m.
i. X is K$(1,2)$
ii. E⊆C(X).
iii. X is K$(2,1)$

Theorem:3

Let X be a zero-symmetric near subtraction semi group admitting mate functions. Then X is a $P_k{}'$ near subtraction semigroup, with E⊆ X_d iff X is a $P_k{}'$ near subtraction semigroup.

Corollary:3.1 The following conditions are equivalent in a near subtraction semi group X with mate functions.

i. X is $P_k{}'$ for any positive integer k.
ii. E ⊆ C(X).
iii. X is a K(m, n), for all positive integers m,n
iv. X is a zero symmetric P_k near subtraction semi group with E ⊆X_d.

Theorem:4
Let X admits a mate function, Then X is stable iff E⊆C (X) .

Theorem:5 Let X be a near subtraction semigroup with a mate function m, then the following conditions are equivalant.
i. X is K$(1,2)$
ii. E⊆C(X).
iii. X is K$(2,1)$

Theorem: 6
Let X admit a mate function "f". Then X is a K (r, m) near subtraction semigroup, for all positive inegers r,m iff X is a K$(1,2)$.

3.Weak Bi-ideals

Definition:3.1
A subalgebra B of $(X, -)$ is said to be a weak bi-ideal if $B^3 \subseteq B$.

Example:
Let X={0, a, b, 1} in which "-" and "." are defined by,

-	0	A	b	1
0	0	0	0	0
a	A	0	a	B
b	B	b	0	0
1	1	b	a	0

•	0	a	b	1
0	0	0	0	0
a	0	a	0	A
b	0	0	b	B
1	0	a	b	1

clearly X is a weak bi - ideal .

Proposition: 3.2
Any homomorphic image of weak bi-ideal is also a weak bi-ideal.

Proof:
Let f: X→X′ be a homomorphisim and B be a weak bi-ideal of X. Let B′= f(B). Let b′∈f(B).Then b'^3 =f $(b)^3$ =f $(b^3$)є f $(B^3$)⊆f(B)= B′.(i.e.) $B'^3 \subseteq B$.
Thus every homomorphic image of a weak bi-ideal is also a weak bi-ideal.

Proposition:3.3
Let X be a left self-distributive s-near subtraction semigroup.Then $B = B^3$,for every weak bi-ideal B of X iff $\sqrt{A}$ =A, for a left X-subalgebra A of X.

Proof:
Assume $B = B^3$,for every weak bi-ideal B of X. By the result 2, strongly regular. Let A be a left X-subalgebra of X. If a ∈$\sqrt{A}$,then a^n∈A,for some positive integer n.since X is strongly regular,$a = xa^2 = xaa$. Also from the fact that X is left self

distributive near subtraction semigroup. We get that $a = xa^2 = xaa = x(axa) = x(axaa) = xaxa^2 = \cdots = xaxa^n \in$ XA ⊆ A.(i.e) a ∈A, therefore$\sqrt{A} \subseteq A$.But obviously A⊆$\sqrt{A}$, therefore A= $\sqrt{A}$, for every left X subalgebra A of X. conversely, A=$\sqrt{A},$ for every left X-subalgebra A of X. Let a∈X, then $a^3 \epsilon Xa^2$,and hence $a \in Xa^2$. Thus X is strongly regular.Therefore by the result 2, $B = B^3$, for every weak bi-ideal B of X.

Preposition: 3.4

Let X be a left self-distributive s-near subtraction semigroup. Then the following conditions are equivalent.

i. $B = B^3$,for every weak bi-ideal B of X with $E \subseteq X_d$.
ii. $aXa = Xa = Xa^2$, for every a ϵ X.
iii. X is a $s_k \, and \, P_k'$ near subtraction semigroup for any positive integer k with $E \subseteq X_d$.
iv. X is stable near subtraction semigroup.
v. X is a K(1,2) near subtraction semigroup and s′-near subtraction semigroup.
vi. X is a K(r,m) and regular near subtraction semigroup.
vii. $B = B^m X B^n$, for every generalized (m,n) bi-ideal B of X.

Proof:

(i)→(ii)

By the result2, X is strongly regular. From this L={0}. By the lemma2.17, en =ene, for all e∈E. since E⊆X_d, by the lemma 2.18, ne=ene, for all e∈E. Hence E⊆C(X).from this X is a GNF and so X is sub commutative. Therefore $aXa = Xaa = Xa^2$.By the theorem 1, X is a left bi-potent and so $Xa = Xa^2$.thus $aXa = Xa = Xa^2$, for all a∈X.

(ii)→(iii)

Since X is a s-near subtraction semigroup, $a \in Xa = Xa^2$. Therefore X is a strongly regular near subtraction semigroup. Therefore X contains no non-zero nilpotent element. Thus eX=eXe. By the assumption eXe=Xe. Therefore eX=eXe=Xe and so by the lemma 2.10, E⊆C(X).Therefore by the theorem2, X is a $P_k{}'$ near subtraction semigroup. Since E⊆C(X), $E \subseteq X_d$. Let xϵX. Since X is regular and let self-distributive, $x = xax = xax^2$ =…….= $xax^k \in Xx^k$ and so X is $s_{k'}$ -near subtraction semigroup.

(iii)→(iv)

Let x∈X, Since X is a s_k - $P_{k`}$ near subtraction semigroup,$x \in x^k$X=xXx and so X is regular. By the theorem3, X is a P_k near subtraction semigroup with E⊆X_d and corollary 3.1, E⊆C(X).Therefore by the theorem 4, X is stable.

(iv)→(v)

Let x∈X. Since X is stable and s-near subtraction semigroup, xϵXx=xXx. (i.e) X is regular. so by the theorem 4,E⊆C(X). Again by the theorem 5,we get that X is a K(1,2)near subtraction semigroup. Since X is regular, we get that X is a s′-near subtraction semigroup.

(v)→(vi)

If X is a s′ and a K(1,2) near subtraction semigroup,then $a \in aX = Xa^2$. Therefore X is strongly regular and so X is a regular near subtraction semigroup. By the theorem 6, X is a K(m,n)near subtraction semigroup.

(vi)→(vii)

Let B be a generalized (m,n) bi-ideal of X. If X is a K(m,n) near subtraction semigroup, then by the theorem 6, and by the theorem 5, E⊆C(X). Let xϵB, since X is regular, $x = xyx = xaxyxax =$

$xa(xyx)ax$ = (xyx) $(ax)^n$ = $x^m a^m$ (xyx) $a^n x^n$ $\in$ $a^n x^n$ $\in$ $x^m X x^n$ $\in$ $B^m X B^n$ .Therefore $B = B^m X B^n$,for every generalized (m,n) bi-ideal B of X.

(vii)→(i)

Assume that B is a bi-ideal of X. since every bi-ideal is a generalized (m,n)bi-ideal of X, B= $B^m X B^n$ ⊆BXB. Thus B=BXB,for every bi-ideal B of X, by the theorem 1, B=B^3, for every weak bi-ideal B of X.

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